Name: KEY Instructor:

Louisiana State University Physics 2101, Exam 2,

September 25, 2007.

• Please be sure to write your name and class instructor above.

• The test consists of 3 questions (multiple choice, no partial credit), and 3 problems.

• For the problems: Show your reasoning and your work. Note that in many of the problems, you can do parts (b), (c) and (d) even if you get stuck on (a) or (b).

• You may use scientific or graphing calculators, but you must derive and explain your answer fully on paper so we can grade your work.

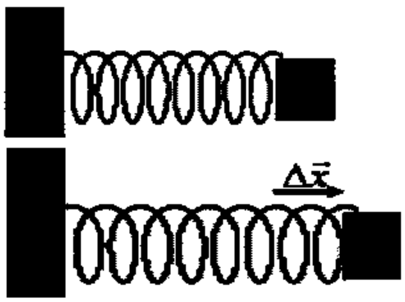
• Feel free to detach, use, and keep the formula sheet pages. No other reference material is allowed during the exam.

Good Luck!

Question 2 (d) point: (not points)

Problem 3: Text ~~while~~ the box m_2

Question 1 (11 points) A stationary block attached to a spring of spring constant K is displaced Δx by an applied force as shown in the figure.
Circle the correct answer below.



(b) (5 points) The spring force, when the block is displaced $\Delta\vec{x}$ from the relaxed position ~~and is stationary at the final position~~, is;

a. $K(\Delta\vec{x})$

b. $-K(\Delta\vec{x})$

c. $\frac{1}{2}K(\Delta\vec{x})$

d. $m\vec{g}$

e. $-\frac{1}{2}K(\Delta\vec{x})$

$$\boxed{\vec{F} = -k\,\Delta\vec{x}}$$

always opposite to the displacement from equilibrium

(b) (6 points) Work done by the applied force to displace the block from the relaxed position to $\Delta\vec{x}$, where the block is stationary at the final position, is;

a. $mg\Delta x$

b. $-mg\Delta x$

c. $\frac{1}{2}K(\Delta x)^2$

d. $K(\Delta x)^2$

e. $-\frac{1}{2}K(\Delta x)^2$

$$\left.\begin{array}{l} W = \Delta K = 0 \\ W = W_s + W_F \end{array}\right\} \quad W_F = -W_s = -\left(-\tfrac{1}{2}k\Delta x^2\right)$$

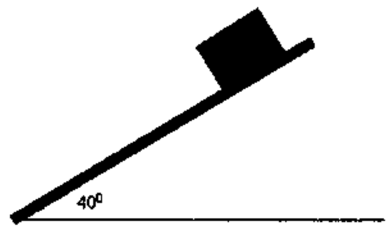
$$W_F = +\tfrac{1}{2}k\Delta x^2$$

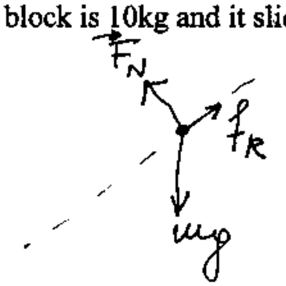
Problem 1 (22 points) In the following figure mass of the block is 10kg and it slides 4m down the ramp, starting from rest.

$m = 10kg$

$\ell = 4m$

$v_0 = 0$





(a)(11 points) What is the coefficient of friction, if the kinetic energy of the block at the bottom of the 4m ramp is 50J? $\overset{0}{\cancel{W}} = \Delta K + \Delta U + \Delta E_{th}$

ch 7: $W_g + W_{f_k} = \Delta K$

$$\Delta K = K_f - \overset{0}{\cancel{K_i}} = K_f = 50J$$

$$\Delta U_g = -mg\,\ell \sin\theta$$

$$\Delta E_{th} = f_k \ell = \mu_k mg\cos\theta\, \ell$$

$$0 = K_f - mg\ell\sin\theta + \mu_k mg\cos\theta\, \ell$$

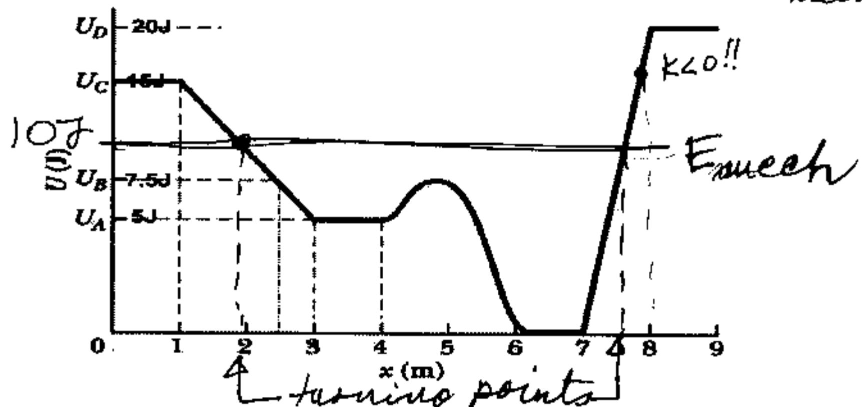
$$\mu_k = \frac{mg\ell\sin\theta - K_f}{\ell mg\cos\theta} = \tan\theta - \frac{K_f}{\ell mg\cos\theta} = \tan 40° - \frac{50J}{4(10)(9.8)\cos 40°}$$

