Physics 2101, Final Exam, **Form A**

December 11, 2007

Name: __________ SOLUTIONS ________________________

Section: (Circle one)

1 (Rupnik, MWF 7:40am)   2 (Rupnik, MWF 9:40am)

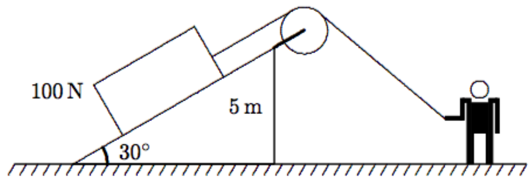
3 (González, MWF 2:40pm)

4 (Pearson, TTh 9:10am)   5 (Pearson, TTh 12:10pm)

- Please turn OFF your cell phone and MP3 player!
- Feel free to detach, use and keep the formula sheet. No other reference material is allowed during the exam. You may use scientific or graphing calculators.
- For the questions, no work needs to be shown (there is no partial credit).
- For the problems, please write as much as you can explaining to us your reasoning. Lonely right answers will not receive full credit, lonely wrong answers will receive no credit at all.
- Please carry units through your calculations when needed, lack of units will result in a loss of points.
- GOOD LUCK!

Question 1 (6 pts)

A man pulls a crate of weight equal to 100 N from the bottom to the top of a frictionless 30° slope which is 5 m high, making the crate move with a **constant speed**.



(a) (2pts) What is the work done by gravity on the crate?

The work done by gravity is equal to minus the change in gravitational potential energy, $W_g = -\Delta U_g = -Mgh = 100\text{N} \times 5\text{m} = -500\text{J}$. It is negative, since the gravitational force points down, and the crate is moving upwards along the incline.

-500 J   -250 J   0   250 J   500 J

(b) (1pt) What is the work done by the man on the crate?

Since the crate is moving with constant velocity, the work done by external forces must be zero (work-energy theorem). The normal force does not do any work since it is perpendicular to the crate's motion. The work done by the man's force must then be equal to the work done by gravity, calculated above:

$0 = W_g + W_m = -500\text{J} + W_m \rightarrow W_m = +500\text{J}$.

-500 J   -250 J   0   250 J   **500 J**

(c) (1 pt) What is the work done by the normal force on the crate?

The normal force does not do any work since it is perpendicular to the crate's motion.

-500 J   -250 J   **0**   250 J   500 J

(d) (2 pts) If there were friction between the crate and the sloped surface, and the man kept pulling up the crate with constant speed, the magnitude of the work done by gravity on the crate would be...

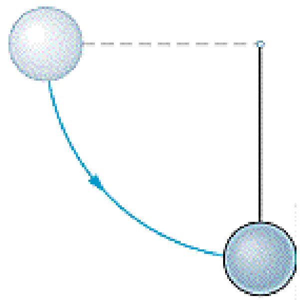
larger than   **the same as**   smaller than

... the work calculated in (a).

Gravity's work is always equal to minus the change in gravitational potential energy, friction or not. If there were friction, the man's work would have to be larger, to compensate for both the work done by gravity and friction forces, since those are both negative.

Question 2 (5 pts)

A steel ball of mass m is fastened to a massless cord of length L, forming a simple pendulum, fixed at the far end. The ball is released from rest when the cord is horizontal.



(a) (2 pts) What is the speed of the ball as the ball moves through the lowest point at the bottom?

$\sqrt{2g/L}$ $\quad\quad$ $\sqrt{\mathbf{2gL}}$ $\quad\quad$ $\sqrt{gL/2}$ $\quad\quad$ $\sqrt{g/(2L)}$

Using conservation of energy between initial position at rest and final position at the bottom oof the swing, we have:

$\Delta KE + \Delta U = \frac{1}{2}mv^2 - mgL = 0 \rightarrow v^2 = 2gL$

(b) (2 pts) What is the tension in the cord as the ball moves through the lowest point at the bottom?

mg $\quad\quad$ $2mg$ $\quad\quad$ **3mg** $\quad\quad$ 0

At the bottom, the forces acting on the pendulum's mass are gravity (down) and tension (up); the net force is a radial force towards the center of the circle of radius L (upwards). The centripetal force produces the radial acceleration, equal to $a = mv^2/L = 2mg$ from above, so

$T - mg = ma = mv^2/L = 2mg \rightarrow T = 3mg$.

