Physics 2101, Final Exam, Spring 2006

May 11, 2006

Name : ____

Section: (Circle one)

1 (Rupnik, MWF 7:40am)

2 (Rupnik, MWF 9:40am)

3 (Rupnik, MWF 11:40am)

4 (Kirk, MWF 2:40pm)

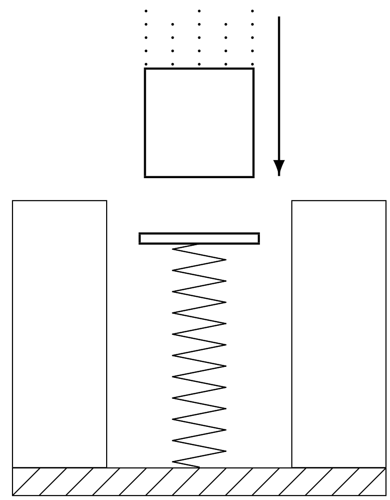
5 (Kirk, TTh 10:40am)

6 (González, TTh 1:40pm)

- Please be sure to write your name and circle your section above.
- For the problems, you *must* show all your work. Explain your thinking clearly. Lonely right answers will not receive full credit, lonely wrong answers will receive no credit.
- For the questions, no work needs to be shown (there is no partial credit).
- Please carry units through your calculations when needed, lack of units will result in a loss of points.
- You may use scientific or graphing calculators, but you must derive your answer and explain your work.
- Feel free to detach, use and keep the formula sheet. No other reference material is allowed during the exam.
- **GOOD LUCK!**

Problem 1 (11 points)

A 180 g block is dropped from rest onto a relaxed vertical spring that has a spring constant of $k = 300\,\mathrm{N/m}$. The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping.



(a) (3 pts) **Calculate the work done on the block by the gravitational force from the time the block falls on the spring until the time the block momentarily stops.**

The work done by the gravitational force $F_g = mg$ force is $W_g = \int_0^{\Delta x} \mathbf{F} \cdot d\mathbf{s} = mg\Delta x$.

Also, the work done by the gravitational force is minus the change in gravitational potential energy. The change in gravitational energy is negative, since the mass moves down a distance Δx:

$\Delta U_g = -mg\Delta x = -0.18\mathrm{kg} \times 9.8\mathrm{m/s}^2 \times 0.12\mathrm{m} = -0.21\mathrm{J}$, and thus $W_g = +0.21J$.

(b) (4 pts) **Calculate work done on the block by the spring force from the time the block falls on the spring until the time the block momentarily stops.**

The work done by the spring force $F_s = -kx$ force is $W_s = \int_0^{|Deltax} \mathbf{F} \cdot d\mathbf{s} = -(1/2)k(\Delta x)^2$.

The work done by the spring is minus the change in spring potential energy. Initially, the spring is not compressed or stretchesd and the potential energy is zero; when the block momentarily stops, the spring potential energy is $(1/2)k(\Delta x)^2$. The change in potential energy is positive: $\Delta U_s = (1/2)k(\Delta x)^2$, and the work done by the spring (which is negative) is $W_s = -(1/2)k(\Delta x)^2 = -0.5 \times 300\mathrm{N/m} \times (0.12\mathrm{m})^2 = -2.16\mathrm{J}$.

(c) (4 pts) **Calculate the height relative to the fully compressed spring from which the block was initially dropped.**

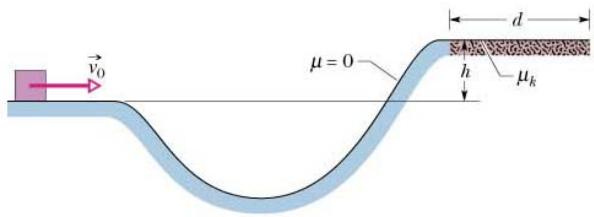
The kinetic energy of the block is zero at the time where it momentarily stops, and it is also zero at the time when it is released from rest at a distance H from the lowest point. Thus, the change in potential energy is zero:

$$0 = \Delta K + \Delta U = 0 + \Delta U = \Delta U_g + \Delta U_s = -mgy + (1/2)k(\Delta x)^2 = 0$$

$$y = \frac{k(\Delta x)^2}{2mg} = \frac{300\mathrm{N/m} \times (0.12\mathrm{m})^2}{2 \times 0.18\mathrm{kg} \times 9.8\mathrm{m/s}^2} = 1.22\mathrm{m}$$

Problem 2 (11 points)

The figure shows a block with mass m and initial speed v_0. The block slides along a track from one level to a higher level passing through an intermediate valley. The track is frictionless until the block reaches the higher level. There a frictional force stops the block in a distance d.



