Physics 2101, Fourth Exam, Fall 2007

November 20, 2007

Name: KEY

Signature: ______________________

Section: (Circle one)

1 (Dr. Rupnik, MWF 7:40 AM)

4 (Dr. Pearson, TTH 9:10 PM)

2 (Dr. Rupnik, MWF 9:40 AM)

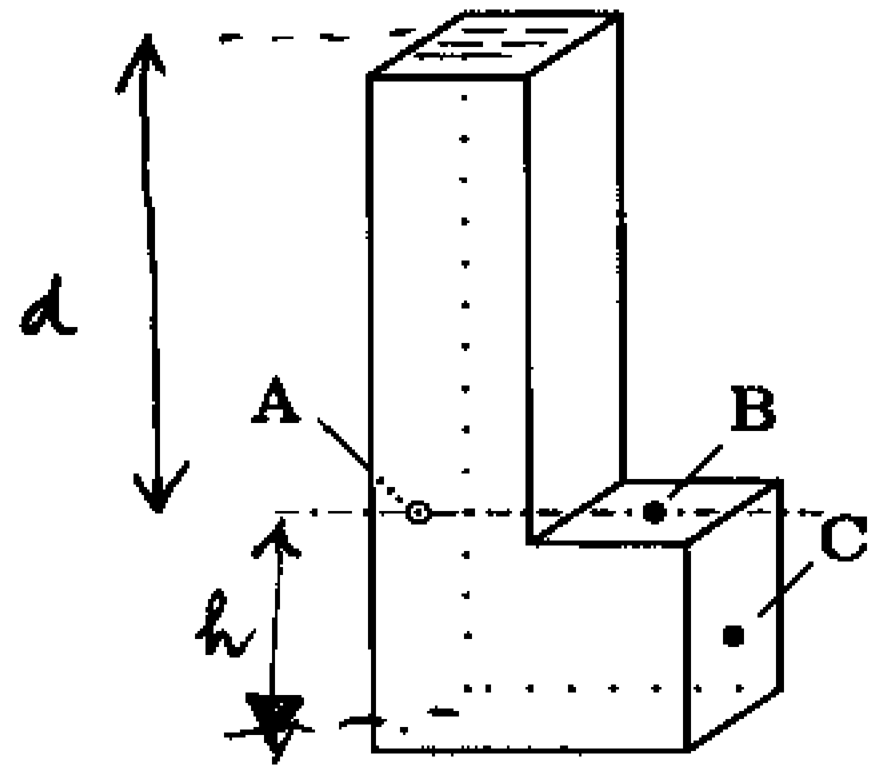
5 (Dr. Pearson, TTh 12:10 AM)

3 (Dr. González, MWF 2:40 PM)

- Please be sure to print and sign your name and circle your section above.
- For the *problems*, you *must* show all your work. Let us know what you were thinking when you solved the problem! Lonely right answers will not receive full credit, lonely wrong answers will receive no credit.
- For the *questions*, no work needs to be shown (there is no partial credit).
- Please, carry units through your calculations when needed, lack of units will result in a loss of points.
- You may use scientific or graphing calculators, but you must derive your answer and explain your work.
- Feel free to detach, use, and keep the formula sheet. No other reference material is allowed during the exam.
- **GOOD LUCK!**

Question 1 (12 points)

The figure below shows an open tank filled with water and three points. Points A and C are on vertical side surfaces and point B is on a horizontal surface, as indicated. Points B and C are centered on their respective surfaces.



(a) (4 pts) The points A and B are at the same height above the bottom surface. Circle the corect statement about the pressure at points A and B.

(i) The pressure at A is greater than the pressure at B.

(ii) The pressure at A is equal to the pressure at B.

(iii) The presure at A is less than the pressure at B.

(iv) There is not enough information to compare the pressures at points A and B.

the same height ⇒ the same depth ⇒ the same pressure

$p_A = p_B = p_0 + \rho g d$

(b) (4 pts) Would the answer change if the container were closed at the top?

