Physics 2101, First Exam, Fall 2007

September 4, 2007

- Please turn OFF your cell phone and MP3 player!
- Write down your name and section number in the scantron form.
- Make sure to mark your answers in the scantron form.
- You can mark your answers in the exam form too, and keep it for reference. We will show the correct answers at the end of the exam, so you can know the grade you will receive.
- Feel free to detach, use and keep the formula sheet. No other reference material is allowed during the exam.
- You may use scientific or graphing calculators.

1. Consider two vectors, $\vec{A} = (2\text{m})\hat{\imath} + (6\text{m})\hat{\jmath} - (3\text{m})\hat{k}$, and $\vec{B} = (4\text{m})\hat{\imath} + (2\text{m})\hat{\jmath} + (1\text{m})\hat{k}$. The product $\vec{A} \cdot \vec{B}$ is...

   (a) $(8\text{m})\hat{\imath} + (12\text{m})\hat{\jmath} - (3\text{m})\hat{k}$
   (b) $(12\text{m})\hat{\imath} - (14\text{m})\hat{\jmath} - (20\text{m})\hat{k}$
   (c) 23m^2
   (d) $17\text{m}^2 \Longleftarrow$
   (e) None of these.

   The scalar product is $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = (2\text{m})(4\text{m}) + (6\text{m})(2\text{m}) + (-3\text{m})(1\text{m}) = 8\text{m}^2 + 12\text{m}^2 - 3\text{m}^2 = 17\text{m}^2$.

2. The value of $\hat{\imath} \cdot (\hat{\jmath} \times \hat{k})$ is:

   (a) 0
   (b) $+1 \Longleftarrow$
   (c) -1
   (d) 3
   (e) $\sqrt{3}$

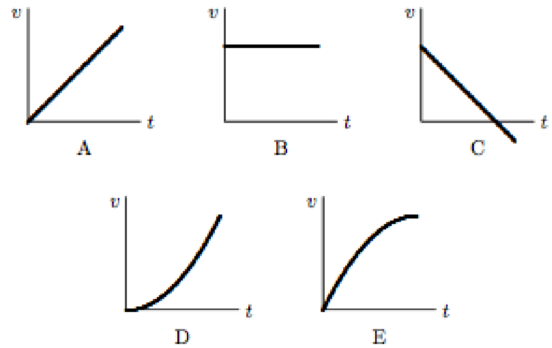
   $\hat{\imath} \cdot (\hat{\jmath} \times \hat{k}) = \hat{\imath} \cdot \hat{\imath} = +1$

3. The coordinate of a particle as a function of time is given by $x(t) = (18\text{m/s})\text{t} - (3.0\text{m/s}^2)\text{t}^2$. The particle is momentarily at rest at $t =$...

   (a) 0
   (b) 2s
   (c) 3s $\Longleftarrow$
   (d) 6s
   (e) 12s

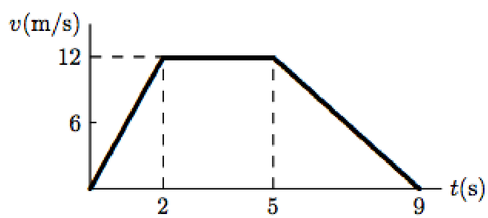
   The particle is momentarily at rest when its velocity is zero. We get the velocity as a function of time from the derivative of the position: $v(t) = 18\text{m/s} - 6\text{m/s}^2 t$. *The velocity is zero when* $t = (18\text{m/s})/(6\text{m/s}^2) = 3\text{s}$.

4. An object is thrown vertically into the air. Which of the following graphs represents the velocity v of the object as a function of the time t? The positive direction is taken to be upward.



An object thrown vertically into the air will have constant negative acceleration $a = -g$, so the velocity will be a linear function of time with negative slope. It will also have a positive value at $t = 0$. The only graph representing such velocity is (c).

5. The graph represents the straight line motion of a car. How far does the car travel between t = 2 s and t = 5 s?



(a) 4m
(b) 12m
(c) 24m
(d) 36m $\Longleftarrow$
(e) 60m

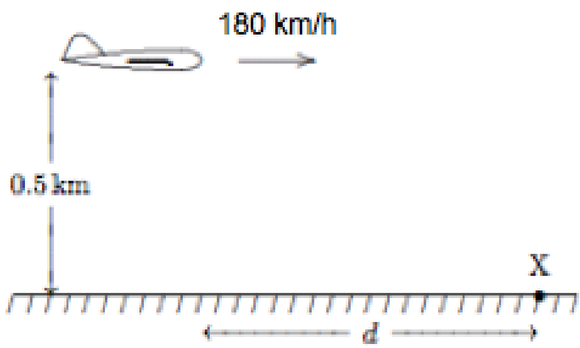
Since $\Delta x = \int v(t)dt$, the distance traveled can be obtained as the area under the curve between $t = 2s$ and $t = 5s$. The area of the rectangle is $\Delta x = 3\text{s} \times 12\text{m/s} = 36\text{m}$.