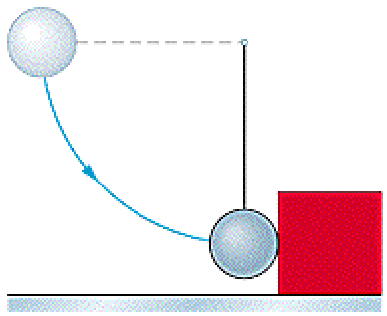
(c) (1 pt) The torque exerted by gravity on the pendulum about the attachment point is largest when the cord is...

horizontal $\quad\quad$ vertical $\quad\quad$ at an angle of 45° $\quad\quad$ gravity's torque is constant

Gravity's force is always vertical and pointing down, so the torque $\vec{\tau} = \vec{F} \times \vec{r}$ will be maximum when the position vector $\vec{r}$ is horizontal: this happens when the cord is horizontal.

Problem 1 (11 pts)

A steel ball of mass 0.5 kg is fastened to a massless cord that is 50cm long and fixed at the far end, forming a simple pendulum. The ball is released when the cord is horizontal. At the bottom of its path, the ball strikes a 3kg block initially at rest. Just after the collision, which is inelastic, the block begins moving to the right with a speed of 0.5 m/s.



(a) (4 pts) What is the speed of the ball just before the collision?

Using conservation of energy between initial position at rest and final position at the bottom oof the swing, we have:

$\Delta KE + \Delta U = \frac{1}{2}mv^2 - mgL = 0 \rightarrow v = \sqrt{2gL} = \sqrt{2 \times 9.8\text{m/s}^2 \times 0.5\text{m}} = 3.13\,\text{m/s}$

(b) (7 pts) What are the magnitude and direction of the steel ball's velocity after the collision?

The linear momentum is conserved in the collision. The initial momentum is that of the pendulum's mass mv_0, where $v0$ was calculated in part (a). The final momentum is the sum of the momentum of the pendulum mass, mv_f, and the block's momentum, MV, where V is given as 0.5 m/s:

$mv_0 = mv_f + MV \rightarrow$

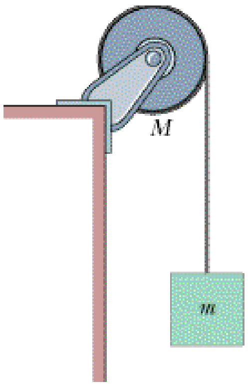
$v_f = (mv_0 - MV)/m = v_0 - (m/M)V = 3.13\,\text{m/s} - (3\text{kg}/0.5\text{kg})0.5\text{m/s}$

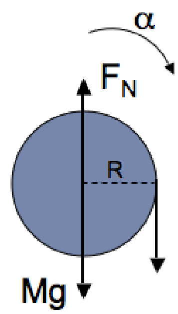
$v_f = 0.13\,\text{m/s}$

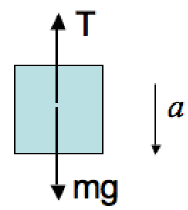
Since we obtained a positive result, the pendulum's mass is moving (slowly) towards the right after collision.

Problem 2 (11 pts)

A mass m is connected with a massless cord to a pulley of mass M and radius R. The pulley is a solid disk. The mass m is let go from rest, and it begins moving down, with the cord unwrapping without slipping.







(a) (3pts) Draw above a free body diagram for the mass, and a free body diagram for the pulley, labeling all the forces.

(b) (4 pts) Write an expression for the magnitude of the acceleration a of the mass m, in terms of g, m, and M.

Newton's law for the pulley and the mass, with $a = \alpha R$ and $I = MR^2/2$ (since the pulley is a disk):

$\tau = I\alpha \Rightarrow TR = \frac{1}{2}MR^2(a/R) = \frac{1}{2}MRa$

$F = ma \Rightarrow mg - T = ma$

From the pulley equation, we obtain $T = Ma/2$, and using this in the mass equation, we obtain:

$mg - Ma/2 = ma \Rightarrow mg = (m + M/2)a \Rightarrow a = g/(1 + (M/2m))$

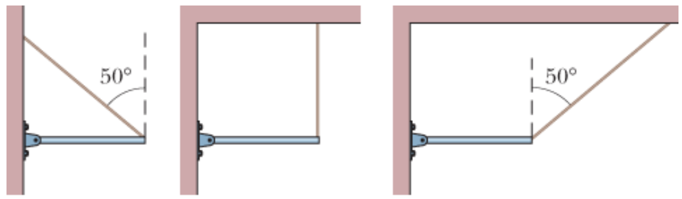
(c) (4 pts) Write an expression for the angular momentum of the pulley L at a time t after the mass m was released from rest, in terms of m, M, g, R and t.