(a) (5 pts) **What is the minimum speed $v_{0,min}$ for the block to be able to make it up to the higher surface? Express your answer in terms of m, g, v_0, and h and numerical constants, as necessary.**

From the initial position until the point at the end the ramp, mechanical energy is conserved, and $\Delta K + \Delta U = K_f - K_0 + \Delta U = (1/2)mv_f^2 - (1/2)mv_0^2 + mgh = 0$. The minimum initial kinetic energy needed for the block to reach the higher ground is the change in potential energy: if $K_0 = mgh$, the kinetic energy (and thus velocity) when it reaches the higher ground will be zero; if it less than that, it will not reach the higher ground; if it is more than that, it will reach the higher ground with non zero velocity, and it will keep moving. Thus,

$$K_{min} = \frac{1}{2}mv_{0,min}^2 = mgh \to v_{0,min} = \sqrt{2gh}.$$

(b) (6 pts) **Assuming $v_0 > v_{0,min}$, what is the maximum distance d the block will travel before it stops due to friction? Express your answer in terms of m, g, v_0, μ_k, and h and numerical constants, as necessary.**

The change in mechanical energy will be equal to the work done by friction. The change in kinetic energy from the initial point until the time it stops is $\Delta K = 0 - \frac{1}{2}mv_0^2 = -\frac{1}{2}mv_0^2$. The change in potential energy is $\Delta U_g = mgh$. The (negative) work done by friction is $W_f = -f_k d = -\mu_k F_N d = \mu_k mgd$. Thus,

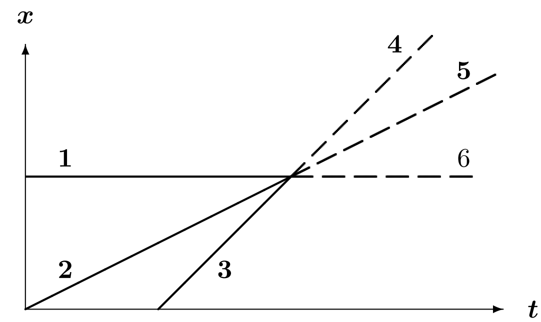
$$\Delta K + \Delta U = W_f \to -\frac{1}{2}mv_0^2 + mgh = -\mu_k mgd$$

$$d = \frac{v_0^2 - 2gh}{2\mu_k g}$$

Notice that the distance is positive only when $v_0^2 > 2gh$, a condition consistent with the answer to part (a).

Question 1 (6 points)

Two bodies have undergone an elastic one-dimensional collision along the x axis. The figure below is a graph of position versus time for those bodies and for their center of mass. Circle the correct answer to the following questions:



(a) (1pt) **Were both bodies initially moving, or was one initially stationary?**

one stationary both moving

Line 1 is horizontal, indicating that the velocity of that body was zero.

(b) (2 pts) **Which line segment corresponds to the motion of the center of mass before the collision?**

1 **2** 3 4 5 6

(c) (1 pt) **Which line segment corresponds to the motion of the center of mass after the collision?**

1 2 3 4 **5** 6

The velocity of the center mass is larger than the smaller velocity of the two bodies, and smaller than the largest velocity of the two bodies. Thus, lines 2 and 5 are the graphs of the center of mass before and after the collision.