(i) Yes.

(ii) No. $p_A = p_B = \rho g d$ ✓

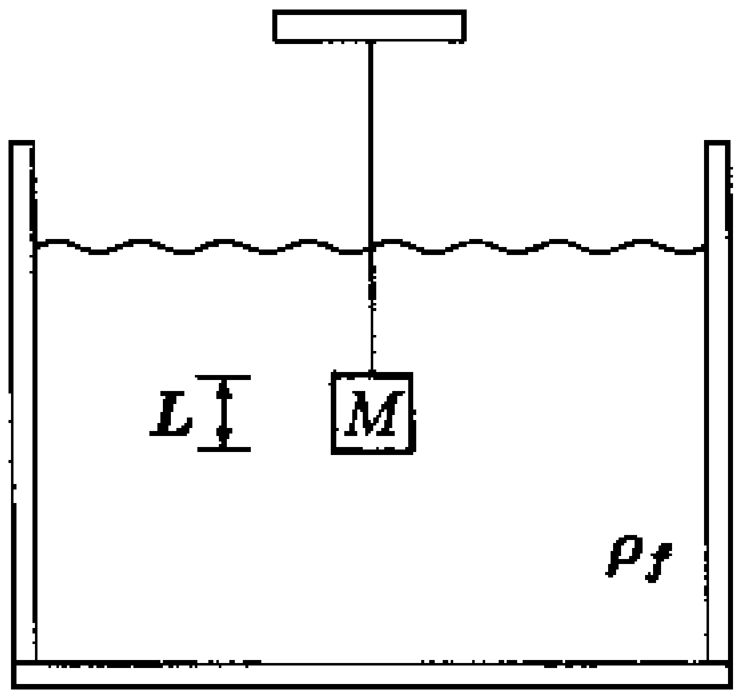
(iii) Not enough information to tell.

(c) (4 pts) Circle the correct direction of the force due to the fluid (hydrostatic force) on the vertical surface at point C.

→	←	↑	↓
(i)	(ii)	(iii)	(iv)

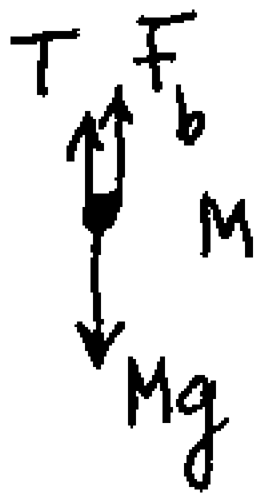
pressure = force per unit area
↳ in static fluid, the force is always perpendicular to the area of the container (or any object within)

Problem 1 - 21 points



A cube with mass M and edge of length L is suspended by a rope in a tank that is filled with fluid of mass density ρ_f, as is illustrated in the figure.

(a) (3 pts) In the space below draw a free-body diagram for the cube. Show the directions of the forces explicitly. Ignore the horizontal forces.



(b) (6 pts) Calculate the buoyant force by which the fluid acts on the block if the magnitude of the tension in the rope is exactly $\frac{1}{3}Mg$. Express the answer in terms of M, g, and numerical constant as needed.

M is at rest $\Rightarrow$ $F_{net} = 0$ $\Rightarrow$ $T + F_b = Mg$

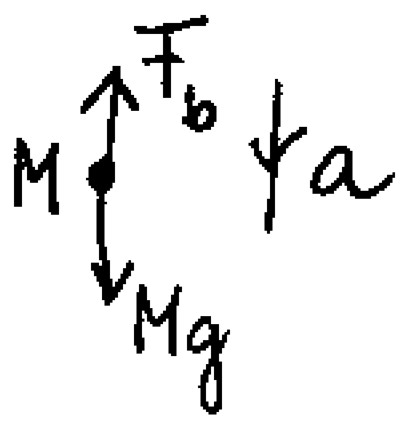
$$\boxed{F_b = Mg - T = Mg - \tfrac{1}{3}Mg = \tfrac{2}{3}Mg}$$

(c) (6 pts) Calculate the density of the fluid ρ_f. Express the answer in terms of M, L, g, and numerical constants as needed.