$$\boxed{\mu_k = 0.672}$$

(b) (11 points) In addition, if an external force is pushing the block down and the kinetic energy of the block at the bottom of the ramp is 100J, find the work done by the external force. (Note: Friction is not zero.)

$$W = \Delta K \quad \text{where} \quad \underset{\text{net work}}{W} = \underbrace{W_g + W_{f_k}}_{\Delta K_{(a)} = 50J} + W_F = 100J \Rightarrow \boxed{W_F = 50J}$$

Question 2 (12 points) The following Figure shows a plot of potential energy U versus x of a 0.200 kg particle that can travel only along an x axis under the influence of a conservative force, where U_A=5J, U_B=7.5J, U_C=15J, and U_D=20J. The particle is released at the point x=2.5m where U= U_B, with kinetic energy 2.5 J. $\Rightarrow E_{mech} = K + U = 2.5 + 7.5 = 10 J$



(a) (4 pts) The particle stops momentarily at, approximately;

(i) x=2.5m and x=7.67m;

(ii) x=2.0m and x=7.5m; ... between the turning points

(iii) x=1.0m and x=8.0m;

(iv) the particle never stops.

(b) (4 pts) The particle has maximum kinetic energy at:

(i) x=6.5m

(ii) x=5m;

(iii) x=2.5m;

(iv) the particle has constant kinetic energy, so it has the same energy at all points.

$E_{mach} = K + U$

$K = K_{max}$ where $U = U_{min}$

(d) (4 pts) The particle is never found at the following points:

(i) x=2.5m

(ii) x=5m

(iii) x=7m

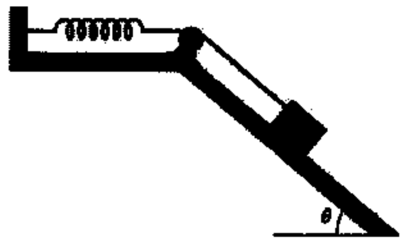
(iv) x=8m

→ beyond turning point

→ Kinetic energy would be negative, there, which is impossible

Problem 2 (22 points) A breadbox of mass M on a <u>frictionless</u> incline surface of angle Θ is connected, by a cord that runs over a pulley, to a light spring of spring constant K, as shown in the figure. The box is released from rest when the spring is unstretched. Assume that the pulley is massless and frictionless.

$v_0 = 0$



(a) (6 pts) Assume the breadbox has moved a distance "d" down the incline (when the spring is not maximally stretched). Write an expression for the speed of the box at that point, in terms of Θ, M, g, d, and K.

$\Delta K + \Delta U = 0$

$\Delta U = \Delta U_g + \Delta U_s$

$\frac{1}{2} m v_f^2 + \frac{1}{2} k d^2 - m g d \sin\theta = 0$

$$\boxed{v_f = \sqrt{2 g d \sin\theta - \frac{k d^2}{m}}}$$

$m = M$

(b) (6 pts) Write an expression, in terms of Θ, M, g and K, for the distance L down the incline that the mass M moves from its point of release to the point where the box <u>stops momentarily</u>.

$\left. \begin{array}{l} v_0 = 0 \\ v_f = 0 \end{array} \right\} \Delta K = 0$

$\Delta K = 0 \Rightarrow \Delta U = 0$

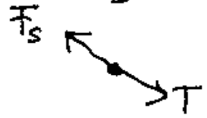
$\frac{1}{2} k L^2 = m g L \sin\theta$

$$\boxed{L = \frac{2 m g \sin\theta}{k}}$$

$m = M$

(c) (5 pts) Draw a free-body diagram showing all the forces on the box.

$|\vec{T}| = |\vec{F}_s| = |kL|$

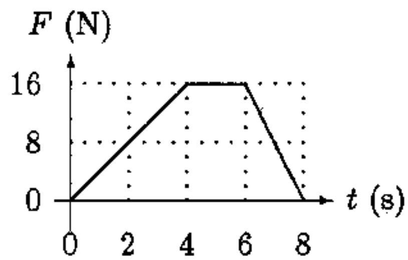


$$\boxed{F_{net} = |kL - Mg\sin\theta|}$$ up the incline

(d) (5 pts) Express the magnitude of the net force on the block M when it has momentarily stopped in term of Θ, M, g, L and K.