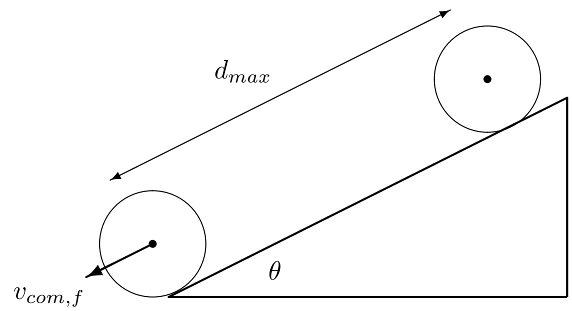
(d) (2 pts) **Is the mass of the body that was moving faster before the collision greater than, less than, or equal to that of the other body?**

greater than less than **equal to**

After the collision, one mass is stationary (line 6) and one mass moves with the same velocity as the intially moving mass (line 4, with the same slope than line 3). Since the masses collided, lines 1 and 4 are for one mass, and lines 3 and 6 for the other mass (if line 1 and 6 are the graphs of the same mass, it means one mass is always stationary and the other mass has always the same velocity). The only way two bodies can "exchange" velocities in a collision is by having the same mass.

Problem 3 (11 points)

A **spherical shell** of mass 4.0 kg rolls without slipping down an incline with an inclination angle of $\theta = 35°$, starting from rest. At the **bottom** of the incline the center of mass of the sphere has a translational speed of $v_{com,f} = 6.0$ m/s. The rotational inertia for a spherical shell about any diameter is $I_{com} = \frac{2}{3}MR^2$.



(a) (5 pts) Calculate the **kinetic energy** of the shell at the **bottom** of the incline.

The kinetic energy of a rolling body has a translational part $(1/2)Mv_{com}^2$ and a rotational part $(1/2)I_{com}\omega^2$. The angular velocity is related to the velocity of the center of ass, $\omega = v_{com}/R$, so

$$K = \frac{1}{2}Mv_{com}^2 + \frac{1}{2}I_{com}\omega^2 = \frac{1}{2}Mv_{com}^2 + \frac{1}{2}\frac{2}{3}MR^2\left(\frac{v_{com}}{R}\right)^2 = \frac{5}{6}Mv_{com}^2$$

$$K_f = \frac{5}{6} \times 4.0\text{kg} \times (6.0\text{m/s})^2 = 120\text{J}$$

(b) (6 pts) How **far**, d_{max}, along the incline, did the shell travel?

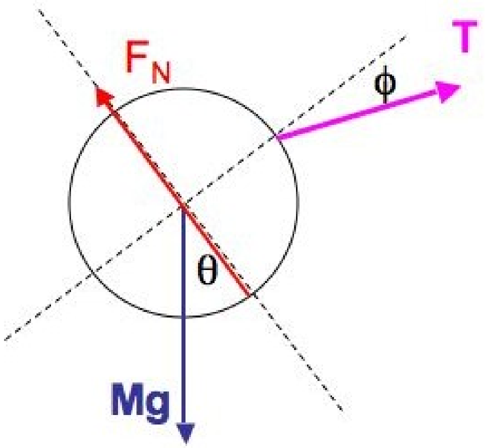
The mechanical energy is conserved in the rolling of the shell down the incline, so the (negative) change in gravitational potential energy plus the (positive) change in kinetic energy is equal to zero. The change in gravitational potential energy is $\Delta U_g = -Mgy = -Mgd_{max}\sin\theta$, and the change in kinetic energy is $\Delta K = K_f - 0 = K_f$, so

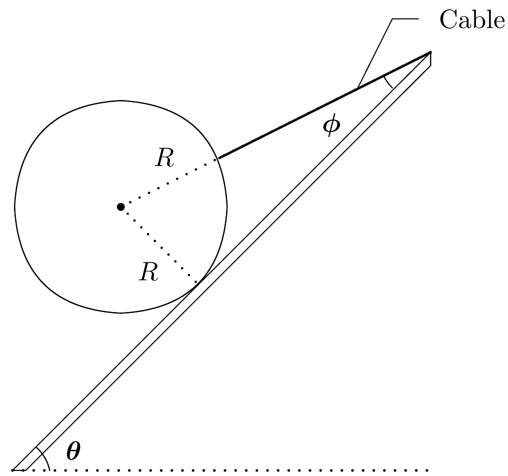
$$\Delta K + \Delta U = K_f - Mgd_{max}\sin\theta = 0$$

$$\Rightarrow d_{max} = \frac{K_f}{Mg\sin\theta} = \frac{120\text{J}}{4.0\text{kg} \times 9.8\text{m/s}^2 \times \sin(35^{\text{o}})} = 5.3\text{m}$$

Problem 4 (11 points)

A sphere of mass M is held stationary on the surface of a frictionless inclined plane by means of a cable, as illustrated in the figure below. The base angle of the inclined plane is θ, and the angle between cable and the surface of the inclined plane is ϕ.





