# Physics 2101, Third Exam, Spring 2006

April 18, 2006

Last Name: KEY

First Name: ______________________

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|---|---|---|
| **Section:** (Circle one) | Section 1 | (Rupnik, MWF 7:40 AM) |
| | Section 2 | (Rupnik, MWF 9:40 AM) |
| | Section 3 | (Rupnik, MWF 11:40 AM) |
| | Section 4 | (Kirk, MWF 2:40 PM) |
| | Section 5 | (Kirk, TuTh 10:40 AM) |
| | Section 6 | (Gonzalez, TuTh 1:40 PM) |

- Please be sure to print your name and circle your section above. Do not write your social security number.
- Examine your paper carefully to be sure it is complete. There should be 3 consecutively numbered problems and 3 questions. The point value of each problem is 23 or 24 and of the question is 10 with total of 100. They are ordered relative to the topics covered.
- You are not required to show any work on the three questions. You may circle the correct answer and move on.
- You **MUST** show your work on the three problems. Failure to show your work will result in the loss of credit.
- Present your solutions in a neat and logically organized manner. Your graders cannot, and will not, award credit to solutions that are illegible.
- For problems with numerical answers, please carry units throughout the calculation.
- You may use scientific or graphing calculators, but you must derive your answer and explain your work.
- Feel free to detach, use, and keep the formula sheet. No other reference material is allowed during the exam.
- **GOOD LUCK!**

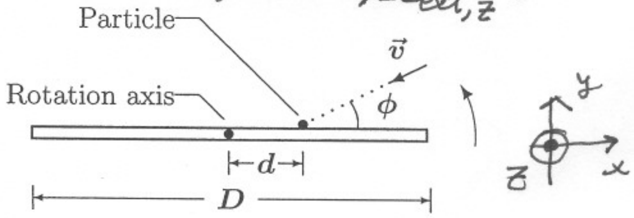
## Problem 1 (23 points)

NOTE: All angular momenta and $I_{str}$ are calculated relative to the axis of rotation.

The figure is an **overhead** view of a thin uniform rod of length $D = 0.80\,\text{m}$ and mass $M = 0.12\,\text{kg}$. The rod is rotating in a counterclockwise direction along a horizontal plane, with an angular velocity of magnitude $\omega = 2.8\,\text{rad/s}$, pointing out of the page. A particle of mass $m = 0.020\,\text{kg}$ and traveling along a straight line, along the same plane, hits the rod and sticks. Just before impact the particle has a velocity $\vec{v}$ of magnitude $15\,\text{m/s}$ at an angle $\phi = 35°$ relative to the rod and the particle is at a distance $d = 0.30\,\text{m}$ from the rod's center.

$\omega = 2.8\,\text{rad/s} = \text{const}$ before the impact $\Rightarrow$ frictionless bearings (no external torque)

the torque of $m\vec{g}$ of the particle does NOT have $z$-component $\Rightarrow \sum \tau_{ext,z} = 0$




system $\equiv$ rod − particle

(a) (4 pts) What is the magnitude **and** direction of the angular momentum of the rod before the impact?

magnitude:

$$L = I\omega = \frac{MD^2}{12}\omega = \frac{(.12\,\text{kg})(.8\,\text{m})^2}{12}\left(2.8\,\frac{\text{rad}}{\text{s}}\right)$$

$$L = 1.79 \times 10^{-2}\,\frac{\text{kgm}^2}{\text{s}}$$

direction: CCW or + or out of page or $+\hat{k}$ (using right-hand-rule)

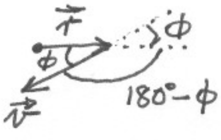
(b) (6 pts) What is the magnitude **and** direction of the angular momentum of the particle before the impact?

particle is treated as a point particle

magnitude:

$$L = m|\vec{r}||\vec{v}|\sin\theta_{\vec{r},\vec{v}} = m\,d\,v\sin\phi$$

$$= (0.02\,\text{kg})(0.3\,\text{m})\left(15\,\frac{\text{m}}{\text{s}}\right)\sin 35° = 5.16 \times 10^{-2}\,\frac{\text{kgm}^2}{\text{s}}$$




$\sin(180°-\phi) = \sin\phi$

direction: CW or − or into the page or $-\hat{k}$ (using right-hand-rule)

(c) (8 pts) Find the final angular speed of the particle-rod system, after the impact.