Question 3 (11 points) A 2.0 kg object is acted upon by a force in the x direction in a manner described by the graph shown. The object is initially at rest.

Circle the correct answer below.



$J = \int_{t_1}^{t_2} F\,dt \equiv$ area under the curve

(a) **(3 pts)** The impulse of the force on the object is (during time $0 \le t \le 8$)

(i) 128 N·s.
(ii) 96 N·s
(iii) 64 N·s
(iv) 80 N·s

$|\vec{J}| = \frac{4\times16}{2} + 2\times16 + \frac{2\times16}{2} = 32+32+16$

$|\vec{J}| = 80$ Ns

(b) **(4 pts)** The momentum acquired by the object in the first four seconds (t=0s to t=4s) is

$\Delta p = J_{0-4} = p_f - \cancel{p_i}^{\,0}$

$p_f = \frac{(4)(16)}{2} = 32$ Ns

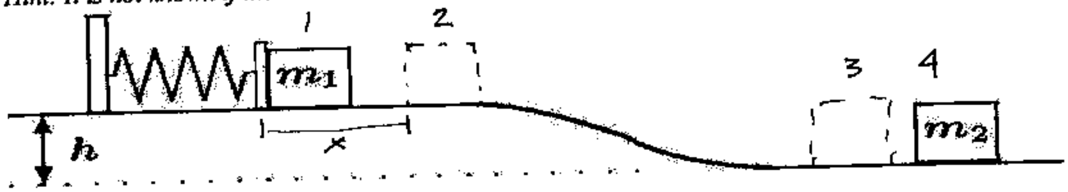
(i) 4 N·s.
(ii) 8 N·s
(iii) 32 N·s
(iv) 64 N·s

(c) **(4 pts)** What is the object doing between t=6s and t=8s?

(i) It is moving in positive x direction and speeding up.
(ii) It is moving in positive x direction and slowing down.
(iii) It is moving in negative x direction and speeding up.
(iv) It is moving in negative x direction and slowing down.

F is positive ⇒ direction of motion did not change
• motion started from rest

Problem 3 (22 points) A spring, with spring constant of 120 N/m, is fixed on a horizontal part of a track that is 2.00 m higher than the rest. A block with mass $m_1 = 0.30$ kg is placed against the spring, compressing it by $x = 0.40$ m. The block is not attached to the spring, so after the spring is released the block slides down the track and hits a box of mass $m_2 = 0.50$ kg that is at rest on the lower portion of the track. The track is frictionless. Just after the collision, [redacted] the box m_2 moves forward with a speed of 2.0 m/s. *Hint: It is not known if the collision is elastic!*



(a) (8 pts) Calculate the speed by which the block m_1 leaves the spring.

$(\cancel{K}^{0} + U_s)_i = (K + \cancel{U_s}^{0})_f$

$\frac{1}{2}kx^2 = \frac{1}{2}m_1 v_{12}^2$

$v_{12} = x\sqrt{\frac{k}{m_1}} = (0.4)\sqrt{\frac{120}{0.3}} = \underline{8.0 \text{ m/s}}$

· conservation of mechanical energy

(b) (7 pts) Calculate the speed of the block m_1 just before the collision.

$1 \to 3$: $(\cancel{K}^{0} + U_s + U_g)_i = (K + \cancel{U_s}^{0} + \cancel{U_g}^{0})_f$

$\frac{1}{2}kx^2 + m_1 g h = \frac{1}{2}m_1 v_{13}^2$

$v_{13} = \sqrt{2gh + \frac{kx^2}{m_1}}$

$= \sqrt{2(9.8)(2) + \frac{120(0.4)^2}{0.3}} = 10.16 \text{ m/s}$

OR $2 \to 3$

$(K+U)_2 = (K+U)_3$

$\frac{1}{2}m_1 v_{12}^2 + m_1 g h = \frac{1}{2}m_1 v_{13}^2$

$v_{13} = \sqrt{v_{12}^2 + 2gh} = \sqrt{64 + 2(9.8)(2)}$

$\underline{v_{13} = 10.16 \text{ m/s}}$

(c) (7 pts) Calculate the speed of the block m_1 just after the collision.

$\sum p_i = \sum p_f$

$m_1 v_{13} = m_1 v_{14} + m_2 v_{24}$

$v_{24} = 2 \text{ m/s}$

$v_{13} = 10.16 \text{ m/s}$

$v_{14} = \frac{m_1 v_{13} - m_2 v_{24}}{m_1} = v_{13} - \frac{m_2}{m_1} v_{24} = 10.16 - \frac{0.5}{0.3}(2) = +6.83 \text{ m/s}$

$\boxed{v_{14} = 6.83 \text{ m/s}}$

· conservation of linear momentum

Formula Sheet for LSU Physics 2101,
Test 2, September 25, Fall '07.