Archimedes' principle: $F_b = m_f g = \rho_f V g$ where $V = L^3$ and $F_b = \frac{2}{3}Mg$

$$\tfrac{2}{3}M\cancel{g} = \rho_f L^3 \cancel{g} \qquad \boxed{\rho_f = \frac{2M}{3L^3}}$$

(d) (6 pts) If the rope breaks, calculate the acceleration of the cube as it begins to sink. Express the answer in terms of g and numerical constants as needed.



• if the rope breaks $\Rightarrow T = 0$

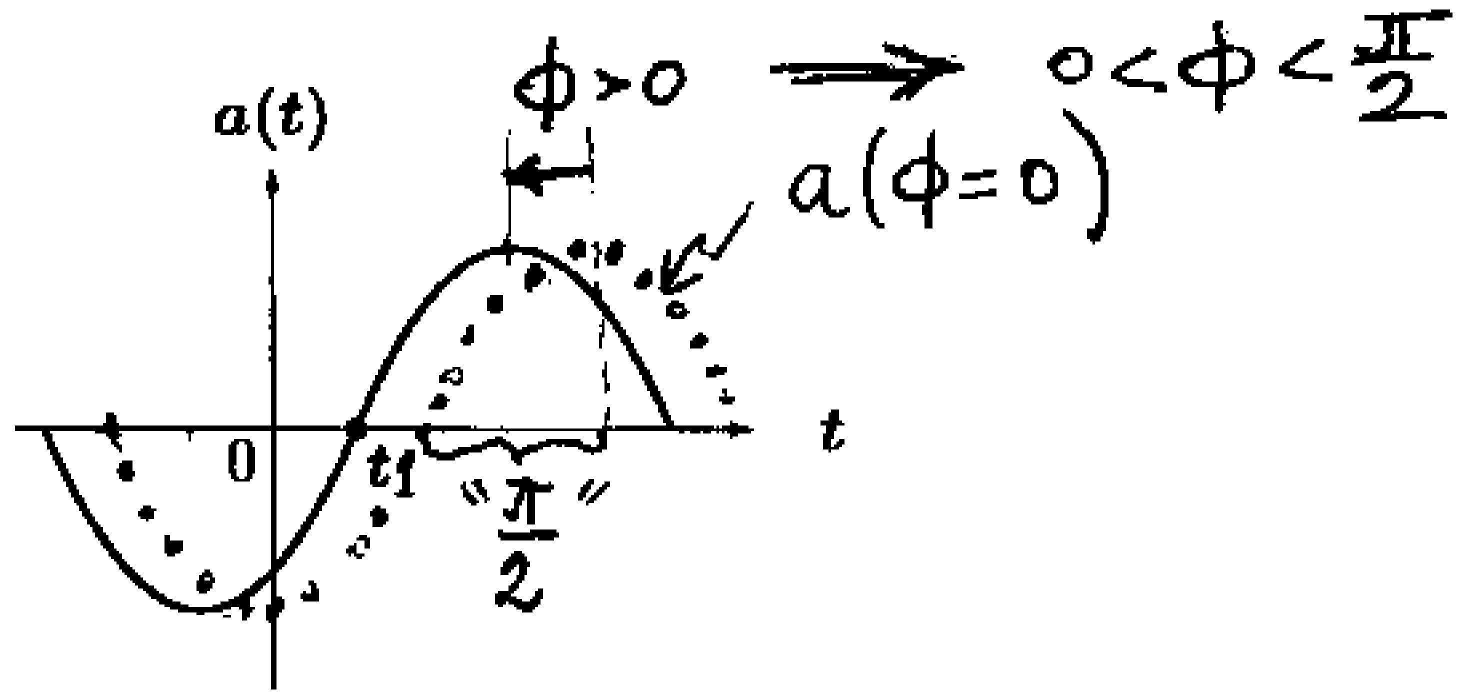
$F_{net} = Ma$

$Mg - F_b = Ma \Rightarrow a = g - \frac{F_b}{M} = g - \frac{\frac{2}{3}\cancel{M}g}{\cancel{M}} = \frac{1}{3}g$