If $\tau_{ext} = 0$ we can use conservation of angular momentum of the system:

$$\sum \vec{L}_i = \sum \vec{L}_f$$

where $\vec{L}_f = I_{str}\vec{\omega}_f$ and where $I_{str} = I_{rod} + I_{particle} = I + md^2$

$$I\omega - m\,d\,v\sin\phi = (I + md^2)\,\omega_f$$

speed: $$\omega_f = \left|\frac{\frac{MD^2}{12}\omega - m\,d\,v\sin\phi}{\frac{MD^2}{12} + md^2}\right| = \left|\frac{1.79\times10^{-2}\,\frac{\text{kgm}^2}{\text{s}} - 5.16\times10^{-2}\,\frac{\text{kgm}^2}{\text{s}}}{\frac{(.12\,\text{kg})(.8\,\text{m})^2}{12} + (.02\,\text{kg})(0.30\,\text{m})^2}\right| = \left|-4.11\,\frac{\text{rad}}{\text{s}}\right| = 4.11\,\frac{\text{rad}}{\text{s}}$$

(d) (5 pts) Has the total kinetic energy of the particle-rod system

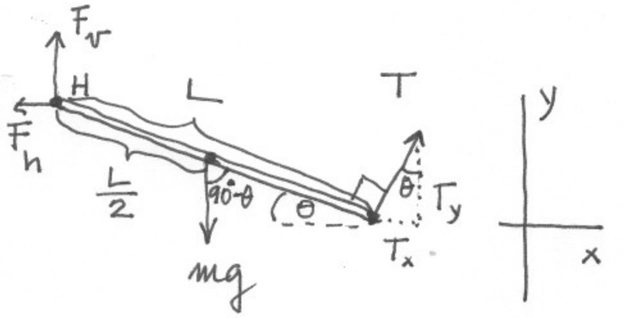
**increased**, **decreased** or **remained the same**? Circle the right answer. **Explain!**

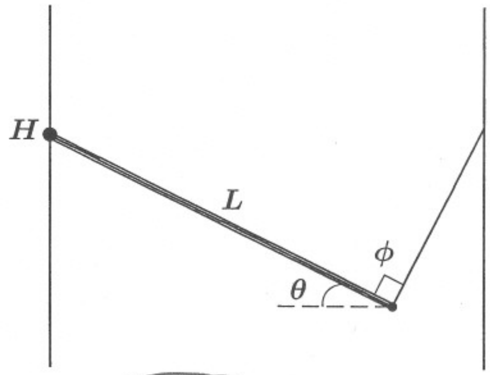
- interaction is a kind of completely inelastic collision $\Rightarrow \Delta K < 0$ (and the loss of kinetic energy is maximal)

## Problem 2 (24 points)

A uniform thin beam has a mass $M$ and a length $L$. The beam is connected by a hinge to a vertical wall on the left and is held in its position by a cable attached to the vertical wall on the right. The cable makes an angle of $\phi = 90.0°$ with respect to the beam. The beam makes an angle of $\theta$ above the horizontal.

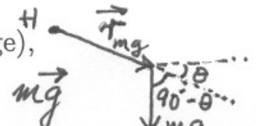
(a) (5 pts) Draw a free-body diagram for the beam.




(b) (3 pts) Relative to the hinge, is the torque of the beam's weight **clockwise** (into the page), **counterclockwise** (out of the page), or **neither**? (*Circle the right answer.*) $\vec{\tau}_{mg,H} = \vec{r}_{mg} \times m\vec{g}$

(use right-hand-rule)

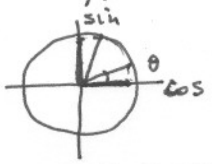



(c) (6 pts) Find the magnitude of the tension in the cable. Express your answer in terms of $M, \theta, L$ and other physical constants as needed. (*HINT: Choose the pivot point at the hinge.*)

$\sum \tau_H = 0$ and using $\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = |\vec{r}||\vec{F}| \sin\theta_{\vec{r},\vec{F}}$:

$\tau_{mg}$ is CW and $\tau_T$ is CCW

$-\frac{\cancel{L}}{2} mg \sin(90° - \theta) + \cancel{L} T \underbrace{\sin 90°}_{1} = 0$




$\sin(90° - \theta) = \cos\theta$

$T = \frac{mg}{2} \cos\theta$

(d) (5 pts) Find the magnitude and direction (up or down) of the **vertical component** of the force exerted on the beam by the hinge. Express your answer in terms of only $M, \theta, L$ and other physical constants as needed.