6. A bullet shot horizontally from a gun:

   (a) never strikes the ground
   (b) travels in a straight line
   (c) strikes the ground much later than one dropped vertically from the same point at the same instant
   (d) strikes the ground much sooner than one dropped from the same point at the same instant
   (e) strikes the ground at approximately the same time as one dropped vertically from the same point at the same instant $\Longleftarrow$

   A bullet shot horizontally will travel with constant vertical acceleration pointing down due to gravity, and horizontal initial velocity, so the vertical component of the initial velocity v_{0y} is zero. The vertical motion will be the same as any other projectile with $v_{0y} = 0$, so the bullet strikes the ground at approximately the same time as one dropped vertically from the same point at the same instant. (The air resistance may change the times slightly).

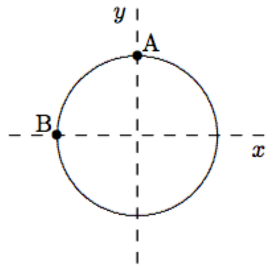
7. The airplane shown is in level flight at an altitude of 0.50 km and a speed of 180 km/h. At what distance d should it release an aid package so it falls at the target location X? Take g = 10m/s^2 .



   (a) 90 m
   (b) 180m
   (c) 360 m
   (d) 500 m $\Longleftarrow$
   (e) 18,000 m

   The horizontal motion of the package will have no acceleration, so it has constant velocity, equal to the horizontal component of the initial velocity. The initial velocity is that of the plane, $v_{0x} = 180\text{km/h}$. Taking the origin of a of a coordinate system at the ground, below the position where the package is dropped, the horizontal displacement is $x(t) = v_{0x}t$, and the vertical distance will be $y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2$, with $y_0 = 0.5\text{km}$ and $v_{0y} = 0$. The pilot wants the horizontal distance to be equal to d at the time the vertical coordinate is zero, which is $t_f = \sqrt{2y_0/g} = \sqrt{2 \times 500\text{m}/10\text{m/s}^2} = 10\text{s}$. At that time the horizontal distance is $d = 180\text{km/h} \times 10\text{s} = (180,000\text{m}/3600\text{s}) \times (10\text{s}) =$ 500m.

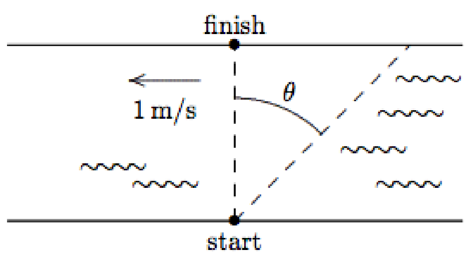
8. A toy racing car moves with constant speed around the circle shown below. When it is at point A its coordinates are x = 0, y = 3 m and its velocity is $\vec{v} = (6\mathrm{m/s})\hat{\imath}$. When it is at point B its velocity and acceleration are:



(a) $\vec{v} = (6\mathrm{m/s})\hat{\jmath}$, $\vec{a} = (12\mathrm{m/s^2})\hat{\imath}$ $\Longleftarrow$
(b) $\vec{v} = (6\mathrm{m/s})\hat{\imath}$, $\vec{a} = (-12\mathrm{m/s^2})\hat{\imath}$
(c) $\vec{v} = (-6\mathrm{m/s})\hat{\jmath}$, $\vec{a} = (12\mathrm{m/s^2})\hat{\imath}$
(d) $\vec{v} = (6\mathrm{m/s})\hat{\imath}$, $\vec{a} = (12\mathrm{m/s^2})\hat{\jmath}$
(e) $\vec{v} = (6\mathrm{m/s})\hat{\jmath}$, $\vec{a} = 0$

Since at point A, the velocity is $\vec{v_A} = (6\mathrm{m/s})\hat{\imath}$ and points towarsd the right, we know the toy car is moving clockwise around the circle. Since the motion has constant speed, the velocity at point B will have the same magnitude than at A, and it will point up: $\vec{v}_B = (6\mathrm{m/s})\hat{\jmath}$. From the coordinate at point A, we know the radius o the circle is r=3m. The magnitude of the acceleration is then $a = v^2/r = (6\mathrm{m/s})^2/3\mathrm{m} = 12\mathrm{m/s^2}$. The acceleration vector always points towards the center of the circle, so at B it will point towards the right: $\vec{a}_B = 12\mathrm{m/s^2}\hat{\imath}$.

