Physics 2101, Second Exam, Spring 2007

March 8, 2007

Name : KEY

Section: (Circle one)

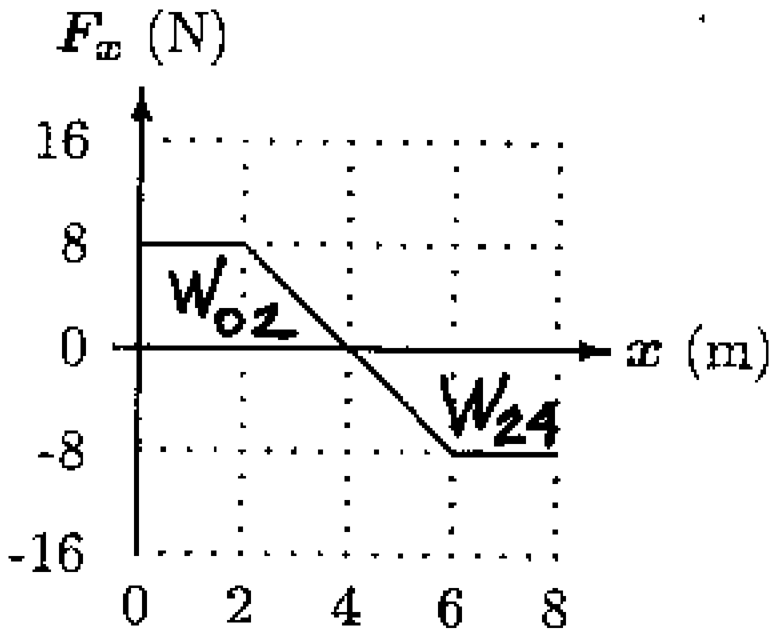
1 (Rupnik, MWF 7:40 am)	2 (Giammanco, MWF 9:40 am)
3 (Rupnik, MWF 11:40 am)	4 (Rupnik, MWF 2:40 pm)
5 (Giammanco, TTh 10:40 am)	6 (González, TTh 1:40 pm)

- Please be sure to write your name and circle your section above.
- For the problems, you *must* show all your work. Let us know what you were thinking when you solved the problem! Lonely right answers will not receive full credit, lonely wrong answers will receive no credit.
- For the questions, no work needs to be shown (there is no partial credit).
- Please carry units through your calculations when needed, lack of units will result in a loss of points.
- You may use scientific or graphing calculators, but you must **derive** your answer and **explain** your work.
- Feel free to detach, use, and keep the formula sheet. No other reference material is allowed during the exam.
- **GOOD LUCK!**

Question 1 - 11 Points

The graph shown below represents the x-component of a force, F_x, acting on a particle that moves along the x axis. The particle starts moving from rest at the position x=0 m.

Circle the correct ending to the statements below.



(a) (4 pts) During first 8 meters of the motion the work done by force F

(i) is positive.
(ii) is negative.
(iii) is zero.
(iv) cannot be determined from the data given.

$W_{02} = -W_{24} \Rightarrow W_{02} + W_{24} = 0$

• Work = area in F-x plot, above +, below −

(b) (4 pts) The kinetic energy of the particle is the largest at

(i) $x = 2$ m.
(ii) $x = 4$ m.
(iii) $x = 6$ m.
(iv) $x = 8$ m.

$W = \Delta K \Rightarrow \Delta K > 0$ as long as $W > 0$
The last position where work is still positive is $x = 4$m; after that the force slows down the object (work is negative)