$\sum F_y = 0$

$F_v - mg + T\cos\theta = 0$

$F_v = mg - T\cos\theta = mg - \frac{mg}{2}\cos^2\theta = mg\left(1 - \frac{\cos^2\theta}{2}\right) > 0$ ✓, up

the choice of direction of $\vec{F}_v$ was right

(e) (5 pts) Find the magnitude and direction (left or right) of the **horizontal component** of the force exerted on the beam by the hinge. Express your answer in terms of only $M, \theta, L$ and other physical constants as needed.

$\sum F_x = 0$

$-F_h + T\sin\theta = 0$

$F_h = T\sin\theta = \frac{mg}{2}\cos\theta\sin\theta = \frac{mg}{4}\sin(2\theta)$ $> 0$ ✓ left

the choice of direction of $\vec{F}_h$ was right

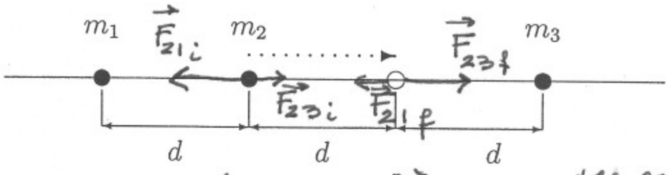
## Question 1 (10 points)

Three particles with equal masses $m_1 = m_2 = m_3 = m$ are positioned along a line, as shown below. Then, you move $m_2$ by $d$ toward $m_3$. The initial and final positions of $m_2$ are drawn in the figure as dark and open circles, respectively.

*Circle the correct answer to each of the questions below.*

system $\equiv m_1 - m_2 - m_3$

(a) (3 pts) The **magnitude** of the net force on $m_2$




(i) increased.

(ii) decreased.

(iii) remained the same.

(iv) Not enough information to answer.

$\leftarrow F_{net,2i}$ $\rightarrow F_{net,2,f}$ the same magnitude!

• masse are equal and positions of $m_2$ are symmetrical

(b) (4 pts) The potential energy of the $m_1$-$m_2$-$m_3$ system (assuming that the zero of gravitational potential energy corresponds to an infinite separation)

(i) increased.

(ii) decreased.

(iii) remained the same.

(iv) Not enough information to answer.

$\Delta U_g = 0$

because the masses are the same and the positions of $m_2$ are symmetrical to each other (or, on average, particles are neither closer or further apart and their masses are equal)

(c) (3 pts) The work done by you on the particle $m_2$, assuming particles are at rest initially and finally, is

(i) positive.

(ii) negative.

(iii) zero.

(iv) Not enough information to answer.

$$W_{you} = W = \cancel{\Delta K}^{0} + \Delta U + \cancel{\Delta E_{th}}^{0} + \cancel{\Delta E_{int}}^{0} \quad \text{... external work}$$

$$\Rightarrow W_{you} = \Delta U = \Delta U_g = 0$$

## Question 2 (10 points)

A penguin of mass $m$ **floats** first in a fluid of density $\rho_1 = \rho_0$, then in a fluid of density $\rho_2 = 0.95\rho_0$, and then in a fluid of density $\rho_3 = 1.1\rho_0$.

*Circle the correct answer to each of the questions below.*

(a) (3 pts) Which liquid exerts the largest buoyant force, $F_b$, on the **floating** penguin?

$F_b = mg$ ... for floating object !!

(i) 1.

(ii) 2.

(iii) 3.

(iv) All three tie.

(b) (3 pts) In which liquid is the **floating** penguin going to displace the most liquid?

→ most volume – the least dense one because (ii)

$F_b = m_{displ.}\, g = V_{displac.}\, \rho_f\, g$

(i) 1.

