**Hour Examination #2**

**March 7, 2006**

**Physics 2101**

**Please print your name in the space below.**

**Do not write your social security number!**

**Last Name:** ______________________

**First Name:** Solutions

Circle your section number

Section #1 – Rupnik, MWF 7:40 AM

Section #2 – Rupnik, MWF 9:40 AM

Section #3 – Rupnik, MWF 11:40 AM

Section #4 – Kirk, MWF 2:40 PM

Section #5 –Kirk, TuTh 10:40 AM

Section #6 – Gonzalez, TuTh 1:40 PM

## Instructions

1. You are not required to show any of your work on the four questions. You may circle the correct response and move on.

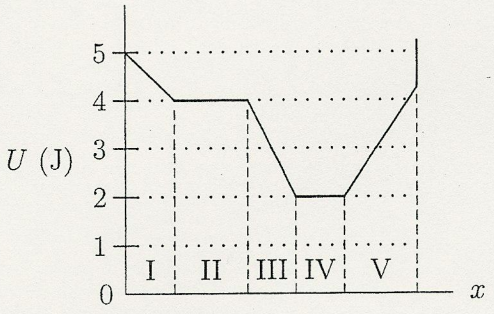
2. You **MUST** show your work on each and every problem. Failure to show your work will result in the loss of credit.

3. Present your solutions in a neat and logically organized manner. Your graders cannot, and will not, award credit to solutions that are illegible.

4. Units are a part of the number. If you solve a problem numerically, then carry along the units line by line.

## Question 1

The figure below shows the potential energy of a particle as a function of its x coordinate. The x axis has been divided into five distinct regions labeled by Roman numerals I through V.




(a) 3 pts In which region or regions is the **magnitude of the force** exerted on the particle the greatest?

(a) I only
(b) I and III
(c) II and IV
(d) III only
(e) V

$F = -\frac{dU}{dx}$

$\Rightarrow$ magnitude of force is greatest where slope is steepest.

(b) 3 pts In which region or regions does the **direction of the force** exerted on the particle point in the + x direction?

(a) I only
(b) I and III
(c) II and IV
(d) III only
(e) V

$F$ is +ve $\Rightarrow -\frac{dU}{dx}$ is +ve

$\Rightarrow \frac{dU}{dx}$ is −ve (regions with negative slope)

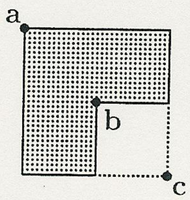
(c) 2 pts What is the largest mechanical energy of the particle if it is to be trapped in regions III, IV, and V?

(a) 1.5 J
(b) 2 J
(c) 4 J
(d) 5 J
(e) No response is possible until the mass and velocity of particle are specified.

$E_{mech} = U + KE = U + \frac{1}{2}mv^2 \geq U(x)$

## Question 2

The figure below shows a metal plate of uniform density and mass M. The plate had been square before 25% of it was removed. Also shown in the figure are three lettered points, a, b, and c. Let the symbols $I_a$, $I_b$, and $I_c$ denote the rotational inertia of the plate about axes that pass through the points a, b, and c, respectively, and are perpendicular to the plate.




(a) 4 pts Which is the largest rotational inertia?

(a) $I_a$
(b) $I_b$
(c) $I_c$
(d) $I_c = I_a$ (tie)
(e) $I_a = I_b = I_c$ (all are equal)

Rotational inertia is largest about an axis that is farther away from the mass.

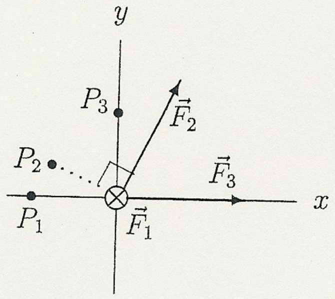
(b) 4 pts Which is the smallest rotational inertia?

(a) $I_a$
(b) $I_b$
(c) $I_c$
(d) $I_c = I_a$ (tie)
(e) $I_a = I_b = I_c$ (all are equal)

Rotational inertia is smallest about the axis where matter is closer.

## Question 3

Three forces, $F_1$, $F_2$, and $F_3$ are applied to a particle that is located at the origin of a coordinate system, as is illustrated in the figure below. The three forces have identical magnitudes but different directions, as is indicated in the figure. In particular note that $F_1$ points into the the paper.




(a) 4 pts Which of the forces establishes a torque of magnitude 0 Nm about $P_1$?

(a) $F_1$ only
(b) $F_2$ only
(c) $F_3$ only
(d) $F_1$ and $F_2$
(e) $F_3$ and $F_1$

$|\vec{r}_1 \times \vec{F}_3| = 0$ because angle between $\vec{r}_1$ and $\vec{F}_3$ is $180°$

(b) 4 pts Which of the forces establishes the torque of least magnitude about point $P_3$?