(c) (3 pts) Between positions $x = 2$ m and $x = 4$ m the particle is

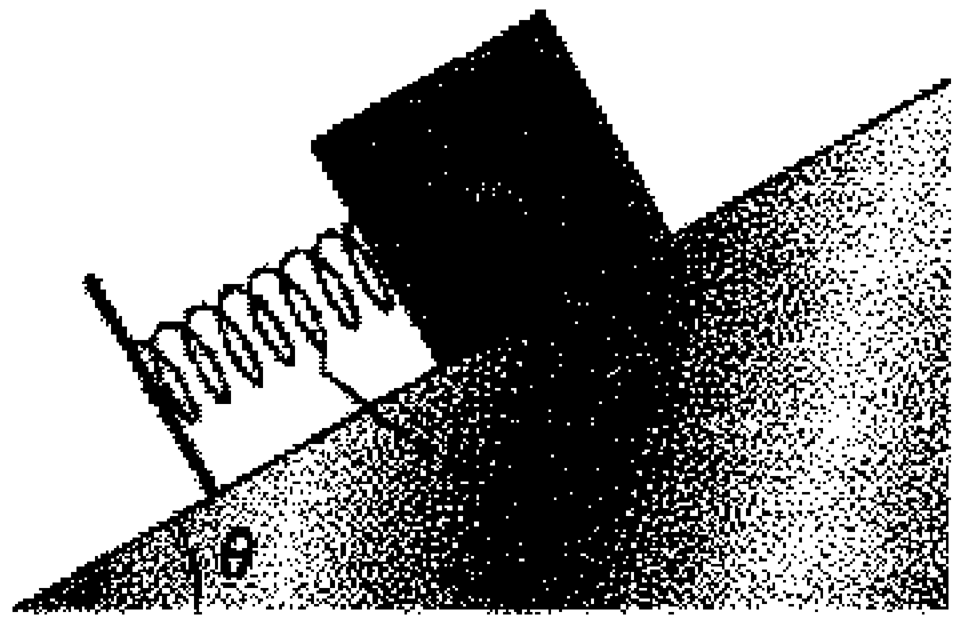
(i) speeding up.
(ii) slowing down,
(iii) moving with constant velocity.

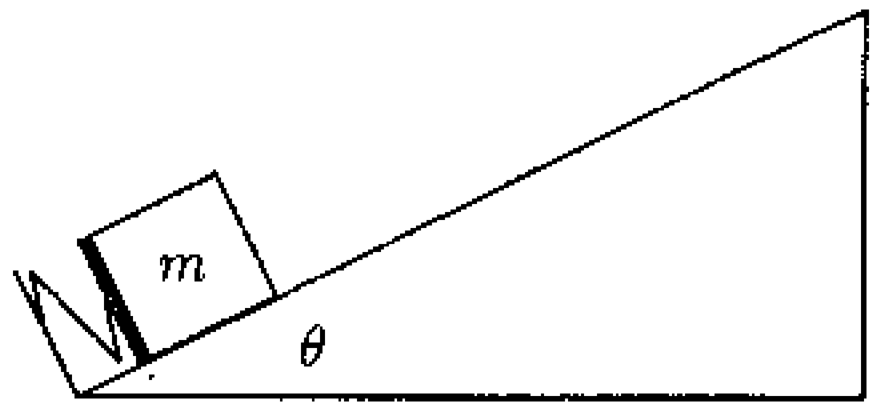
• the particle is speeding up but with smaller and smaller acceleration (force is decreasing)

Question 2 - 11 points

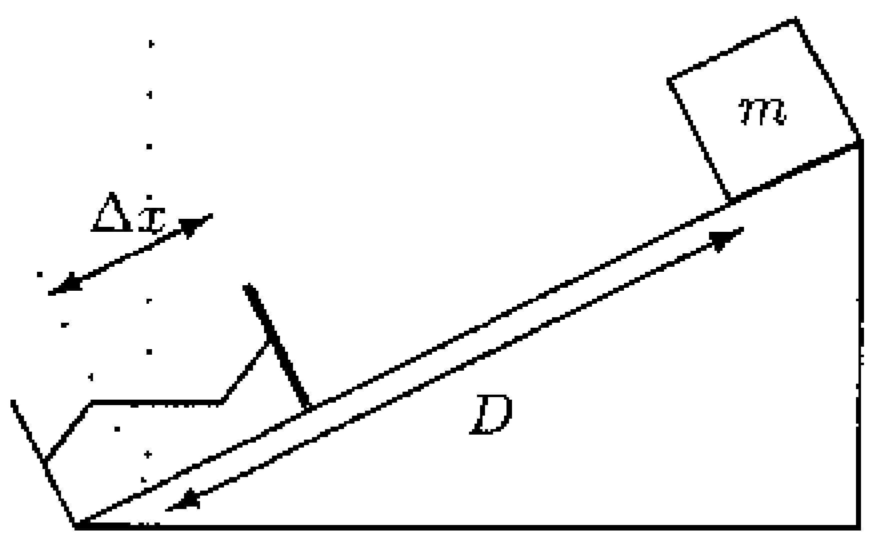
A block with mass m is placed against a spring on an incline with angle θ and a coefficient of kinetic friction μ_k, $\mu_k > 0$. The block is not attached to the spring. The spring, with spring constant k, is compressed by Δx and then released. The block moves a distance $D, D > \Delta x$, along the incline before it momentarily stops.