9. A girl wishes to swim across a river to a point directly opposite as shown. She can swim at 2 m/s in still water and the river is flowing at 1 m/s. At what angle θ with respect to the line joining the starting and finishing points should she swim?



(a) 30° $\Longleftarrow$
(b) 45°
(c) 60°
(d) 90°
(e) It depends on the width of he river.

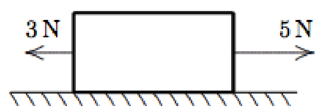
The motion of the girl can be thought of her swimming at an angle θ in a still river at a speed v =2m/s, and then the river taking her in a horizontal direction at v_r =1m/s to the finish point. Therefore, $v \sin\theta = v_r$, and $\sin\theta = v_r/v = 0.5$, so $\theta = 30°$.

10. Two objects, one having twice the mass of the other, are dropped from the same height. Neglecting air resistance, which of the following statements is true?

    (a) The lighter object reaches the ground first.
    (b) The heavier object reaches the ground first.
    (c) Both objects reach the ground with the same velocity. $\Longleftarrow$
    (d) The heavier object falls with a larger acceleration.
    (e) Both objects fall with the same, constant, velocity.

    The vertical position of an object in free fall is determined by the acceleration, the initial vertical velocity, and the initial height. Thus, two objects dropped from the same height will reach the ground at the same time and with the same velocity, independent of their masses. This is what presumably Galileo proved dropping objects from the Tower of Pisa.

11. The block shown moves with constant velocity on a horizontal surface. Two of the forces on it are shown. A frictional force exerted by the surface is the only other horizontal force on the block. The frictional force is:



    (a) 0
    (b) 2 N, leftward $\Longleftarrow$
    (c) 2 N, rightward
    (d) slightly more than 2 N, leftward
    (e) slightly less than 2 N, leftward

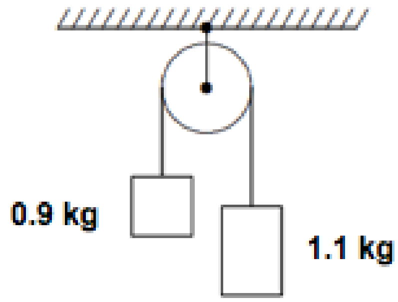
    Since the block moves with constant velocity, its acceleration is zero. Since the acceleration is zero, the net force must be zero too. We know that friction must be then a force of magnitude 2N, pointing to the left. The object is moving to the right.

12. A constant force of 8.0 N is exerted for 4.0 s on a 16-kg object initially at rest. If no other forces are acting on the object, the change in speed of this object will be:

    (a) 0.5 m/s
    (b) 2 m/s $\Longleftarrow$
    (c) 4 m/s
    (d) 8 m/s
    (e) 32 m/s

    The acceleration of the the object is $a = F/m = 8.0\text{N}/16\text{kg} = 0.5\text{m/s}^2$. *The velocity of the object at* $t = 4s$ *is* $v = at = 0.5\text{m/s}^2 \times 4.0\text{s} = 2.0\text{m/s}$.

13. Two blocks are connected by a string and pulley as shown. Assuming that the string and pulley are massless, and that the acceleration of gravity is $g = 10\text{m/s}^2$, the magnitude of the acceleration of each block is:



(a) 0.05m/s^2

(b) 0.02m/s^2

(c) 0.01m/s^2

(d) 0.5m/s^2

(e) 1m/s^2 $\Longleftarrow$

The pulley will rotate clockwise, with the right, heavier, block moving down and the left,lighter, block moving up. The acceleration of both blocks has the same magnitude, a, down on the right block and up on the left block. Newton's law on the left block is $T - m_1 g = m_1 a$, *and on the right block is* $m_2 g - T = m_2 a$. *Adding up both equations, we obtain* $(m_2 - m_1)g = (m_1 + m_1)a$, *or* $a = g(m_2 - m_1)/(m_2 + m_1) = (10\text{m/s}^2)(0.2\text{kg}/2.0\text{kg}) = 1\text{m/s}^2$.