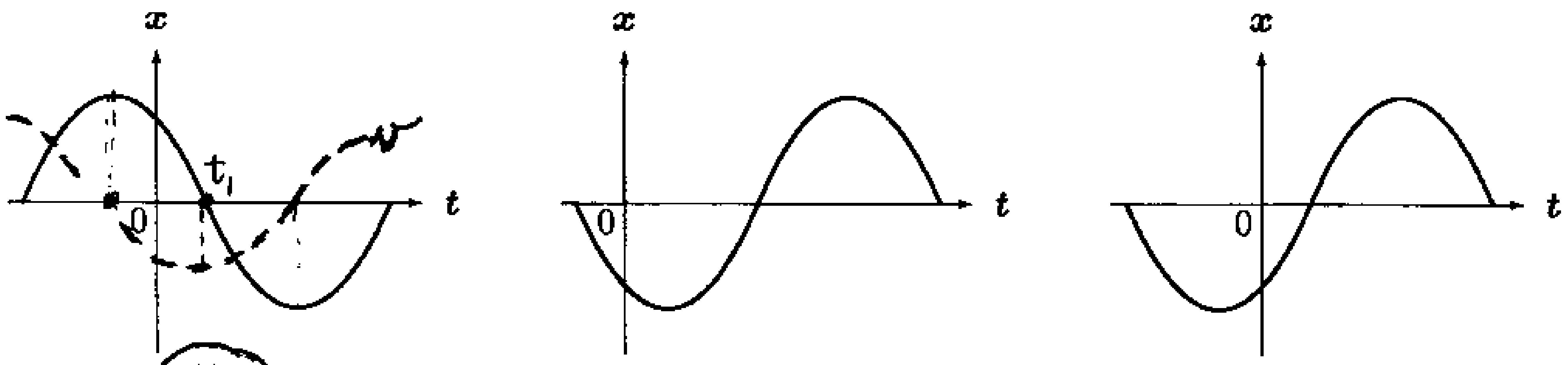
$$\boxed{a = \frac{g}{3}}$$

Question 2 - 12 points

The **acceleration** $a(t)$ of a particle undergoing simple harmonic motion is graphed in the figure below.



(a) (4 pts) Circle the plot below that represents the **position** $x(t)$ for the same particle?



(i) (ii) (iii)

(b) (4 pts) At time t_1, the velocity of the particle is

positive negative zero

• $v(t)$ is drawn as dashed line in (i) (derivative of $x(t)$ with respect to t)

(c) 4 pts) Circle the correct range for the phase constant for this harmonic oscillation, if the position function $x(t)$ has the form $x = x_m \cos(\omega t + \phi)$.

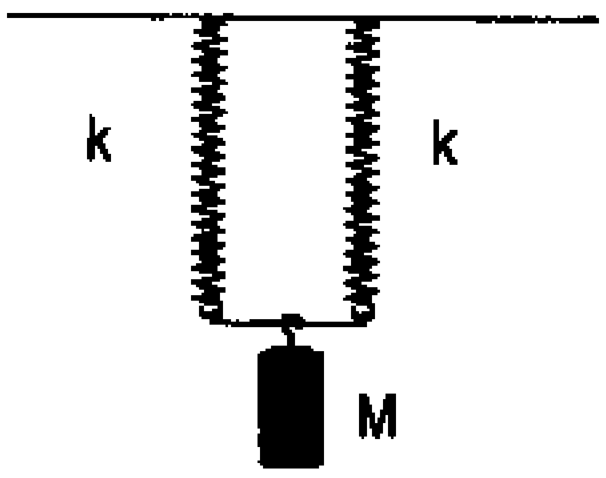
$0 < \phi < \frac{\pi}{2}$ $\frac{\pi}{2} < \phi < \pi$ $\pi < \phi < \frac{3\pi}{2}$

if $x(t) = X_m \cos(\omega t + \phi) \Rightarrow a(t) = -X_m \omega^2 \cos(\omega t + \phi)$ ... represents our plot

