Physics 2101, Third Exam, Fall 2007

October 23, 2007

Name: KEY

Signature: _____

Section: (Circle one)

1 (Dr. Rupnik, MWF 7:40 AM)

2 (Dr. Rupnik, MWF 9:40 AM)

3 (Prof. González, MWF 2:40 PM)

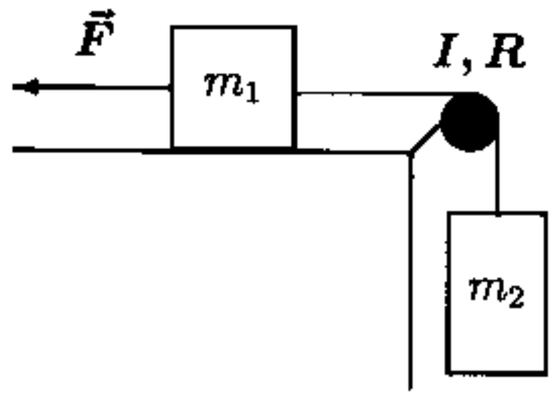
4 (Prof. Cherry, TTH 12:10 PM)

5 (Prof. González, TTh 9:10 AM)

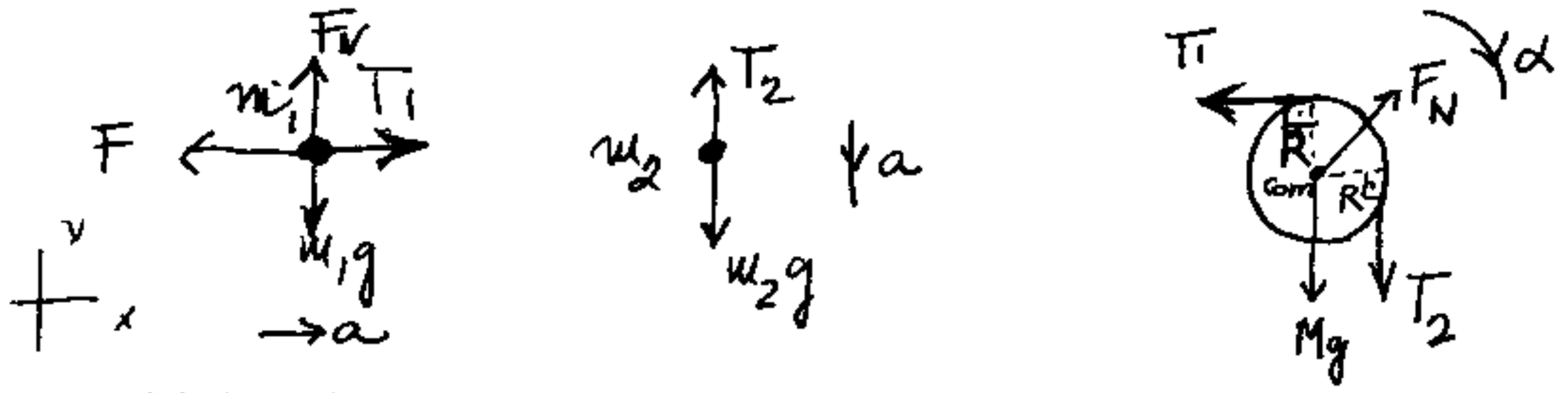
- Please be sure to print and sign your name and circle your section above.
- For the *problems*, you *must* show all your work. Let us know what you were thinking when you solved the problem! Lonely right answers will not receive full credit, lonely wrong answers will receive no credit.
- For the *questions*, no work needs to be shown (there is no partial credit).
- Please, carry units through your calculations when needed, lack of units will result in a loss of points.
- You may use scientific or graphing calculators, but you must derive your answer and explain your work.
- Feel free to detach, use, and keep the formula sheet. No other reference material is allowed during the exam.
- **GOOD LUCK!**

Problem 1 - 21 points

The figure shows two blocks, having masses m_1 and m_2, connected by a massless string over a pulley of radius R and rotational inertia I about its center of mass. In addition, a horizontal force F pulls on block m_1 and block m_2 has a downward acceleration $\vec{a}$.
The string does not slip on the pulley; there is no friction between the surface and block m_1; the pulley's axis is frictionless.