(a) (3 pts) **Draw a free body diagram for the sphere, showing all external forces. Label each force. (You may draw directly on the diagram above).**

(b) (4 pts) **Calculate the tension in the cable, expressing your answer in terms of M, g, θ, ϕ and numerical constants, as necessary.**

The sum of the forces along the incline add up to zero in equilibrium:

$$0 = F_{\|} = +T\cos\phi - Mg\sin\theta = 0 \Rightarrow T = Mg\frac{\sin\theta}{\cos\phi}$$

(c) (4 pts) **Calculate the normal force exerted on the sphere by the surface of the inclined plane. Express your answer in terms of M, g, θ, ϕ and numerical constants, as necessary.**

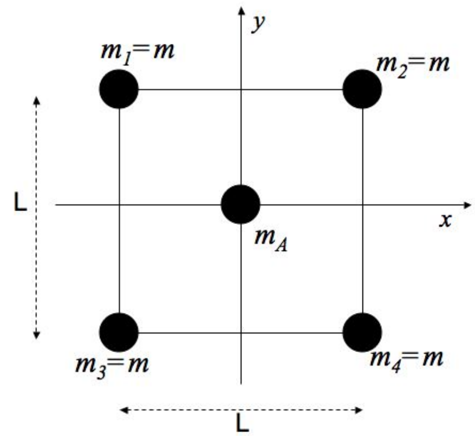
The sum of the forces perpendicular to the incline add up to zero in equilibrium:

$$0 = F_{\perp} = +F_N - Tsin\phi - Mg\cos\theta$$

$$\Rightarrow F_N = T\sin\phi + Mg\cos\theta = Mg\frac{\sin\theta}{\cos\phi}\sin\phi + Mg\cos\theta = Mg\frac{\cos(\phi-\theta)}{\cos\phi}$$

Question 2 (5 points)

The figure shows a square of edge length L formed by four spheres of equal mass m. A fifth mass, m_A, is at the center of the square.



(a) (1pt) **What is the direction of the gravitational force on the mass m_A at the center of the square produced by the pair of masses m_1, m_4?**

m_1 and m_4 are at the same distance from m_A, so the forces have the same magnitude; since the forces will have opposite directions, they will cancel each other.

$\hat{\imath}$ $\quad -\hat{\imath}$ $\quad \hat{\jmath}$ $\quad -\hat{\jmath}$ $\quad$ **the magnitude is zero**

(b) (2 pt) **What is the direction of the gravitational force on the mass m_A at the center of the square produced by the pair of masses m_1, m_3?**

m_1 and m_3 are at the same distance from m_A, so the forces have the same magnitude; the force F_1 will point towards the upper left corner, and the force F_3 towards the lower left corner. The vertical components will cancel, and the resulting horizontal component will point in the negative x direction.

$\hat{\imath}$ $\quad \underline{-\hat{\imath}}$ $\quad \hat{\jmath}$ $\quad -\hat{\jmath}$ $\quad$ the magnitude is zero

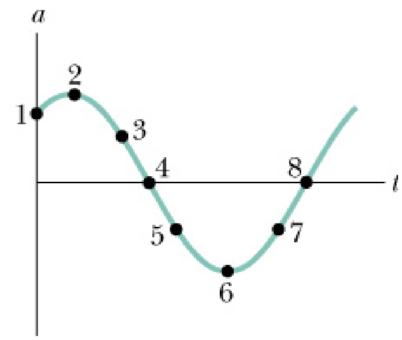
(c) (2 pts) **What is the magnitude of the work done by an external agent to bring mass m_A to the center of the square, with all the other four masses in place?**

The magnitude work done by an external agent to bring the mass m_A to the center of the square is equal to the mass m_A time the magnitude of the potential energy at the center of the square, produced by the other masses. The potential of any one of the masses at the center is $U_i = -Gm/r = -Gm/(L/\sqrt{2}) = -\sqrt{2}Gm/L$. The magnitude of the total potential energy at the center is $|U| = 4\sqrt{2}Gm/L$ and the work done to bring mass m_A is $W = 4\sqrt{2}Gmm_A/L$.