$a(\phi=0) = -X_m \omega^2 \cos \omega t$ ... drawn as dotted line in a-t plot above

• to get positive ϕ, one moves $a(\phi=0)$ toward left, untill it coincide with $a(t)$ with ϕ

Problem 2 - 22 points



A block of mass $M = 0.60\,\text{kg}$ hangs from two identical, parallel springs stretching them by $d = 12.0\,\text{cm}$ relative to their relaxed length. The block is at rest, as shown in the figure.

(a) (3 pts) Draw a free body diagram for the block M, as depicted in the figure above.

$kx \uparrow \uparrow kx \quad x = d$

M

$\downarrow Mg$

• Kinematics of SHM is going to be the same as for the horizontal spring-block where the new equilibrium position will be at $x_e = d$.

(b) (6 pts) Calculate the spring constant k for each spring.

the block is at rest $\Rightarrow F_{net} = 0 \Rightarrow 2kd = Mg$

$$k = \frac{Mg}{2d} = \frac{(0.60\,kg)(9.8\,m/s^2)}{2(0.12\,m)} = \underline{24.5\,N/m}$$

(c) (7 pts) The block M is pulled down a farther distance of $x = 5.0\,\text{cm}$ and released from rest. Calculate the period of the resulting simple harmonic oscillation.

$a = -\omega^2 x$ for SHM $\Rightarrow F_{SHM} = -M\omega^2 x$ ... restoring force of SHM

here: $F = -kx - kx = -2kx$ ... relative to the equilibrium point

$\Rightarrow -M\omega^2 \cancel{x} = -2k\cancel{x} \Rightarrow \omega = \sqrt{\frac{2k}{M}}$

$$\omega = \sqrt{\frac{2k}{M}} = \sqrt{\frac{2}{M}\left(\frac{Mg}{2d}\right)} = \sqrt{\frac{g}{d}} = \sqrt{\frac{9.8\,m/s^2}{0.12\,m}} = \underline{9.0\,\frac{rad}{s}}, \quad \text{or } \omega = \sqrt{\frac{2(24.5\,\frac{N}{m})}{0.6\,kg}} = 9.0\,\frac{rad}{s}$$

(d) (4 pts) Calculate the maximum speed of block M during these oscillations.