The angular momentum is $L = I\omega$. The pulley is rotating with a constant angular acceleration $\alpha = a/R$, so the angular velocity a time t after starting from rest is $\omega = \alpha t$:

$$L = I\omega = (MR^2/2)(at/R) = \frac{1}{2}(MR^2)\frac{gt/R}{1 + (M/2m)} = \frac{MRgt}{2 + (M/m)}$$

Question 3 (6 pts)

The figure shows three situations in which the same horizontal rod is supported by a hinge on one wall at one end, and by a cord at its other end.



(1) (2) (3)

The forces on the rod are the hinge force at the left end of the rod (with horizontal and vertical components F_h and F_v), gravity mg at the center of the rod pointing down, and tension T at the right end of the rod.

In equilibrium, the torque about any point is zero; the torque about the end of the rod is the torque of the vertical component of the rod (clockwise, $F_v L$), and the torque of gravity (counterclockwise, $mgL/2$), so the hinge's vertical force is $F_v = mg/2$, same in all cases.

The torque about the left end of the rod is $mgL/2$ clockwise from gravity, and $TL\cos\theta$ counterclockwise from tension, with $\theta = 50°$ in the first and third case, and $0°$ in the middle case. The magnitude of tension is then $T = mg/2\cos\theta$, smallest in the middle case, and equal in the first and third cases.

The net horizontal force is zero, so the hinge's horizontal force must be equal to the tension's horizontal component: it is zero in the middle case, and has equal magnitude in the first and third cases, but it points to the right in the first case, and to the left in the third case.

(a) (2 pts) In which situation is the magnitude of the tension force on the cord the **largest**?

(i) Tension in (1) is largest.

(ii) Tension in (2) is largest.

(iii) Tension in (3) is largest.

(iv) **Tension in (1) and (3) tie, tension in (2) is smaller**.

(v) Tension is the same in all.

(b) (2 pts) In which situation is the vertical force on the rod from the hinge the **largest**?

(i) Hinge vertical force in (1) is largest.

(ii) Hinge vertical force in (2) is largest.

(iii) Hinge vertical force in (3) is largest.

(iv) Hinge vertical force in (1) and (3) tie, hinge vertical force in (2) is smaller.

(v) **Hinge vertical force is the same in all**.

(c) (2 pts) In which situation is the magnitude of the horizontal force on the rod from the hinge the **smallest**?

(i) Hinge horizontal force in (1) is smallest.

(ii) **Hinge horizontal force in (2) is smallest**.

(iii) Hinge horizontal force in (3) is smallest.

(iv) Hinge horizontal force in (1) and (3) tie, hinge horizontal force in (2) is smaller.

(v) Hinge horizontal force is the same in all.

Problem 3 (11pts)

Consider a star with a mass $M = 2 \times 10^{30}$kg, about the mass of our Sun.

(a) (4 pts) What is the minimum radius the star can have for which the escape velocity is below the speed of light, $c = 3 \times 10^8$ m/s? (If the star is a black hole, such radius is the radius of the "event horizon").

The escape velocity is $v_e = \sqrt{2GM/R}$, so if $v_e^2 = 2GM/R \leq c^2$, we must have
$R \geq 2GM/c^2 = 2 \times 6.67 \times 10^{-11}\mathrm{m}^3/(\mathrm{kg\,s}^2) \times 2 \times 10^{30}\mathrm{kg}/(3 \times 10^8\mathrm{m/s})^2 = 2.96$ km.

(b) (4 pts) What speed is needed for a shuttle orbiting the star in a circular orbit at a radius of 100km from the center of the star?

In a circular orbit, the centripetal force is the gravitational force between the shuttle and the star, $F = GmM/r^2$, and it is equal to the mass times the centripetal acceleration, $F = ma = mv^2/r$, so we have

$F = ma \Rightarrow GMm/r^2 = mv^2/r \Rightarrow v = \sqrt{GM/r}$

$v = \sqrt{6.67 \times 10^{-11}\mathrm{m}^3/(\mathrm{kg\,s}^2) \times 2 \times 10^{30}\mathrm{kg}/(10^5\mathrm{m})} = 3.65 \times 10^7\mathrm{m/s}$.