(ii) 2.

(iii) 3.

(iv) All three tie.

→ most mass – all the same (iv) because $m_{displ.} = m$ for floating object !!

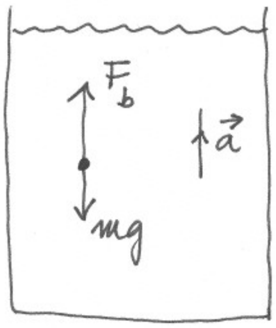
(c) (4 pts) The penguin spots a fish and dives in to catch it. After missing the fish the penguin decides to let the liquid brings him to the surface. Neglect the viscosity of the liquid (drag or friction). The constant $g$ used is the free fall acceleration close to the Earth surface.
If $F_b$ is the buoyant force on the penguin during the ascent, which relation represents the correct form of the magnitude of the penguin's acceleration, $a$, during ascent?

(i) $a = g$.

(ii) $a = \dfrac{F_b}{m} - g$.

(iii) $a = g + \dfrac{F_b}{m}$.

(iv) $a = \dfrac{F_b}{m}$.




2nd Newton's law: $\vec{F}_{net} = m\vec{a}$

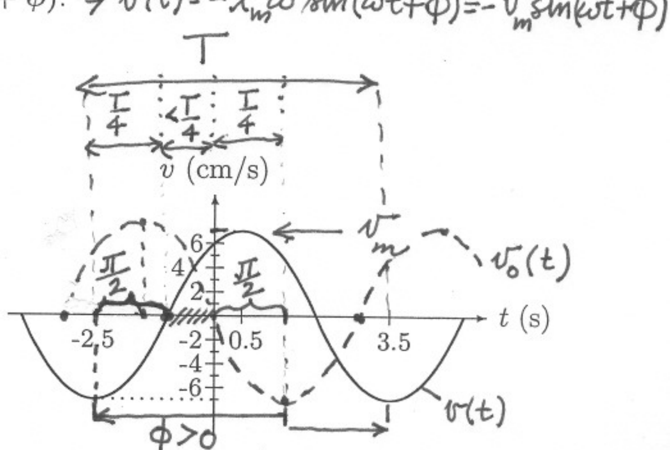
$F_b - mg = ma$

$a = \dfrac{F_b}{m} - g$

## Question 3 (10 points)

The graph shows the velocity function $v(t)$ of a linear **simple harmonic oscillator**. Assume that the position function $x(t)$ has the form $x(t) = x_m \cos(\omega t + \phi)$. $\Rightarrow v(t) = -x_m \omega \sin(\omega t + \phi) = -v_m \sin(\omega t + \phi)$

*Circle the correct answer to each of the questions below.*




(a) (3 pts) What is the maximum speed, $v_m$?

- (i) 2.0 cm/s.
- (ii) 4.0 cm/s.
- (iii) 6.0 cm/s.
- (iv) 7.0 cm/s.

(b) (3 pts) What is the period, $T$?

T... time between two points with the same x and v

- (i) 0.5 s.
- (ii) 1.0 s.
- (iii) 3.5 s.
- (iv) 6.0 s.

(c) (4 pts) What is the phase constant, $\phi$? $v(t)$ compared to $v_0(t) = -v_m \sin\omega t$

- (i) $0 \text{ rad} < \phi < \frac{\pi}{2}$ rad.
- (ii) $\frac{\pi}{2} \text{ rad} < \phi < \pi$ rad.
- (iii) $\pi \text{ rad} < \phi < \frac{3\pi}{2}$ rad.
- (iv) $\frac{3\pi}{2} \text{ rad} < \phi < 2\pi$ rad.