$$v_m = X_m\omega = (0.05\,m)(0.90\,\tfrac{rad}{s}) = \underline{0.45\,m/s}$$

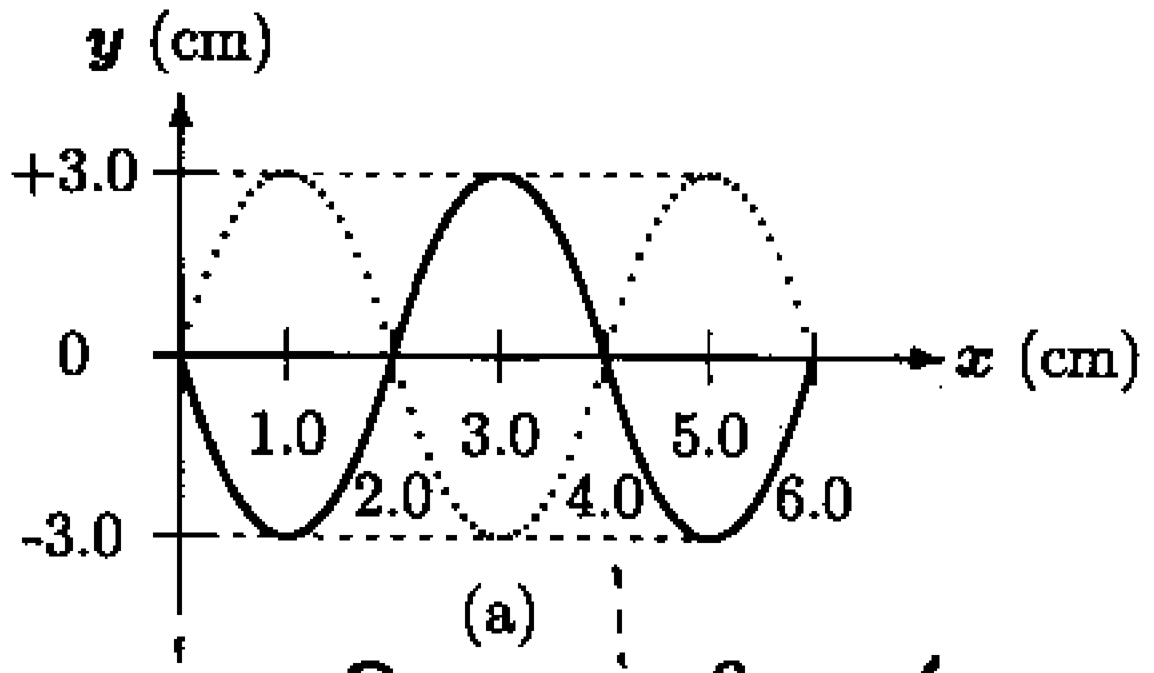
• amplitude of SHM: $X_m = 5.0\,cm$

(e) (2 pts) How much time does it take block M to first reach its maximum speed after release?

$$t = \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{2\omega} = \frac{\pi}{2(9.0\,\frac{rad}{s})} = \underline{0.175\,s}$$

Question 3 - 11 points

Two sinusoidal waves with the same amplitude and wavelength travel through each other along a string that is stretched along an x axis. The figure (a) shows two snapshots of their resultant wave on a string in its 3^{rd} harmonic. The wave speed is 6.0 cm/s.



(a)

$\leftarrow \lambda \rightarrow$ $\lambda = 4$ cm $= 0.04$ m

(a) (3 pt) What is the **wavelength** of the 3^{rd} harmonic shown in figure (a)?

(i) 2.0 cm.

(ii) 3.0 cm.

(iii) 4.0 cm.

(iv) not enough info to tell.

(b) (3 pts) What is the frequency f of the wave?

(i) 1.0 Hz.

(ii) 1.5 Hz.

(iii) 2.0 Hz.

(iv) 3.0 Hz.

$v = \frac{\lambda}{T} = \lambda f \Rightarrow f = \frac{v}{\lambda} = \frac{6.0\ \text{cm/s}}{4.0\ \text{cm}} = 1.5\ s^{-1} = 1.5\ \text{Hz}$