0 $\quad 2G\frac{mm_A}{L}$ $\quad \sqrt{2}G\frac{mm_A}{L}$ $\quad \underline{\mathbf{4\sqrt{2}G\frac{mm_A}{L}}}$ $\quad 4G\frac{mm_A}{L}$

Question 3 (6 points)

The acceleration $a(t)$ of a particle that executes simple harmonic motion with amplitude x_m along the x-axis is plotted as a function of time in the figure. Eight points along the plot have been labeled with numbers.



The position as a function of time $x(t)$ for a simple harmonic oscillator is related to the acceleration as $a(t) = -\omega^2 x(t)$, so the plot will look opposite to the one shown above: the position will be maximum at point 2, minimum at point 6, and zero (going through equilibrium) at points 4 and 8. The velocity is maximum when the particle goes through equilibrium, it will be positive when the position is increasing (point 4) and negative when the position is decreasing (point 8).

(a) (2 pts) Which of the labeled points, if any, corresponds to the particle's being located at $x = -x_m$?

(a) points 5 and 7
(b) points 4 and 8
(c) **point 2**
(d) point 6
(e) none of the above

(b) (2 pts) At labeled point 4, the velocity of the particle

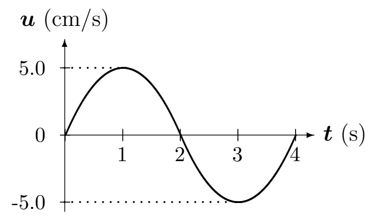
(a) **points in the direction of $+\hat{\imath}$.**
(b) points in the direction of $-\hat{\imath}$.
(c) is 0m/s in magnitude
(d) cannot be determined from the information provided

(c) (2 pts) At labeled point 5 the particle is located at

(a) $x = +x_m$.
(b) $x = -x_m$.
(c) $x = 0$m.
(d) between $x = -x_m$ and $x = 0$m.
(e) **between $x = 0$m and $x = +x_m$.**

Question 4 (6 points)

A certain transverse longitudinal wave $y(x,t) = y_m \sin(kx - \omega t + \phi)$ has a wavelength of 10cm, and is moving along a string in the positive direction of an x-axis. The transverse velocity of the particle in the string at $x = 0$ as a function of time, $u(t)$, is shown in the figure. Circle the right answers to the following questions:



The plot shown is the velocity of the point at the origin, where $y(x = 0, t) = y_m \sin(-\omega t + \phi)$. The plot is then $u = -y_m \omega \cos(\omega t + \phi)$. These are equations for a simple harmonic oscillator of angular frequency ω, so we can read the period from the plot, $T = 4s$. The maximum value of the transverse velocity is $u_m = y_m \omega = y_m 2\pi/T$, so we can find the maximum displacement value $y_m = u_m T/2\pi = 5.0\text{cm} \times 4\text{s}/2\pi = 10/\pi\text{cm/s}$. The wave speed is $v = \lambda/T = 10\text{cm}/4\text{s} = 2.5\text{cm/s}$.

(a) (2 pts) What is the period T of the wave?

2s **4s** 5s Cannot be determined from the information given.

(b) (2 pts) What is the wave speed v?

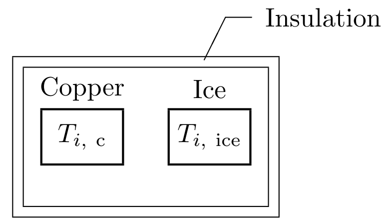
2.5 cm/s 5 cm/s 4 cm/s Cannot be determined from the information given.

(c) (2 pts) What is the wave amplitude y_m?

20cm 10cm **$10/\pi$ cm** Cannot be determined from the information given.