1st method (using the plot above)

$\phi = \frac{\pi}{2} + \frac{\pi}{2} +$ //// reagon, which is less than $\frac{\pi}{2}$

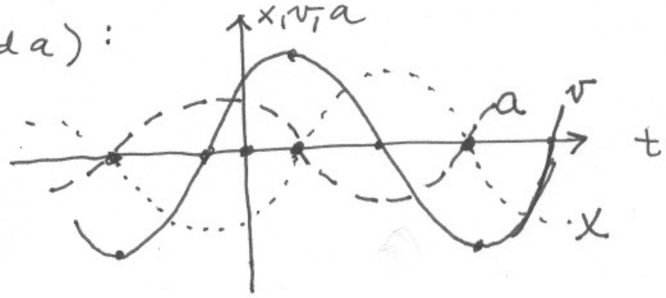
$\Rightarrow \pi < \phi < \frac{3\pi}{2}$

2nd method (use drawings of x, v, and a):

from the plot on right:

- at $t=0$ $x<0$, $v>0$ (also, $a>0$)

which is true in the third quadrant


3rd method (using values on t axis):

$\frac{T}{2} < \Delta t < \frac{3T}{4}$

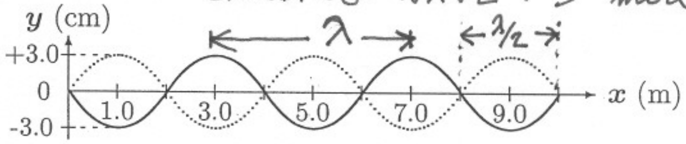
using $\frac{\Delta t}{T} = \frac{\phi}{2\pi}$ $\Rightarrow \Delta t = \frac{T}{2\pi}\phi \Rightarrow \phi = \Delta t \frac{2\pi}{T} \Rightarrow \pi < \phi < \frac{3\pi}{2}$

## Problem 3 (23 points)

Two waves are generated on a string of length 10.0 m to produce a five-loop **standing wave** with amplitude of 3.0 cm, as shown in the figure below (notice the different units for the distance $x$ along the string and vertical displacement $y$). The wave speed is 65 m/s. Let the equation for one of the waves be of the form $y(x,t) = y_m \sin(kx + \omega t)$.

(a) In the equation for the other wave, what are the numerical values for the following quantities?

STANDING WAVE : $5^{th}$ mode




(i) (5 pts) The amplitude, $y_m$?

$$y_m = \frac{y_m'}{2} = \frac{3.0\,cm}{2} = \underline{\underline{1.5\ cm}}$$

$$Y = y' = y_1 + y_2 = 2\,y_m \sin kx \cos\omega t$$

where $Y_m = y_m' = 2y_m$

(ii) (6 pts) The wave number, $k$?

- by inspecting the plot :

$$5\frac{\lambda}{2} = L \Rightarrow \lambda = \frac{2L}{5} = \frac{2(10m)}{5} = \underline{\underline{4\,m}} \qquad \underline{\underline{k}} = \frac{2\pi}{\lambda} = \frac{2\pi}{4\,m} = \underline{\underline{\frac{\pi}{2}\frac{rad}{m}}}$$

- or just by reading the distance between, for example, two crests:

$\lambda = 4\,m$

(b) For the standing wave on the string drawn above

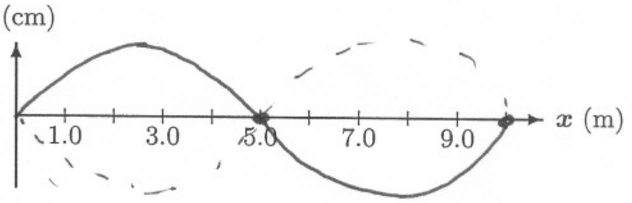
(i) (7 pts) find the maximum displacement of the string element at a position $x = 1.5$ m.

$$Y(x,t) = y'(x,t) = (3.0\ cm)\sin\left[\left(\frac{\pi}{2}\frac{rad}{m}\right)x\right]\cos\omega t$$

$Y(x,t) = y'(x,t)$ is maximal when $\cos\omega t = 1$, therefore:

$$Y(x,t) = y'(x=1.5m) = (3.0\ cm)\sin\left[\left(\frac{\pi}{2}\frac{rad}{m}\right)(1.5\,m)\right] = \underline{\underline{2.12\ cm}}$$

(ii) (5 pts) Draw the <u>second harmonic</u> on this plot.




- second harmonic has two loops or segments
- it should, also, have a larger amplitude $\left(P_{avg} = \frac{1}{2}\mu v\left(\frac{4\pi^2}{T^2}\right)y_m^2\right)$