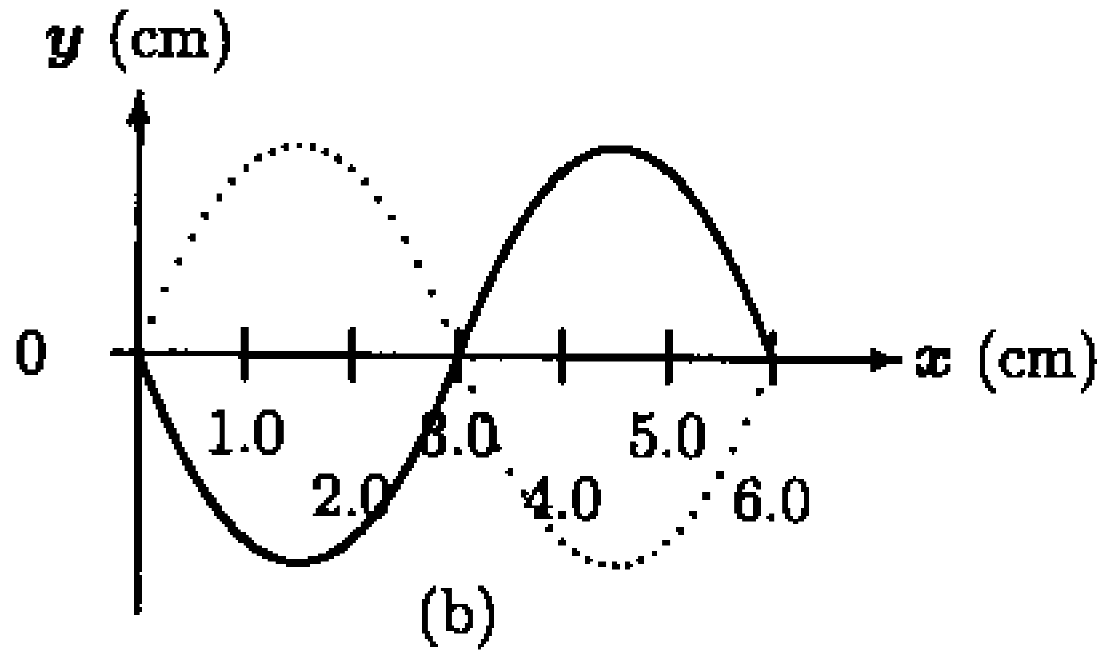
(c) (3 pts) The harmonic shown in figure (b) is

(i) first (1^{st}).

(ii) second (2^{nd}).

(iii) fourth (4^{th}).

(iv) sixth (6^{th}).



(b)

(d) (2 pts) What are the directions of each wave along the string, forming the pattern in figure (a), relative to each other?

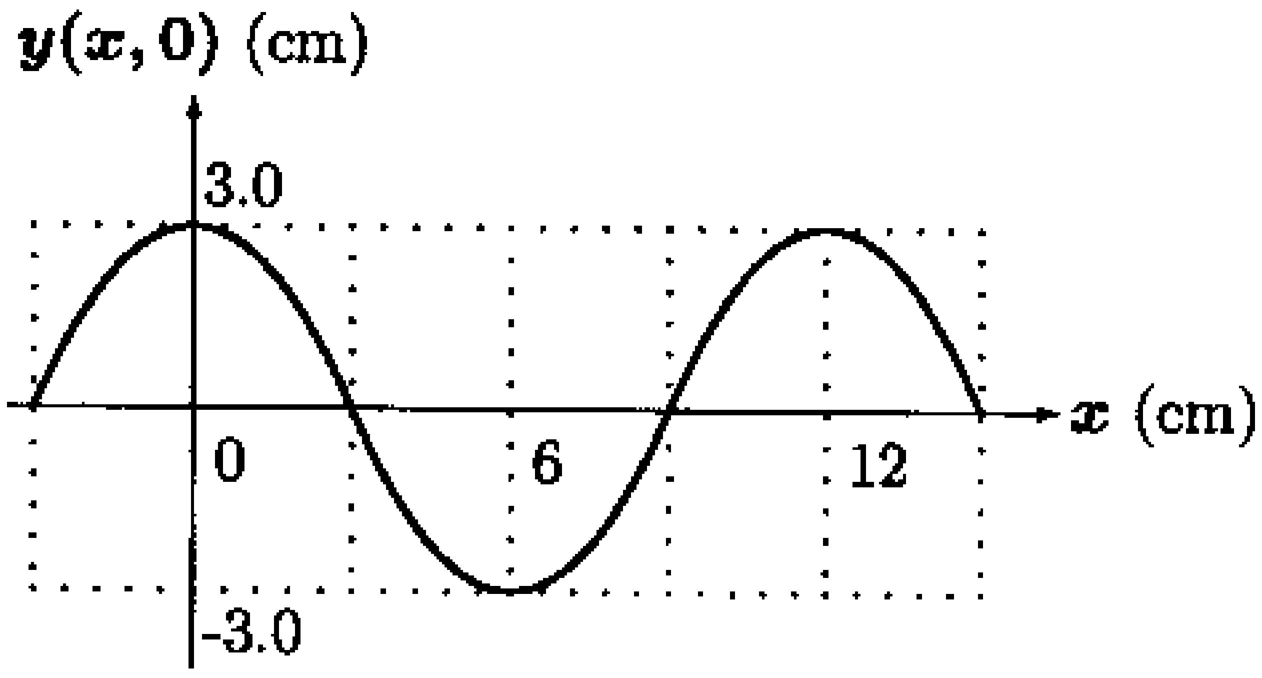
(i) The same directions.