(a) (7 pts) Draw free-body diagrams for m_1, m_2, and the **pulley** in the space below.



(b) (6 pts) Apply second Newton's law for m_1, m_2, and the **pulley** for this situation and write the appropriate net force and net torque equations in terms of m_1, m_2, I, R, F, g, and numerical constants, as needed.

m_1:
$\Sigma F_x = m_1 a$
$\boxed{T_1 - F = m_1 a}$ (1)
$\Sigma F = 0$
$m_1 g = F_N$

m_2:
$\Sigma F = m_2 a$
$\boxed{m_2 g - T_2 = m_2 a}$ (2)

pulley:
$\Sigma \tau_{com} = I_{com}\alpha$ where $\alpha = \frac{a}{R}$
$(T_2 - T_1)R = I\frac{a}{R}$ $/:R$
$\boxed{T_2 - T_1 = I\frac{a}{R^2}}$ (3)

(c) (6 pts) Find the magnitude of the acceleration $|\vec{a}|$. Express it in terms of m_1, m_2, I, R, F, g, and numerical constants, as needed.

To find a we'll sum all three equations: (1)+(2)+(3)

$$m_2 g - F = \left(m_1 + m_2 + \frac{I}{R^2}\right)a$$

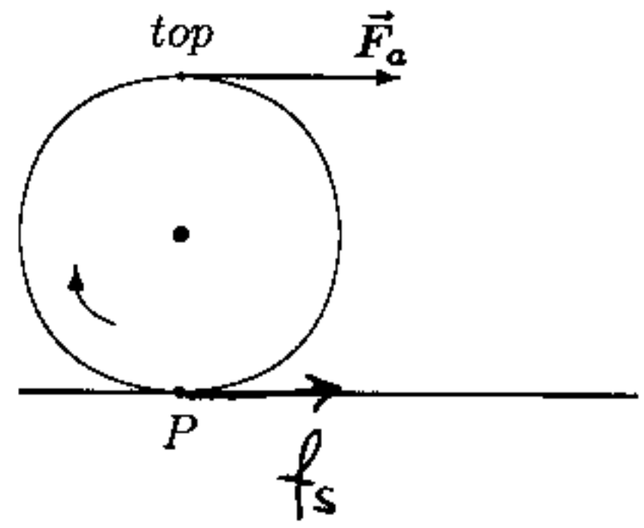
$$\boxed{a = \frac{m_2 g - F}{m_1 + m_2 + \frac{I}{R^2}}}$$

(c) (2 pts) What is the relation between the magnitude of the tension in the cord to the left of the pulley, $|\vec{T}_1|$, and below the pulley, $|\vec{T}_2|$? *Circle the right answer.*

(a) $|\vec{T}_1| > |\vec{T}_2|$ (b) $|\vec{T}_1| = |\vec{T}_2|$ (c) $|\vec{T}_1| < |\vec{T}_2|$

• check equation (3)

Question 1 - 11 points



In the figure, a constant horizontal force $\vec{F}_a$ is applied to a rim of a disk, by a fishing line. The disk rolls smoothly (without slipping) along the horizontal surface.

(a) (3 pts) The speed of the point P is

(i) 0.

(ii) v_{com}.

(iii) $2v_{com}$.

(iv) $v_{com}/2$.

(b) (4 pts) The speed of the point at the top is

(i) 0.

(ii) v_{com}.

(iii) $2v_{com}$.

(iv) $v_{com}/2$.