(a) $F_1$
(b) $F_2$
(c) $F_3$
(d) $F_1 = F_2$ (tie)
(e) $F_3 = F_1$ (tie)

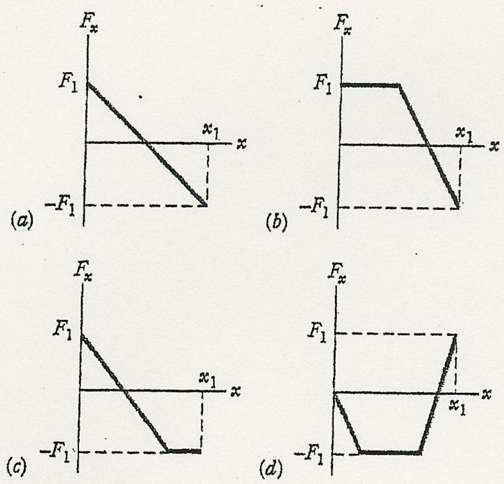
$|\vec{r}_3 \times \vec{F}_1| = |\vec{r}_3 \times \vec{F}_3| = |r_3| F$

$|\vec{r}_3 \times \vec{F}_2| = |r_3| F \sin\theta < |r_3| F$

$\Rightarrow$ $F_1$ and $F_3$ tie, and produce a larger torque than $F_2$

## Question 4

A force $\vec{F} = \hat{\imath}F(x)$ is applied to a particle that is located at the origin of some coordinate system. Graphs (a), (b), (c), and (d) in the figure below show four possible functions for $F(x)$.




$W = \int F\,dx$ = area under the curve.

Let the quantities $W_a$, $W_b$, $W_c$, and $W_d$ denote the work performed by the force in each graph during a displacement of the particle from the origin to $x_1$.

(a) 3 pts Which of the quantities $W_a$, $W_b$, $W_c$, and $W_d$ is positive? Circle all corect answers on the line below.

$W_a$ $W_b$ $W_c$ $W_d$

(b) 3 pts Which of the quantities $W_a$, $W_b$, $W_c$, and $W_d$ is negative? Circle all corect answers on the line below.

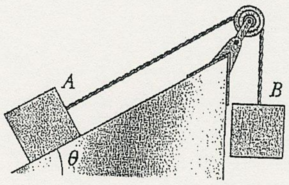
$W_a$ $W_b$ $W_c$ $W_d$

(c) 2 pts Which of the quantities $W_a$, $W_b$, $W_c$, and $W_d$ is 0 J? Circle all corect answers on the line below.

$W_a$ $W_b$ $W_c$ $W_d$

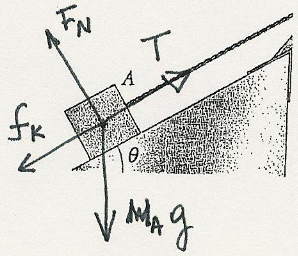
## Problem 1

Blocks A and B, with masses $M_A = 5$ kg and $M_B = 10$ kg, are connected by a massless rope that passes over a massless pulley, as is illustrated below. The pulley is mounted on frictionless bearings. The base angle of the inclined plane is $\theta = 30°$, and the coefficient of kinetic friction between the surface of the inclined plane and the bottom of block A is $\mu_k = 0.25$.

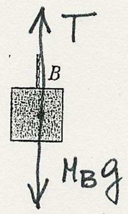



In the space below draw free body diagrams for bodies A an B. Label the forces clearly and show their directions explicitly.

(a) 2 pts




(b) 1 pt




(c) 3 pts Consider the system formed by block A plus block B plus the interconnecting rope. Which of the forces you identified in parts (a) and (b) are **internal** forces? You may ignore the force exerted on the rope by the pulley.

Only tension is an internal force between blocks A and B.

Blocks A and B are released from rest with the rope taut, and block B then descends through a distance L. The rope neither stretches nor breaks during the motion.

(d) 5 pts Calculate the work performed by each of the external forces you identified in part (c) on the system during the descent of block B.

- Normal force on block A does not do any work because it is perpendicular to displacement
- Work of friction on mass A: $W_f = -f_k L = -\mu_k F_N L = -\mu_k M_A g \cos\theta L$
  $= -5.3\,J$
- Work of gravity on mass A: $W_{gA} = -M_A g L \sin\theta = -12.3\,J$
- Work of gravity on mass B: $W_{gB} = +M_B g L = +49.0\,J$

(e) 6 pts Calculate the speed of block B after it descends through distance L as described above.