Circle the correct ending to the statements below, related to the **motion of the block up the incline to the highest point.**





initial state of the system



final state of the system

(a) (4 pts) The change of spring potential energy is

(i) $\Delta U_s > 0$.
(ii) $\Delta U_s < 0$.
(iii) $\Delta U_s = 0$.
(iv) Not enough info to answer.

$\Delta U_s = U_{s,f} - U_{s,i} = 0 - \frac{1}{2}k(\Delta x)^2 < 0$

(b) (3 pts) The change of gravitational potential energy of the block is

(i) $\Delta U_g > 0$.
(ii) $\Delta U_g < 0$.
(iii) $\Delta U_g = 0$.
(iv) Not enough info to answer.

. the block's height increases during motion $\Rightarrow \Delta U_g > 0$

or $\Delta U_g = U_{gf} - U_{gi} = mgD\sin\theta - 0 > 0$

(c) (4 pts) The correct relation, between the magnitudes of the change of spring and gravitational potential energies, is:

(i) $|\Delta U_s| > |\Delta U_g|$.
(ii) $|\Delta U_s| < |\Delta U_g|$.
(iii) $|\Delta U_s| = |\Delta U_g|$.
(iv) Not enough info to answer.

• initial spring potential energy was partially transferred to gravitational pot. energy and partially transferred to thermal energy $\Rightarrow |\Delta U_s| > |\Delta U_g|$,

because: $\Delta U_g + \Delta U_s + \Delta E_{th} = 0$

$\Rightarrow \Delta U_s = -\Delta U_g - \Delta E_{th}$

$\Rightarrow |\Delta U_s| = |\Delta U_g| + |\Delta E_{th}| > |\Delta U_g|$

Question 3 - 12 points

A dog and a boat are initially at rest, as shown in the figure. Then, the dog starts walking along the boat toward a shore. Consider the dog and the boat to form an isolated system, that is, neglect the friction between the boat and the water.

isolated system : $\vec{F}_{net} = 0$
$\Rightarrow \sum \vec{P}_{\text{before interaction}} = \sum \vec{P}_{\text{after interaction}}$

Circle the correct ending to the statements below, related to the dog's walk.

(a) (4 pts) While the dog walks toward the shore the boat

(i) is at rest.

(ii) moves toward the shore.

(iii) moves away from the shore.

(iv) moves in a direction that cannot be determined from the data given.

$0 = \vec{P}_{dog} + \vec{P}_{boat}$
$\Rightarrow \vec{P}_{boat} = -\vec{P}_{dog}$ ... opposite direction !!

(b) (4 pts) While the dog walks toward the shore the center of mass of the dog+boat system

(i) is at rest.

(ii) moves toward the shore.

(iii) moves away from the shore.

(iv) moves in a direction that cannot be determined from the data given.

• the dog+boat system is at rest before interaction $\Rightarrow$
• therefore, it is at rest during and after interaction, within isolated system $v_{com} = 0$, becaus only net extenal force can change v_{com}!

(c) (4 pts) At any moment, the magnitude of the linear momentum of the dog

(i) is equal to the magnitude of the linear momentum of the boat.