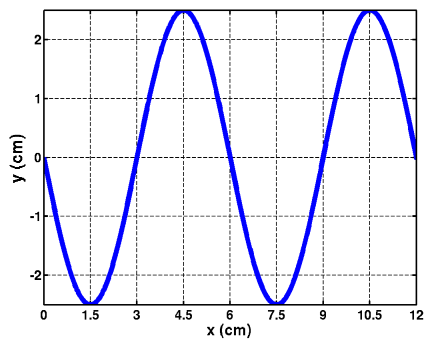
This is 12% the speed of light, very fast!

(c) (3 pts) A probe begins to fall from the shuttle towards the star, in a straight linear path towards the center of the star. As the probe approaches the star, does the probe's acceleration get larger or smaller, or is it constant? Explain your answer.

As the probe falls towards the star, the force producing the acceleration is the gravitational force from the planet, $F = ma = GMm/r^2$, so $a = GM/r^2$. As the probes gets closer to the planet, the distance r decreases, so the acceleration will increase.

Question 4 (6 pts)

The figure shows a snapshot of a traveling wave, taken at $t = 0$. The wave is traveling towards the right. The wave has amplitude $y_m = 2.5$ cm and period T=10 s.



From the drawing, the wavelength is $\lambda = 6$ cm, and the wavenumber is $k = 2\pi/\lambda$.

The points on the string are moving up and down in simple harmonic motion, and at $t = 0$, the points at the maximum and minimum amplitude will be momentarily at rest; the ones between $x = 0$ and $x = 1.5$cm and between $x = 4.5$cm and $x = 6$cm will be moving up; the ones between $x = 1.5$ cm and $x = 4.5$cm will be moving down.

At t=3T/4, the wave will have traveled a distance $3\lambda/4 = 4.5$cm towards the right, so the bottom peak will have moved from $x = 1.5cm$ to $x = 6$cm, and there will be another point at $y = -y_m$ a wavelength to the left, at $x = 0$cm. The string element at $x = 0$cm was at $y = 0$ at $t = 0$, moved up to y_m at $t = T/4$, down to zero at $t = T/2$, and down to $-y_m$ at $t = 3T/4$.

(a) (1 pt) At $t = 3T/4$, which of the following points will be at $y = -y_m$?

x = 0 cm $x = 1.5$ cm $x = 3$ cm $x = 4.5$ cm

(b) (2 pts) At $t = 0$, which of the following points has an downwards transverse speed?

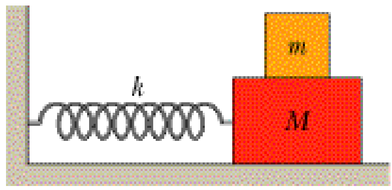
$x = 0$ cm $x = 1.5$ cm $x = 3$ cm   **x = 4.5** cm

(c) (1 pt) What is the wavenumber k of the traveling wave?

2π/(6cm) 2π 6 cm $2\pi/(12$ cm)

Problem 4 (11pts)

A mass M is attached to a spring of spring constant k, and a smaller mass m is set on top of the larger mass. The mass M is on a horizontal, frictionless surface, but there is friction between the two blocks, with coefficients μ_k and μ_s. The spring-blocks system is set into simple harmonic oscillations, by compressing the spring a distance x_m and releasing the system from rest.



(a) (3 pts) Derive an expression for the period T of the oscillations, in terms of k, M and m.

We take as the masses M and m as a single system, the external forces are gravity, $(M+m)g$, canceled by the normal force on M, and the spring force, producing simple harmonic motion: $F = -kx = (M+m)a = -(M+m)\omega^2 x$, so $\omega = \sqrt{k/(M+m)}$ and

$T = 2\pi/\omega = 2\pi/\sqrt{k/(M+m)} = 2\pi\sqrt{(M+m)/k}$.

(b) (4 pts) Assuming the block m does not slip, derive an expression for the maximum force on the mass m, in terms of m, M, k and x_m. (This is the friction force, producing the oscillation of the mass m).