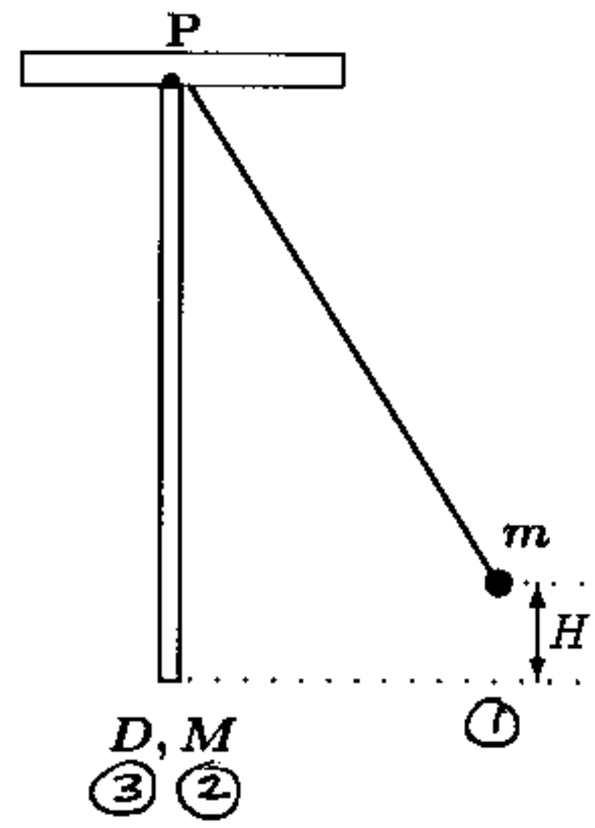
(c) (4 pts) The force of static friction is in the direction

(i) that is the same as the direction of $\vec{F}_a$.

(ii) that is opposite to the direction of $\vec{F}_a$.

(iii) that cannot be determined from the data given.

Problem 2 (23 points)



A thin rod of length $D = 1.2\,\text{m}$ and mass $M = 3.8\,\text{kg}$ is suspended from one end as indicated in the figure. A pendulum with a massless string of the same length D and with the bob's mass $m = 0.10\,\text{kg}$ is pulled to the side so that its center of mass is $H = 0.20\,\text{m}$ higher than its lowest position and then released from rest. At its lowest point the pendulum bob hits the rod. Just after the collision the bob moves backwards with a speed of $v_f = 2.0\,\text{m/s}$. $= v_3$

(a) (3 pts) What is the rotational inertia of the rod about the rotational axis at one end of the rod?

$$I_p = I_{com} + M\left(\frac{D}{2}\right)^2 = \frac{MD^2}{12} + \frac{MD^2}{4} = \frac{MD^2}{3} = \frac{(3.8\,\text{kg})(1.2\,\text{m})^2}{3}$$

$$I_p = 1.824 \simeq 1.82\ \text{kg m}^2$$

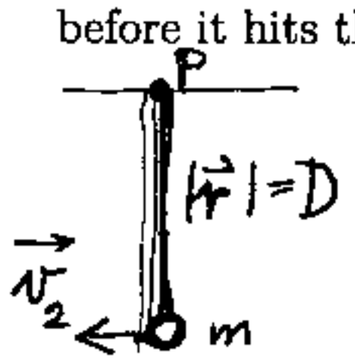
(b) (3 pts) Calculate the bob's speed just before it hits the rod.

① → ② mechanical energy is conserved

$$(\cancel{K}^{0} + U)_1 = (K + \cancel{U}^{0})_2$$

$$mgH = \frac{1}{2}mv^2 \Rightarrow v_2 = \sqrt{2gH} = \sqrt{2(9.8\,\text{m/s}^2)(0.20\,\text{m})} = \underline{\underline{1.98\ \text{m/s}}}$$

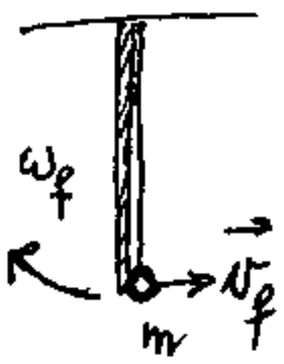
(c) (5 pts) What is the bob's angular momentum about point **P** (magnitude and direction) just before it hits the rod?



$$L_{bob} = |\vec{r}||\vec{v}_{bob}|m\sin\theta_{r,v} = Dv_2 m \sin 90^\circ = (1.2\,\text{m})(1.98\,\tfrac{\text{m}}{\text{s}})(0.10\,\text{kg})$$

$|\vec{L}_{bob}| = 0.238\ \text{kg m}^2/\text{s}$, direction $\equiv$ CW, into the page, or negative

(d) (10 pts) Calculate the rod's angular speed, ω_f, just after the collision.