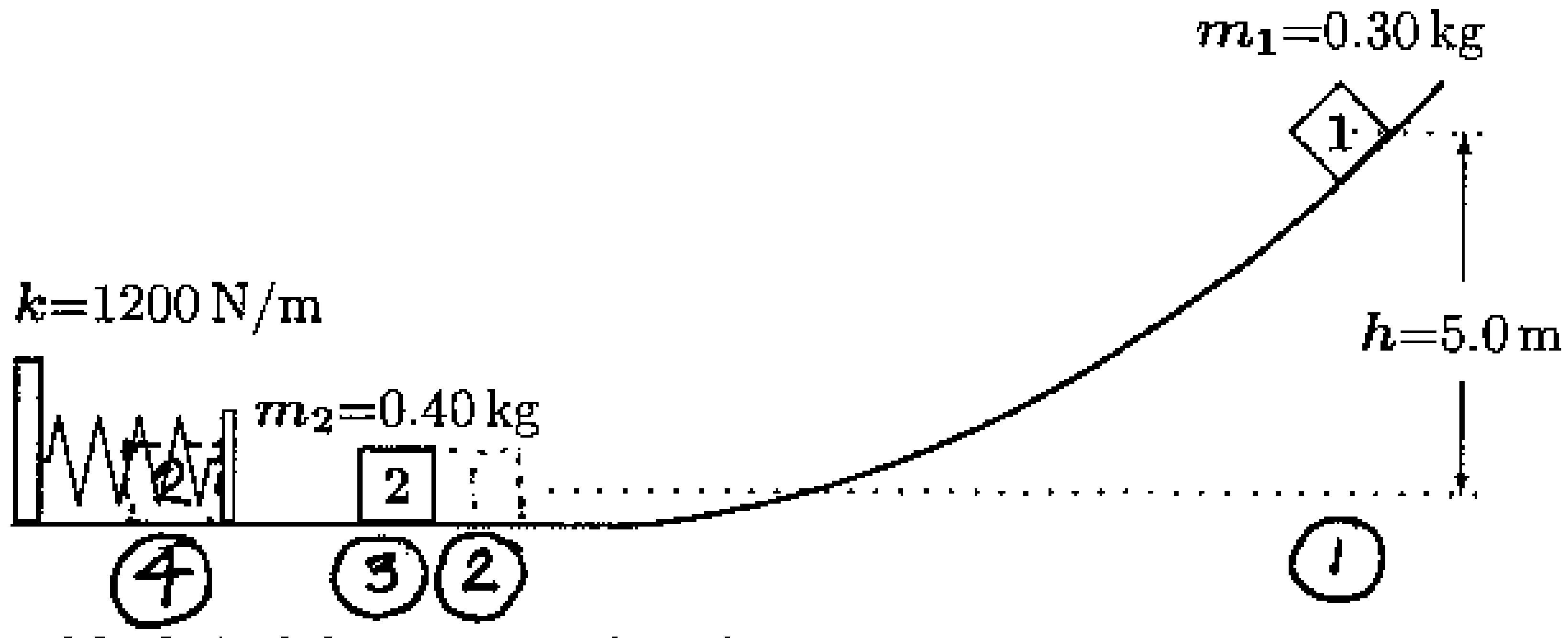
(ii) is larger than the magnitude of the linear momentum of the boat.

(iii) is smaller than the magnitude of the linear momentum of the boat.

(iv) cannot be compared to the magnitude of the linear momentum of the boat.

$\vec{P}_{boat} = -\vec{P}_{dog}$
$\Rightarrow |\vec{P}_{boat}| = |\vec{P}_{dog}|$

Problem 1 - 22 points



A block 1 of mass m_1=0.30 kg slides from rest along a **frictionless** ramp from height h=5.0 m, as shown. At the bottom of the ramp it collides with a stationary block 2, which has mass m_2=0.40 kg. Just after the collision, block 1 moves back toward the ramp with a speed of $v_{1,\text{after}}$. After the collision, block 2 slides into a relaxed spring and stops momentarily after compressing it by $x_{\max}$=0.40 m. The spring constant (force constant) is k=1200 N/m.
Hint: It is not known if the collision is elastic!