Quadratic formula: for $ax^2 + bx + c = 0$, $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Dot Product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = |\vec{a}| \left|\vec{b}\right| \cos(\phi)$ (ϕ is smaller angle between $\vec{a}$ and $\vec{b}$)

Cross Product: $\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$, $\left|\vec{a} \times \vec{b}\right| = |\vec{a}| \left|\vec{b}\right| \sin(\phi)$

Equations of Constant Acceleration:

linear equation	missing
$v = v_o + at$	$x - x_o$
$x - x_o = v_o t + \frac{1}{2} a t^2$	v
$v^2 = v_o^2 + 2a(x - x_o)$	t
$x - x_o = \frac{1}{2}(v_o + v)t$	a
$x - x_o = vt - \frac{1}{2} a t^2$	v_o

Vector Equations of Motion for Constant Acceleration: $\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2}\vec{a} t^2$, $\vec{v} = \vec{v}_o + \vec{a} t$

Projectile Motion:

$x = v_{ox} t$ $\quad y = v_{oy} t - \frac{1}{2} g t^2 \quad R = \dfrac{v_o^2 \sin(2\theta_o)}{g}$

$v_x = v_{ox} = \text{constant}$ $\quad v_y = v_{oy} - gt$

Force of Friction: Static: $f_s \leq f_{s,max} = \mu_s F_N$, Kinetic: $f_k = \mu_k F_N$

Elastic (Spring) Force: Hooke's Law $F = -kx$ (k = spring (force) constant)

Kinetic Energy (nonrelativistic): Translational: $K = \frac{1}{2} m v^2$ Rotational: $K = \frac{1}{2} I \omega^2$

Work:
$W = \vec{F} \cdot \vec{d}$ (const. force), $W = \int_{x_i}^{x_f} F(x)dx$ (variable 1D force), $W = \int_{r_i}^{r_f} \vec{F}(\vec{r}) \cdot d\vec{r}$ (variable 3D force)

Work - Kinetic Energy Theorem: $W = \Delta K = K_f - K_i$

Work done by weight (gravity close to the Earth surface): $W = m\,\vec{g} \cdot \vec{d}$

Work done by spring force F = - k x: $W = -k \int_{x_i}^{x_f} x\, dx = -k \left(\frac{x_f^2}{2} - \frac{x_i^2}{2} \right)$

Power:
Average: $P_{avg} = \dfrac{W}{\Delta t}$, $P = \vec{F} \cdot \vec{v}_{avg}$ (const. force) Instantaneous: $P = \dfrac{dW}{dt}$, $P = \vec{F} \cdot \vec{v}$ (const. force)

Potential Energy Change: $\Delta U = -W$ (conservative force)

Potential-Force Relation: $F(x) = -\dfrac{dU(x)}{dx}$

Gravitational (near Earth) Potential Energy: $U(y) = mgy$ (at the height y)

Elastic (Spring) Potential Energy: $U = \dfrac{1}{2}kx^2$ (relative to the relaxed spring)

Conservation of Energy: $W = \Delta K + \Delta U + \Delta E_{th} + \Delta E\text{int}$, where W is the *external* work done on the system, and $\Delta E_{th} = f_k d$.

Center of Mass: $M = \sum_{i=1}^{N} m_i$, $x_{com} = \dfrac{1}{M}\sum_{i=1}^{N} m_i x_i$, $y_{com} = \dfrac{1}{M}\sum_{i=1}^{N} m_i y_i$, $z_{com} = \dfrac{1}{M}\sum_{i=1}^{N} m_i z_i$

$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i \qquad \vec{v}_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{v}_i \qquad \vec{a}_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{a}_i = \frac{1}{M}\sum_{i=1}^{N} \vec{F}_i$$

Definition of Linear Momentum: one particle: $\vec{p} = m\vec{v}$, system of particles: $\vec{P} = \sum_{i=1}^{N} \vec{p}_i = M\vec{v}_{com}$

Newton's 2nd Law for a System of Particles: $\vec{F}_{net} = M\vec{a}_{com} = \dfrac{d\vec{P}}{dt}$

Conservation of Linear Momentum of an Isolated System: $\sum \vec{p}_i = \sum \vec{p}_f$

Impulse - Linear Momentum Theorem (one dimension): $\Delta\vec{p}_1 = \vec{J}_{12} = \int_{t_1}^{t_2} \vec{F}_{12}(t)dt = \vec{F}_{avg,12}\Delta t$

Elastic Collision (1 Dim): $v_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2}v_{1i} + \dfrac{2m_2}{m_1 + m_2}v_{2i}$, $v_{2f} = \dfrac{2m_1}{m_1 + m_2}v_{1i} + \dfrac{m_2 - m_1}{m_1 + m_2}v_{2i}$