$\sum \vec{L}_i = \sum \vec{L}_f$ ⇐ ② → ③ collision within an isolated system

$$\underset{CW}{\vec{L}_{bob,i}} = \underset{CW}{\vec{L}_{rod,f}} + \underset{CCW}{\vec{L}_{bob,f}} \Rightarrow |\vec{L}_{bob,i}| = |\vec{L}_{rod,f}| - |\vec{L}_{bob,f}|$$

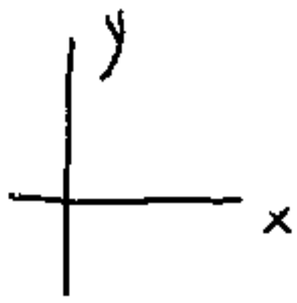
where $|\vec{L}_{bob,f}| = Dv_f m$

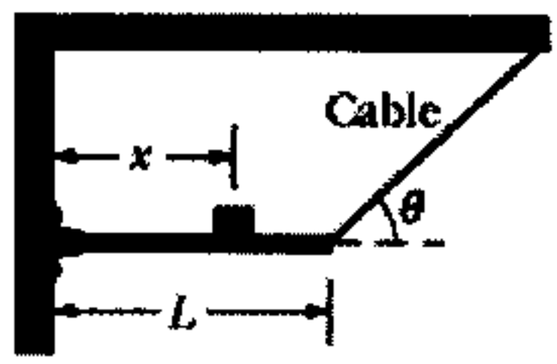
$$Dv_2 m = I_p\omega_f - Dv_f m$$

$$\omega_f = \frac{mD(v_2 + v_f)}{I_p} = \frac{(0.10\,\text{kg})(1.2\,\text{m})(1.98 + 2)\,\text{m/s}}{1.82\ \text{kg m}^2} = \underline{\underline{0.26\ \text{rad/s}}}$$

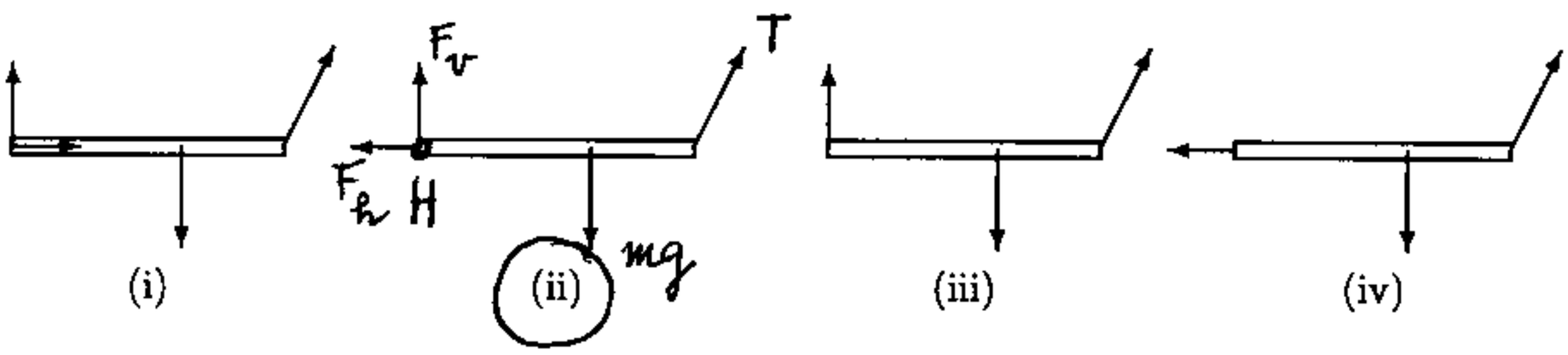
Question 2 - 11 points

In the figure, a massless horizontal beam of length L is hinged at its left end and attached to a cable at its right end. A package of mass m is positioned on the beam at a distance x from the left end. The angle between the cable and horizontal is θ.