(a) (7 pts) Find the speed (magnitude of velocity) of block 1 just before the collision, $v_{1,\text{before}}$.

- motion of 1 from 1 to 2: $m_1 g$ is the only force with nonzero work
$\Rightarrow$ mechanical energy is conserved:

$\Delta K = -\Delta U_g$ ... for block 1, initiall at rest ($K_{11}=0$)

$K_{12} - \cancel{K_{11}}^{\,0} = -(-m_1 g h)$

$\frac{1}{2} m_1 v_{12}^2 = m_1 g h$

v_{12}: 1 = block, 2 = position

$$v_{12} = \sqrt{2gh} = \sqrt{2(9.8\tfrac{m}{s^2})(5.0m)} \simeq 9.9\ m/s$$

(b) (7 pts) Find the speed (magnitude of velocity) of block 2 just after the collision, $v_{2,\text{after}}$.

- motion from 3 to 4 of block 2: F_s is the only force with nonzero work
$\Rightarrow$ mechanical energy is conserved

$\Delta K = -\Delta U_s$ ... for block 2 finally at rest $\Rightarrow K_{24}=0$

$\cancel{K_{24}}^{\,0} - K_{23} = -\left(\frac{1}{2} k x_{max}^2 - 0\right)$

$\frac{1}{2} m_2 v_{23}^2 = \frac{1}{2} k x_{max}^2$

$$v_{23} = x_{max}\sqrt{\frac{k}{m_2}} = (0.40m)\sqrt{\frac{1200\ N/m}{0.40\,kg}} = \underline{\underline{21.9\ m/s}}$$

(c) (8 pts) Find the speed (magnitude of velocity) of block 1 just after the collision, $v_{1,\text{after}}$.

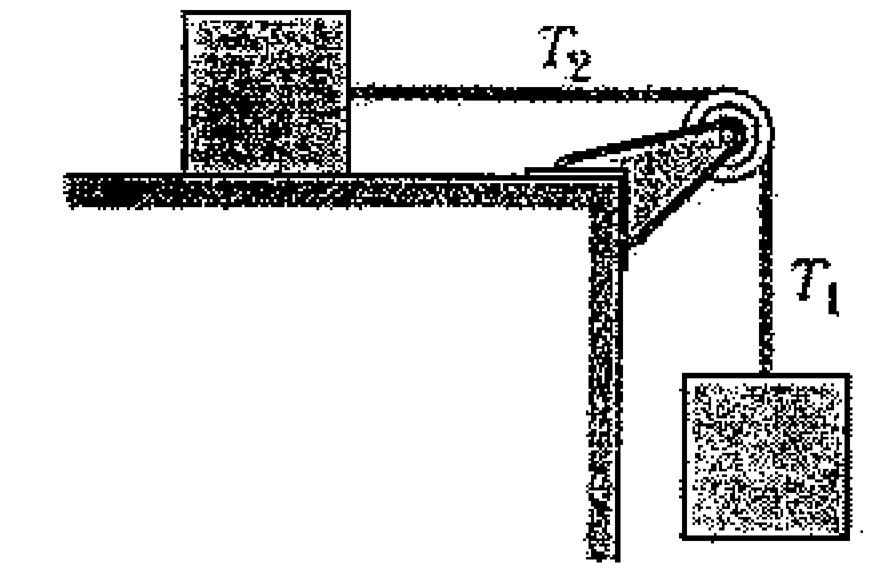
- during the "collision" the $m_1 + m_2$ system is isolated $\Rightarrow$ the total linear momentum of the system is conserved: $\sum p_{before} = \sum p_{after}$
- the text claims that block 1 moves back after the "collision"
- using as + direction toward right, we have:

$$-m_1|\vec{v}_{12}| = +m_1|\vec{v}_{13}| - m_2|\vec{v}_{23}|$$

$$\Rightarrow \underline{\underline{|\vec{v}_{13}|}} = \frac{m_2|\vec{v}_{23}| - m_1|\vec{v}_{12}|}{m_1} = \frac{m_2}{m_1}|\vec{v}_{23}| - |\vec{v}_{12}| = \frac{0.40\,kg}{0.30\,kg}(21.9\tfrac{m}{s}) - 9.9\tfrac{m}{s} = \underline{\underline{19.3\ldots}}$$

NOTE: The obvious increase in Kinetic energy after "collision" (both velocities have larger magnitudes than initial velocity) makes this "collision", or interaction, an explosion, where $\Delta K > 0$: there was a gunpowder on the surfaces of the blocks, to accomplish that.