Problem 5 (11 pts)

A block of copper whose mass m_c is 80 g is heated in a laboratory oven to a temperature of $T_{i,\ \mathrm{c}} = 300\,^\circ\mathrm{C}$ and put into the thermally isolated container together with a piece of ice of mass $m_{\mathrm{ice}} = 250\,\mathrm{g}$ and temperature $T_{i,\ \mathrm{ice}} = -10^\circ\mathrm{C}$. Eventually the two substances come to thermal equilibrium at final temperature T_f. The specific heat of copper is 386 J/kg·K.



(a) (3 pts) **How much energy would be released if the copper block cools down to 0°C?**

$$Q_c = m_c c_c \Delta T_c = 0.08\mathrm{kg} \times 386\mathrm{J/kg^oC} \times (0^o\mathrm{C} - 300^o\mathrm{C}) = -9,264\mathrm{J}$$

The negative sign indicates that the copper releases heat when cooling down.

(b) (4 pts) **How much energy would the ice absorb if it would warm up and melt at 0°C?**

$$Q_{ice} = m_{ice} c_{ice} \Delta T_{ice} + m_{ice} L_{melt}$$

$$Q_{ice} = 0.25\mathrm{kg} \times 2220\mathrm{J/kg^oC} \times 10^o\mathrm{C} + 0.25\mathrm{kg} \times 3.33 \times 10^5\mathrm{J/kg} = 5,550\mathrm{J} + 83,250\mathrm{J} = 88,800\mathrm{J}$$

(c) (4 pts) **What is T_f, the final equilibrium temperature? Explain.**

Since the heat released by copper cooling down to ^{o}C, 9,264 J, is enough to warm up the ice to melting temperature (which needs 5,550 J), but not to melt it all (a process that needs 83,250 J), the final temperature will be zero degrees Celsius.

The mass of ice melted will be $m_{ice,melted}$ =(9,264 J - 5,550 J)/3.33 × 10^5 J/kg = 1.1 × 10^{-2}kg = 11 g, a very small fraction of the initial mass.

Problem 6 (11 points)

A bubble of air with initial volume V_0 is located at the bottom of a lake of depth D. The temperature at the bottom of the lake is T_b, and the temperature at the surface of the lake is T_s, with both temperatures measured in Kelvin. The bubble rises to the surface. During its ascent, the temperature of the bubble is equal to the temperature of the water that surrounds it. Treat the air in the bubble as an ideal gas.

(a) (3 pts) **Find the pressure at the bottom of the lake. Express your answers in terms of atmospheric pressure p_a, density of water ρ, acceleration of gravity g and D.**

From the equation or pressure in fluids, we know that

$$p_b = p_a + \rho g D$$

(b) (3 pts) **Find the number of molecules in the bubble, expressing your answer in terms of atmospheric pressure p_a, density of water ρ, acceleration of gravity g, D, V_0, T_b, Boltzmann constant k and numerical factors, as necessary.**

At the bottom of the lake, the pressure on the bubble is p_b, the volume is V_0, and the temperature is T_b. We can use the law for ideal gases:

$$p_b V_0 = NkT_b \Rightarrow N = \frac{p_b V_0}{kT_b} = (p_a + \rho g D)\,\frac{V_0}{kT_b}$$

(c) (5 pts) **Calculate the volume V_f of the bubble just as it reaches the surface of the lake, expressing your answer in terms of atmospheric pressure p_a, density of water ρ, acceleration of gravity g, D, V_0, T_b, T_s, Boltzmann constant k and numerical factors, as necessary.**

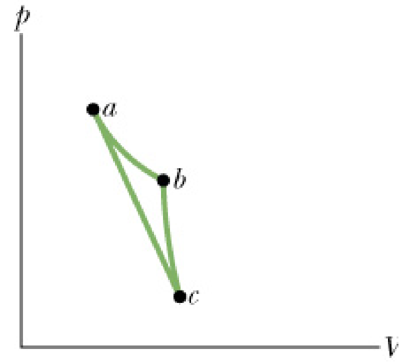
We use again the law for ideal gases, this time at the surface of the lake:

$$p_a V_f = NkT_s \Rightarrow V_f = \frac{NkT_s}{p_a} = (p_a + \rho g D)\,\frac{V_0}{kT_b}\,\frac{NkT_s}{p_a}$$

$$V_f = \left(1 + \frac{\rho g D}{p_a}\right)\frac{T_s}{T_b} V_0$$

Question 5 (5 points)

In the pV diagram shown below, the gas does 10 J of work when taken along the isotherm *ab* and 6 J when taken along the adiabat *bc*.