(a) (3 pts) Circle the free body diagram that most appropriately represents the beam.



If we move the package leftward, keeping the beam horizontal and in equilibrium, do the following physical quantities increase, decrease, or stay the same? *Circle the right answer.*

(b) (4 pts) The tension in the cable

$\tau_{mg,H} = x\,mg$ ... becomes smaller, (CW)

$\tau_{T,H} = L\,T\sin\theta$ ... needs to decrease to keep the balance (CCW)

increases decreases stays the same

(c) (4 pts) The vertical component of the force in the hinge

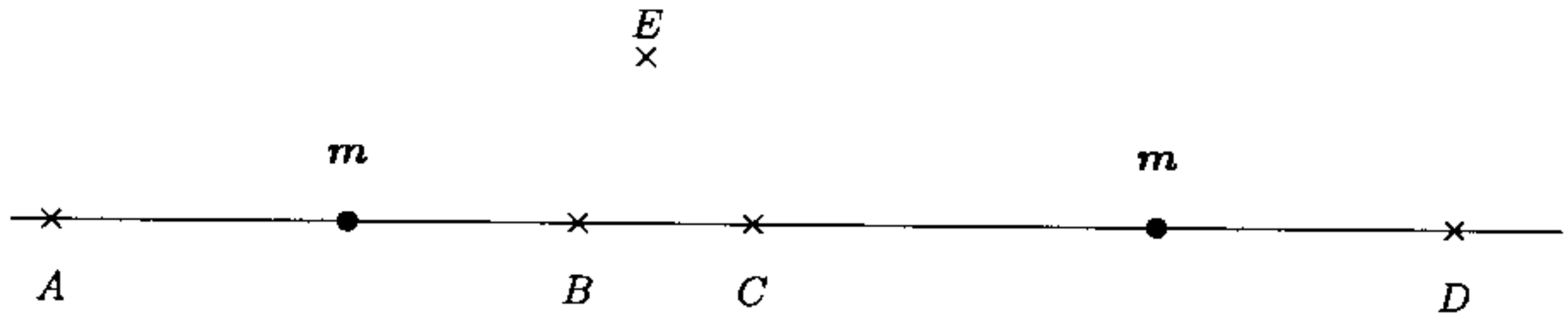
increases decreases stays the same

$\Sigma F_y = 0 \quad \begin{cases} F_v + T\sin\theta - mg = 0 \\ F_v = mg - T\sin\theta \end{cases}$ ... mg is the same and T decreases

or: closer the package is to the hinge, hinge has to support more and cable less

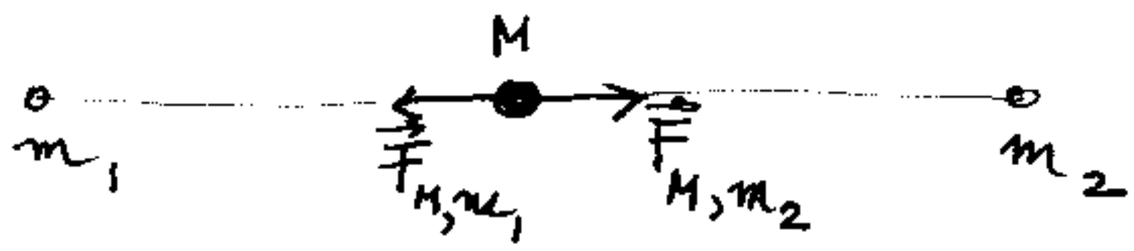
Question 3 - 12 points

Two particles of equal mass m are shown in the figure. Consider bringing a third particle of mass M to one of the five positions shown, A, B, C, D, or E.
Circle the correct answer to the questions below.