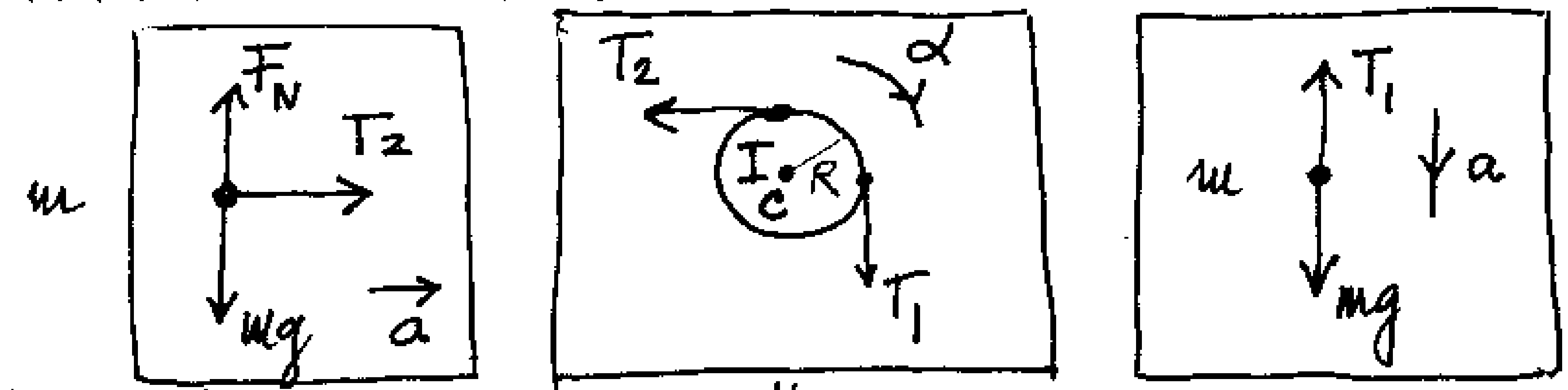
Problem 2 - 22 points



In the figure, two 2.00 kg blocks are connected by a massless string over a pulley of radius 3.00 cm and rotational inertia of 9.00×10^{-4} kg·m^2. The string does not slip on the pulley and the pulley's axis is frictionless. The system is released from rest and the horizontal surface is **frictionless**.

$R = 3cm = 3 \times 10^{-2} m$

(a) (5 pts) Draw free-body diagrams for each block and the pulley in the space below.



horizontal block　　pulley　　hanging block

(b) (10 pts) Calculate the magnitude of either block's linear acceleration.

To find a, T_1, and T_2 we will apply second Newton's laws: $\Sigma F = ma$ for each block and $\Sigma \tau_c = I_c \alpha$ for the pulley along direction of acceleration a for blocks and along angular acceleration α for pulley; also, cord does not slip $\Rightarrow |\vec{a}| = |\vec{\alpha}| R$

horiz. block: $|\vec{T_2}| = m|\vec{a}|$ (1)

hanging block: $mg - |\vec{T_1}| = m|\vec{a}|$ (2)

pulley: $|\vec{T_1}|R - |\vec{T_2}|R = I_c|\vec{\alpha}| = I\frac{|\vec{a}|}{R} \Rightarrow |\vec{T_1}| - |\vec{T_2}| = \frac{I_c}{R^2}|\vec{a}|$ (3)

Summing equations (1), (2), and (3) we get:

$$mg = \left(2m + \frac{I_c}{R^2}\right)|\vec{a}|$$

$$|\vec{a}| = \frac{mg}{2m + \frac{I_c}{R^2}} = \frac{1}{2 + \frac{I_c}{mR^2}} g = \frac{1}{2 + \frac{9 \times 10^{-4} kgm^2}{(2.0kg)(3 \times 10^{-2} m)^2}} \left(9.8 \frac{m}{s^2}\right) = 3.92 \ m/s^2$$

(c) (4 pts) Calculate the magnitude of string tension on the hanging block, T_1.