The acceleration of mass m is the same as that of the system $M+m$, and has a maximum value $a_m = \omega^2 x_m = kx_m/(M+m)$; the maximum force on mass m will then be

$F_{max} = ma_{max} = mkx_m/(M+m)$

(c) (4 pts) Derive an expression for the amplitude of the oscillations that puts the smaller block on the verge of slipping over the larger block, in terms of m, M, μ_s, k and g.

The friction force cannot be larger than $f_{s,max} = \mu_s F_N$. The normal force on the mass m is mg, so if the maximum force on mass m (from the previous answer) is equal to the maximum possible static friction force, we have

$F_{max} = ma_{max} = mkx_m/(M+m) = \mu_s mg$, and

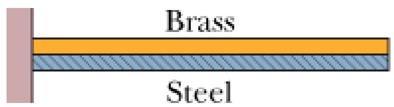
$x_m = \mu_s(g/k)(M+m)/m$.

Question 5 (5 pts)

A brass and a steel rod have equal lengths L_0 at temperature T_0, and are glued as shown in a bimetal strip. Brass has a larger coefficient of thermal expansion than steel.

When the temperature is increased, the brass will have a large length, $L_b = L_0(1 + \alpha_b \Delta T)$ than the steel, $L_s = L_0(1 + \alpha_s \Delta T)$, so the strip will bend downwards, with the brass arc longer than the steel arc.

The fractional change in length is $\Delta L/L_0 = \alpha \Delta T$, independent of length or thickness.



(a) (3 pts) If the temperature is increased, the strip will ...

(i) elongate, but stay straight.

(ii) **elongate and bend downwards.**

(iii) elongate and bend upwards.

(iv) stay the same length.

(v) get shorter.

(b) (2 pts) If the strips are thicker, the fractional change in length $\Delta L/L_0$ for each strip due to a change in temperature will ....

(i) **be the same as when the strips are thinner.**

(ii) be larger than when the strips are thinner.

(iii) be smaller than when the strips are thinner.

(iv) depend on the length of the strips.

Problem 5 (11pts)

25 grams of ice at $-10°$C are added to cool down 100 cm^3 of soda. The soda is initially at room temperature, 25°C. Assume the ice-soda system is insulated from the rest of the world, and that the soda has the same properties as water.

(a) (2 pts) What would be the heat absorbed by the ice warming up to melting temperature?

$$Q = mc_{ice}\Delta T = 0.025\text{kg} \times 2220\text{J}/(\text{kg}°) \times (0° - (-10°) = +555\text{J}.$$

(b) (2 pts) What would be the heat the ice needs to absorb to completely melt?

$$Q = mL_{fusion} = 0.025\text{kg} \times 3.33 \times 10^5\text{J/kg} = +8,325\text{J}.$$

(c) (2 pts) What is the heat that the soda releases to the system if it were to cool down to freezing temperature?

The mass of the soda is $M = \rho V = 10^3\text{kg/m}^3 \times 100(10^{-2}\text{m})^3 = 0.1\text{kg}$.

$$Q = Mc_{water}\Delta T = 0.1\text{kg} \times 4190\text{J}/(\text{kg}°) \times (0° - 25°) = -10,475\text{J}$$

(d) (3pts) What is the final temperature of the system?

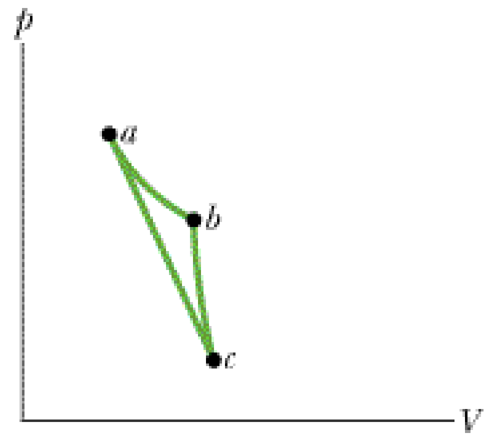
$$\Delta Q = 0 = mc_{ice}(0° - (-10°) + mL + mc_{water}(T_f - 0°) + Mc_{water} \times (T_f - 25°)$$
$$0 = 555J + 8,325J + c_{water}(M + m)T_f - 10,475J$$
$$T_f = 1595\text{J}/(4190\text{J}/(\text{kg}°\text{C})0.125\text{kg}) = 3.05°C$$

(e) (2 pts) Is the change of entropy in the ice+soda system positive, negative or zero? Why?