(ii) Opposite directions.

(iii) Not enough info to tell.

Problem 3 (22 points)

The figure shows a snapshot of a sinusoidal transverse wave of period of 2.0 s, at $t = 0$, that travels along a string in the positive direction of the x axis. The wave equation is of the form $y(x,t) = y_m \sin(kx - \omega t + \phi)$.



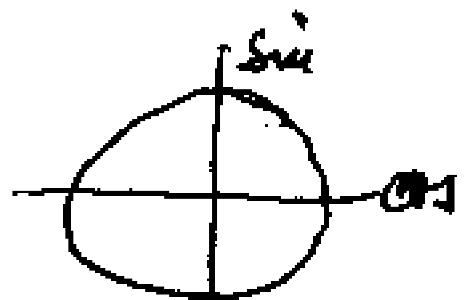
$|\leftarrow \lambda \rightarrow|$ $\lambda = 12\,\text{cm}$

(a) (4 pts) Calculate the angular wave number k.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{12\,\text{cm}} = 0.524\,\text{rad/cm} = 52.4\,\frac{\text{rad}}{\text{m}}$$

(b) (6 pts) Calculate the speed v of this wave.

$$v = \frac{\omega}{k} = \frac{2\pi}{Tk} = \frac{2\pi}{(2.0\,\text{s})(52.4\,\frac{\text{rad}}{\text{m}})} = \underline{\underline{0.06\,\text{m/s}}} = 6\,\text{cm/s}$$



(c) (5 pts) Calculate the phase constant ϕ.

$$\left.\begin{array}{l} y_m = 3\,\text{cm} \\ y(0,0) = 3\,\text{cm} \end{array}\right\} \quad y(0,0) = y_m \sin\phi \Rightarrow \sin\phi = \frac{y(0,0)}{y_m} = \frac{3}{3} = 1 \Rightarrow \phi = \frac{\pi}{2}, \frac{5\pi}{2}, \ldots$$

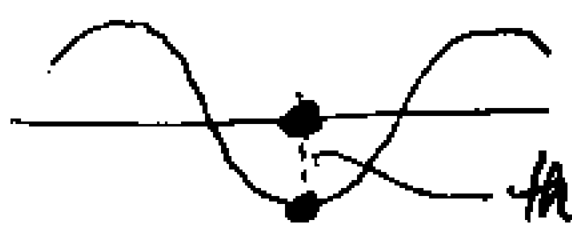
$\text{slope}(0,0) = 0$

$$\text{slope}(x,t) = \frac{\partial y}{\partial x} = k y_m \cos(kx - \omega t + \phi)$$

$$\left.\begin{array}{l} \text{slope}(0,0) = k y_m \cos\phi \\ \text{using slope}(0,0) = 0 \end{array}\right\} \phi = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$$

$$\boxed{\phi = \frac{\pi}{2}}$$

(d) (2 pts) After how much time will the string element at $x = 6$ cm first reach the position $y = 0$ cm?



the time for that is $t = \frac{T}{4} = \frac{2.0\,\text{s}}{4} = \underline{\underline{0.5\,\text{s}}}$

(e) (3 pts) Calculate the maximum transverse speed of the string element at $x = 6$ cm.

$$u_x = y_m \omega = (3.0\,\text{cm})\frac{2\pi}{2.0\,\text{s}} = 9.4\,\text{cm/s} = \underline{\underline{0.094\,\text{m/s}}}$$

$\omega = \frac{2\pi}{T}$

• regardless of the position along the string!