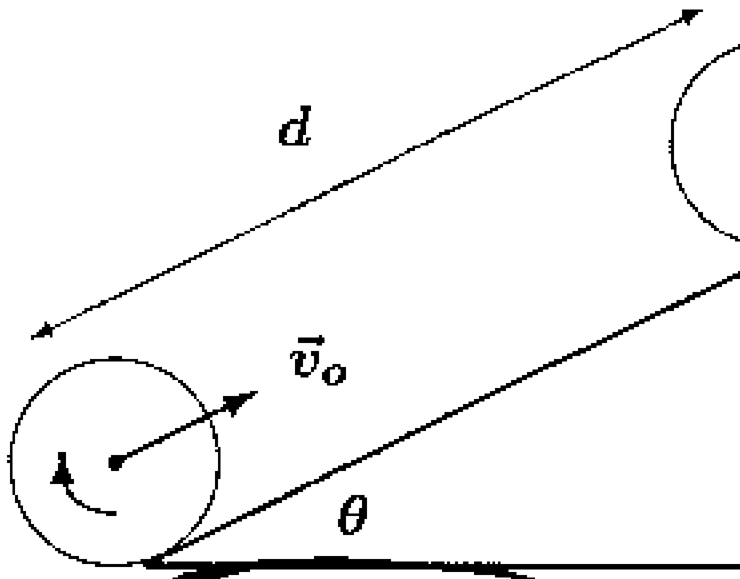
$$|\vec{T_1}| = m(g - |\vec{a}|) = (2.0kg)\left(9.8\frac{m}{s^2} - 3.92\frac{m}{s^2}\right) = 11.76 N \simeq 11.8 N$$

(d) (3 pts) Circle the correct relation between magnitude of tensions T_1 and T_2

$T_1 < T_2$　　$T_1 = T_2$　　$T_1 > T_2$

• the tension T_1 has to turn the pulley and sustain the apparent weight of the hanging block, while tension T_2 only pulls on horizontal block sustaining only its inertia

Problem 3 - 22 points



An object with a circular cross section and rotational inertia $I = \beta m R^2$, $0 < \beta \leq 1$, rolls smoothly (no slipping) up an incline with an inclination angle θ. Initially, at the **bottom** of the incline, the rolling object has the center of mass speed v_o.

smooth rolling $\Rightarrow v_{com} = \omega R$ and $a_{com} = \alpha R$, here $v_{com} = v_0$

(a) (7 pts) Obtain a symbolic expression for the **kinetic energy** of this rolling ~~object at~~ the **bottom** of the incline, in terms of m, v_o, β, and numerical constants.

$$\underline{\underline{K}} = \frac{1}{2} m v_{com}^2 + \frac{1}{2} I_{com} \omega^2 = \frac{1}{2} m v_{com}^2 + \frac{1}{2}(\beta m R^2)\left(\frac{v_{com}}{R}\right)^2 = \underline{\frac{1}{2} m v_{com}^2 (1+\beta)}$$

• using the pure rotation about the contact point P: $K = \frac{1}{2} I_P \omega^2$, we have:

$$I_P = I_{com} + mR^2 = \beta m R^2 + mR^2 = mR^2(1+\beta)$$

$$\underline{\underline{K}} = \frac{1}{2}\left(mR^2(1+\beta)\right)\left(\frac{v_{com}}{R}\right)^2 = \underline{\frac{1}{2} m v_{com}^2 (1+\beta)}$$ ...and the result is the same as above

(b) (8 pts) Obtain a symbolic expression for the distance d, representing how **far up** along the incline the shell travels before stopping, in terms of v_o, β, θ, g, and numerical constants. Here, g is the free-fall acceleration.