(a) (4 pts) The net force on particle M due to the other two particles of mass m is zero if it is positioned at the point

A B C D E



and $\vec{F}_{Mm_1} = -\vec{F}_{Mm_2}$ so they cancel each other

(b) (4 pts) Assuming zero gravitational potential energy when the particles are infinitely apart, the gravitational potential energy of the three particle system is

positive negative zero

• each pair of particles has negative gravitational potential energy, so the sum will be negative, too

(c) (4 pts) Assuming the third particle is at rest in its initial and final positions, work done by you while bringing the third particle from infinity to its present position is

$\boxed{\Delta U_g = U_{fg} - U_{fi} < 0}$ because particles are closer together as the result

$\boxed{W_{you} = \Delta U_g}$ because $\Delta K = 0$

$\Rightarrow W_{you} < 0$

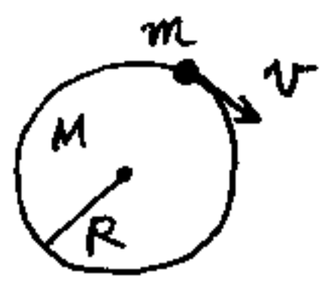
positive negative zero.

$U_{g,max} = 0$ ... when particles are infinitely apart

Problem 3 (22 points)

A planet has a mass of $M = 1.5 \times 10^{22}$ kg, a radius of $R = 3.0 \times 10^5$ m, and no atmosphere. A space probe of mass $m = 200$ kg is to be launched from its surface.

(a) (8 pts) If the probe is launched horizontally and is to stay in orbital motion with the radius equal to R, just above the surface of the planet, calculate the **horizontal** speed (orbital speed) needed for the probe.



uniform circular motion:

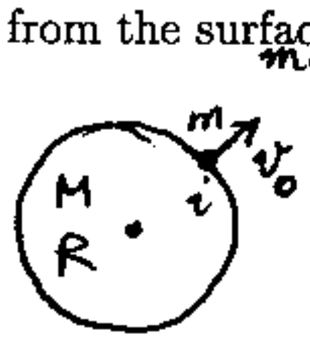
$$F_g = F_{cp} \Rightarrow G\frac{\cancel{m}M}{R^{\cancel{2}}} = \cancel{m}\frac{v^2}{R} \Rightarrow v = \sqrt{\frac{GM}{R}}$$

$$v = \sqrt{\frac{(6.67\times10^{-11}\,Nm^2/kg^2)(1.5\times10^{22}\,kg)}{3.0\times10^5\,m}} = 1.83\times10^3\,m/s = 1.83\,km/s$$

Kepler's law:

or: $T = \sqrt{\frac{4\pi^2}{GM}R^3}$ and $v = \frac{2\pi R}{T} = \frac{2\pi R}{\sqrt{4\pi^2R^3/GM}} = \sqrt{\frac{GM}{R}}$ ... the same as above

(b) (8 pts) If the probe is launched **vertically** from its surface and is to achieve a maximum distance of 8.0×10^5 m from the center of the planet, with what initial **speed** must the probe be launched from the surface of the planet?



m: $v_f = 0$

$r_i = R$

$r_f = 8.0\times10^5\,m$

• conservation of mechanical energy:

$$(K+U)_i = (\overset{0}{K}+U)_f$$

$$\frac{1}{2}\cancel{m}v_0^2 - G\frac{\cancel{m}M}{R} = -G\frac{\cancel{m}M}{r_f}$$

$$v_0 = \sqrt{2GM\left(\frac{1}{R}-\frac{1}{r_f}\right)} = \sqrt{2(6.67\times10^{-11}\frac{Nm^2}{kg^2})(1.5\times10^{22}\,kg)\left(\frac{1}{3\times10^5\,m}-\frac{1}{8\times10^5\,m}\right)}$$

$$v_0 = 2.04\times10^3\,m/s = 2.04\,km/s$$

(c) (6 pts) Calculate the minimum initial kinetic energy needed if the probe is going to move infinitely far away from the surface of the planet.

so that

$r_f = \infty$

$\left.\begin{array}{l} K_f = 0 \\ U_f = 0 \end{array}\right\} E_{mec_f} = 0$

$r_i = R$

$K_i + U_i = 0$

$$K_i = -U_i = G\frac{mM}{R} = (6.67\times10^{-11}\frac{Nm^2}{kg^2})\frac{(200\,kg)(1.5\times10^{22}\,kg)}{3\times10^5\,m}$$

$$K_i = 6.67\times10^8\,J$$