(a) (3 pts) **What is the change in the internal energy of the gas when it is taken along the straight path from a to c?**

If we consider a reversible cycle *abca*, we know the net change in internal energy is zero, so $0 = \Delta E_{ab} + \Delta E_{bc} + \Delta E_{ca}$, and thus $\Delta E_{ac} = -\Delta E_{ca} = \Delta E_{ab} + \Delta E_{bc}$. Since the process *ab* is along an isotherm, $\Delta E_{ab} = 0$. Since the process *bc* is adiabatic, we know $Qbc = 0 = \Delta E_{bc} + W_{bc}$ and then $\Delta E_{bc} = -W_{bc} = -6$ J.

(a) −4 J
(b) +4 J
(c) +6 J
(d) **−6 J**
(e) 16 J
(f) none of the above

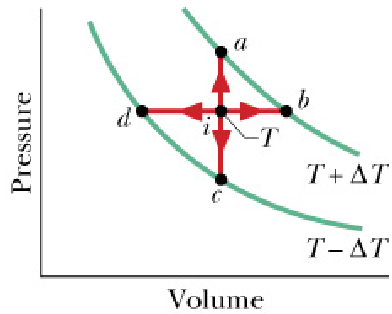
(b) (2 pts) **The work delivered by the gas to the environment when taken along the straight line from a to c is**

(a) exactly 16 J
(b) greater than 16 J
(c) **less than 16 J**
(d) negative in sign (meaning work is performed on the gas, not by the gas)
(e) none of the above

The work is the are under the curve in the process; thus the work done under the curve *ac* is smaller than the work done under the curves *abc*, which is equal to 10J+6J=16J.

Question 6 (6 points)

Point i in the figure represents the initial state of an ideal gas at temperature T, which is separately taken to four possible end states, a, b, c and d.



(a) (3 pts) In what processes does the gas entropy *decrease* ? (Circle all that apply)

The entropy change in the constant volume processes is $\Delta S = nC_V \ln(T_f/T_i)$, and in the constant pressure processes, $\Delta S = nC_P \ln(T_f/T_i)$. Thus, the change in entropy will be negative when the temperature decreases, which happens in $i \to c$ and $i \to d$.

a *b* **c** **d**

(b) (3 pts) For which process is the *magnitude* of the change in gas entropy, $|\Delta S|$, the largest?

For a monoatomic gas, $C_P = 5R/2$ and $C_V = 3R/2$, so for a fixed ratio of final and initial temperature, the change in entropy is largest in a constant pressure process than in a constant volume process. The entropy increase in $i \to b$ is $\Delta S_{ib} = nC_P \ln((T + \Delta T)/T) > 0$; and the entropy decrease in $i \to d$ is $\Delta S_{id} = nC_P \ln((T - \Delta T)/T) < 0$. The ratio $T/(T - \Delta T)$ is larger than the ratio $(T + \Delta T)/T$, and thus the change in entropy for id is largest.

The difference of the magnitudes of changes in entropy is

$$|\Delta S_{ib}| - |\Delta S_{id}| = nC_P \ln\left(\frac{T + \Delta T}{T}\right) - \left(-nC_P \ln\left(\frac{T - \Delta T}{T}\right)\right)$$

$$= nC_P \left(\ln\left(\frac{T + \Delta T}{T}\right) + \ln\left(\frac{T - \Delta T}{T}\right)\right) = nC_P \ln \frac{(T + \Delta T)(T - \Delta T)}{T^2}$$

$$|\Delta S_{ib}| - |\Delta S_{id}| = nC_P \ln \frac{T^2 - (\Delta T)^2}{T^2} < 0$$

so the change in entropy in the cooling process id is larger than in the warming process ib.

A simpler proof: if we use $\Delta T \ll T$, then the temperature is approximately constant and equal to the average temperature, and the entropy change is $\Delta S \approx \Delta Q/T_{avg} = nC_P \Delta T/T_{avg}$. The average temperature is smaller in the cooling process id than in the warming process ib, and thus the change in entropy is largest in id.

a *b* *c* **d**