• during smooth rolling, along the incline, only mg has nonzero work ($W_{F_N} = 0$ and $W_{f_s} = 0$ because there is no relative motion between the contact point P and the incline surface)

$\Rightarrow$ we can use conservation of mechanical energy:

$$(K + \cancel{U}^{0})_i = (\cancel{K}^{0} + U)_f, \quad v_f = 0 \text{ and } h_i = 0, \; h_f = d \sin\theta$$

$$\frac{1}{2}\cancel{m} v_{com}^2 (1+\beta) = \cancel{m} g d \sin\theta$$

$$\underline{\underline{d = \frac{v_{com}^2 (1+\beta)}{2g \sin\theta}}}$$

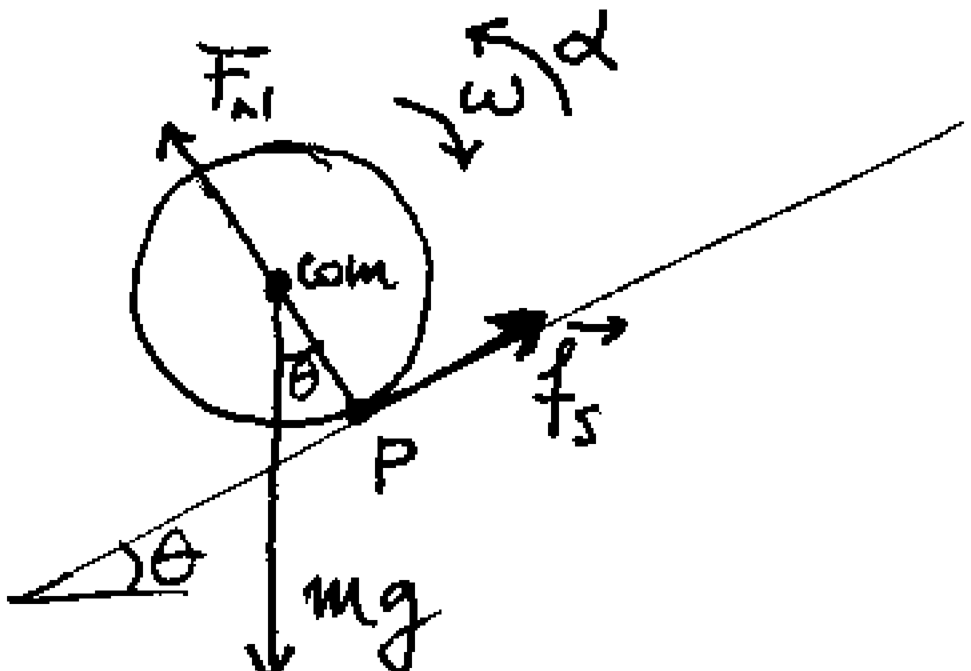
because $a = $ const one can find d using v^2 formula, but then one must find a!!

$$\cancel{v^2}^{0} = v_0^2 + 2ad \Rightarrow d = \frac{v_0^2}{2|\vec{a}|}$$

$$\left.\begin{array}{l} \Sigma F = ma \Rightarrow mg\sin\theta - f_s = ma \\ \Sigma \tau_{com} = I_{com}\alpha \Rightarrow R f_s = I_{com}\frac{a}{R} \Rightarrow f_s = I_{com}\frac{a}{R^2} \end{array}\right\} |\vec{a}| = \frac{g\sin\theta}{1 + \frac{I_{com}}{mR^2}}$$

$$|\vec{a}| = \frac{g\sin\theta}{1+\beta}$$

(c) (4 pts) Draw a free-body diagram for the shell as it rolls smoothly up the incline.



$\vec{\alpha}$ is in the opposite direction of $\vec{\omega}$ because the motion slows down

(d) (3 pts) Does friction do any work on the shell? Explain in a complete sentence.

No, the static friction, in contact point P, during smooth rolling does not do any work, because there is no relative motion between the contact point and surface (no displacement) and no angular displacement. The com is being displaced, so $W_g \neq 0$! but $W_{F_N} = 0$ and $W_{f_s} = 0$!!