14. A 32-N force, parallel to the incline, is required to push a certain crate at constant velocity up a frictionless incline that is 30° above the horizontal. The mass of the crate is:

(a) 3.3 kg

(b) 3.8 kg

(c) 5.7 kg

(d) 6.5 kg $\Longleftarrow$

(e) 160 kg

Since the crate is moving at a constant velocity, its acceleration is zero, and the net force must be zero. The components of the forces along the plane are $F - mg\sin\theta = 0$, *so* $m = F/(g\sin\theta) = (32\text{N})/(9.8\text{m/s}^2)(0.5) = 6.5\text{kg}$

15. An object rests on a horizontal frictionless surface. A horizontal force of magnitude F is applied. This force produces an acceleration:

    (a) only if F is larger than the weight of the object
    (b) only while the ob ject suddenly changes from rest to motion
    (c) always $\Longleftarrow$
    (d) only if the inertia of the ob ject decreases
    (e) only if F is increasing

    The weight and the normal force are both in the vertical direction; if there is no friction a force F would be the only force with a horizontal component, and thus it will always produce an acceleration.

16. A ball with a weight of 1.5 N is thrown at an angle of 30° above the horizontal with an initial speed of 12 m/s. At its highest point, the net force on the ball is:

    (a) 9.8 N, 30° below horizontal
    (b) zero
    (c) 9.8N, up
    (d) 9.8 N, down
    (e) 1.5 N, down $\Longleftarrow$

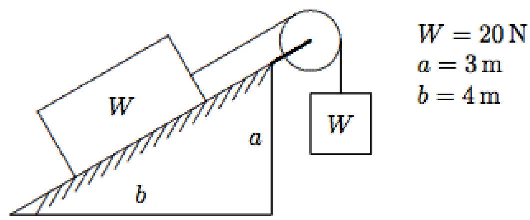
    The only force on the ball is the gravitational force, which is constant, has magnitude mg =1.5 N, and points down.

17. A brick slides on a horizontal surface. Which of the following will increase the magnitude of the frictional force on it?

    (a) Putting a second brick on top $\Longleftarrow$
    (b) Decreasing the surface area of contact
    (c) Increasing the surface area of contact
    (d) Decreasing the mass of the brick
    (e) None of the above

    Since the block is moving, the frictional force will be kinetic friction, and will have magnitude $f_k = \mu_k F_N$, independent of the surface area of contact. The normal force will be equal to the weight, so putting a second brick on top of the first will increase the weight and thus the frictional force.

18. The system shown remains at rest. Each block weighs 20 N. The force of friction on the upper block (the one on the incline) is:



(a) 4N
(b) 8N $\Longleftarrow$
(c) 12N
(d) 16N
(e) 20N

The system is in equilibrium, so the net force is zero. If there were no friction, the hanging block would fall down, so the static friction force f_s must be holding the system pointing downwards the incline.

The components of the force along the incline are $T - W\sin\theta - f_s = 0$, so $f_s = T - W\sin\theta$.

The angle of the incline is $\theta = \tan^{-1} a/b$, and $\sin\theta = a/(a^2 + b^2) = 3/5$.

The force equation on the hanging block is $T - W = 0$, so we know $T =$20 N.

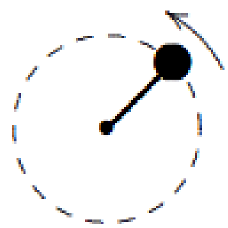
Thus, $f_s = T - W\sin\theta = 20\text{N} - (3/5)20\text{N} = 8\text{N}$.

19. An object moves around a circle with constant speed. If the radius is doubled keeping the speed the same then the magnitude of the centripetal force must be:

(a) twice as great
(b) half as great $\Longleftarrow$
(c) four times as great
(d) one-fourth as great
(e) the same

The force that produces uniform circular motion is $F = ma = mv^2/R$. Doubling the radius R, and keeping the mass and speed constant, will thus need a force half as great.

20. The iron ball shown is being swung in a vertical circle at the end of a 0.7-m long string. How slowly can the ball go through its top position without having the string go slack?



(a) 1.3 m/s

(b) 2.6 m/s $\Longleftarrow$

(c) 3.9 m/s

(d) 6.9 m/s

(e) 9.8 m/s

Assuming the string is not slack, the forces at the top will be the weight (down) and the tension (down). The net force $F = mg + T$ *is equal to the centripetal force producing the circular motion, and it is then equal to the mass times the centripetal acceleration:* $F = ma = mv^2/r$. *The tension can be deduced from* $mg + T = mv^2/R$, *or* $T = m(v^2/R - g)$. *If the ball is moving too slowly, we obtain a negative number, which is not physical, since th string cannot push up, it can only pull down at the top. The minimum speed to obtain a positive answer for tension is then* $v = \sqrt{gR} = \sqrt{(9.8\mathrm{m/s^2})(0.7\mathrm{m})} = 2.6\mathrm{m/s}$.