Since we consider the soda+ice system as an isolated system, the entropy cannot decrease, so the change in entropy can only be zero or larger than zero. The melting of the ice is an irreversible process, so the entropy must increase: the change in entropy is positive. Notice that the entropy of the soda decreases, since it gets cooler, but the entropy of the ice increases by a larger amount.

Question 6 (6 pts)

In the pV diagram shown in the figure, the gas is taken from a to b keeping the temperature constant, from b to c in an adiabatic process, and back from c to a in a straight line. Fill in the table with 0 (zero), + (positive) or − (negative), indicating the sign of the quantity in each process.



When the volume increases in a process, the work done by the gas is positive: this is true for $a \to b$ and for $b \to c$. The work done by the gas in the process $c \to a$ is negative.

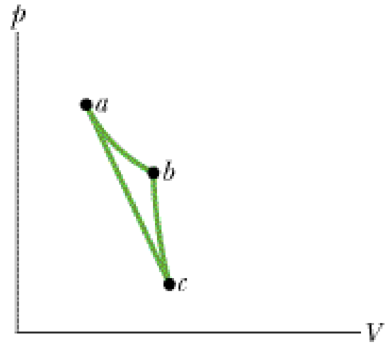
The $a \to b$ process is isothermal, so the change in internal energy is zero, and $\Delta E_{int} = 0 = Q - W$, so $Q = W$, and since W_{ab} is positive, so is Q_{ab}. The $b \to c$ process is adaibatic, so $Q_{bc} = 0$.

The change in entropy is given by $\Delta S = \int dQ/T = nRln(V_f/V_i) + nC_V ln(T_f/T_i)$. In the process $a \to b$, temperature is constant and volume increases, so $\Delta S_{ab} > 0$. In the adiabatic process $b \to c$, there is no heat exchange, so $\Delta S = 0$.

	$a \to b$	$b \to c$
Work done by the gas	+	+
Heat absorbed or released by the gas	+	0
Change of Entropy	+	0

Problem 6 (11pts)

In the pV diagram shown in the figure, a diatomic gas is taken from a to b keeping the temperature constant, from b to c in an adiabatic process, and back from c to a in a straight line. At point a, the temperature is 300K, the pressure is 50 kPa, and the volume is 0.1m^3. At point b, the volume is $0.15\ \text{m}^3$. At point c, the volume is $0.3\ \text{m}^3$, and the temperature is 227 K. Assume all the processes are reversible.



(a) (2 pts) How many moles are there in the gas?

$pV = nRT \Rightarrow$

$n = p_a V_a/(RT_a) = 5 \times 10^4\text{Pa} \times 0.1\text{m}^3/(8.31(\text{J/mol K})\,300\text{K}) = 2.0\text{mol}.$

(b) (4 pts) What are the work done by the gas, W_{ab}, and the heat absorbed by the gas, Q_{ab}, when taken from a to b?

From a to b, T is constant, so

$W_{ab} = nRT_a \ln(V_b/V_a) = 2\text{mol} \times 8.31(\text{J/mol K})) \times 300\text{K} \times \ln(0.15/0.1)$

$\text{W}_{\text{ab}} = +2.0\text{kJ}$

Since T is constant, $\Delta E_{int} = 0 = Q_{ab} - W_{ab}$ and $Q_{ab} = +2.0\text{kJ}$

(c) (3 pts) What are the work done by the gas, W_{bc}, and the heat absorbed by the gas, Q_{bc}, when taken from b to c?

From b to c, the process is adiabatic, so $Q_{bc} = 0$. Since $\Delta E_{int,bc} = Q_{bc} - W_{bc}$, then

$W_{bc} = -\Delta E_{int,bc} = -nC_V \Delta T = -n(5R/2)(T_c - T_b)$

$W_{bc} = -2\text{mol} \times (5/2)8.31(\text{J/(molK)}) \times (227\text{K} - 300\text{K})$

$W_{bc} = +3.0\text{kJ}.$

(d) (2 pts) Will the efficiency of this cycle be smaller, larger or equal to that of a Carnot engine operating between the lowest and the highest temperature of this cycle? Why?

The efficiency of any heat engine is equal or smaller than that of a Carnot engine. A Carnot engine has two adiabatic process, and two isothermal processes; so this is not a Carnot engine, and its efficiency must be smaller.