Use work energy theorem:

$$W_{TOT} = \Delta KE = \frac{1}{2} M_A V_A^2 + \frac{1}{2} M_B V_B^2 - 0$$

$$= \frac{1}{2}(M_A + M_B)V^2 \qquad (\text{since } V_A = V_B)$$

$$\Rightarrow V = \sqrt{\frac{2\,W_{TOT}}{M_A + M_B}}$$

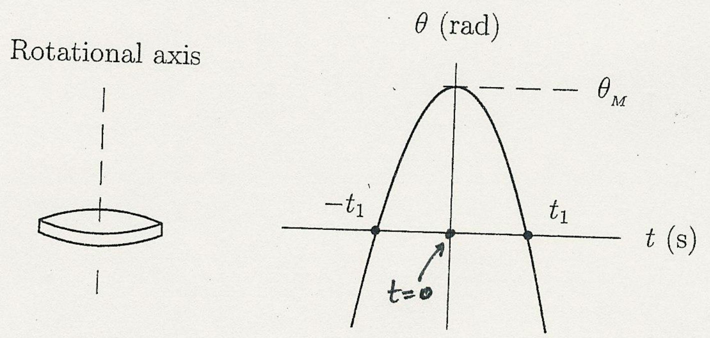
$$W_{TOT} = -5.3\,J - 12.3\,J + 49.0\,J = 31.4\,J$$

(from previous part)

$$\Rightarrow V = 2.0\ m/s$$

## Problem 2

A disk with moment of inertia $I_0$ is set into motion with the direction of its angular velocity vector initially pointing in the $+\hat{k}$ direction. The graph shown below shows the angular position of a point on the surface of the rotating disk.




The function $\theta(t)$ depicted above is a parabola of the form:

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_0 t^2,$$

where the symbols $\theta_0$, $\omega_0$, and $\alpha_0$ are constants.

Express your answers to parts (a) through (d) in terms of numerical constants, $\theta_M$, $t_1$, and $I_0$ as needed.

(a) 3 pts What is $\theta_0$ for the function $\theta(t)$ depicted in the graph above?

$\theta_0 = \theta(t=0) = \theta_M$ (from inspection of graph)

(b) 4 pts What is $\omega_0$ for the function $\theta(t)$ depicted in the graph above?

$\omega(t) = \dfrac{d\theta}{dt}$ is the slope of the curve shown in the graph

$= \omega_0 + \alpha_0 t$

$\omega_0 = \omega(t=0) = 0$ (again, from graph inspection: slope is zero at $t=0$)

(c) 7 pts What is $\alpha_0$ for the function $\theta(t)$ depicted in the graph above?

We now know $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_0 t^2$

$= \theta_M + \frac{1}{2}\alpha_0 t^2$

From the graph, we also know $\theta(t = t_1) = 0$

or $\theta_M + \frac{1}{2}\alpha_0 t_1^2 = 0 \Rightarrow \alpha_0 = -\frac{2\theta_M}{t_1^2}$

(d) 3 pts Calculate the torque $\vec{\tau}$ that is being exerted on the rotating disk. Specify both the magnitude and the direction of the torque.

If $\theta(t) = \theta_M + \frac{1}{2}\alpha_0 t^2 = \theta_M - \theta_M \frac{t^2}{t_1^2}$

then the angular acceleration is constant:

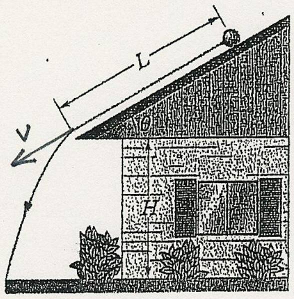
$$\alpha(t) = \frac{d^2\theta}{dt^2} = \alpha_0 = -\frac{2\theta_M}{t_1^2}.$$

Newton's law : $\tau = I\alpha$

$$\Rightarrow \vec{\tau} = I_0 \alpha_0 \hat{k} = -I_0 \frac{2\theta_M}{t_1^2} \cdot \hat{k}$$

## Problem 3

As illustrated in the figure below a solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance L = 6.0 m down a roof that is inclined at angle $\theta = \pi/6$ radians, or 30°.




(a) 10 pts What is the angular speed of the cylinder about its center as it leaves the roof?

Conservation of mechanical energy:

$$mgL\sin\theta = \frac{1}{2} m V_{com}^2 + \frac{1}{2} I_{com}\omega^2 \qquad V = \omega R$$

$$mgL\sin\theta = \frac{1}{2}(I_{com} + mR^2)\omega^2 \qquad I_{com} = \frac{1}{2} m R^2$$

$$\omega = \sqrt{\frac{2\cancel{m}gL\sin\theta}{3/2\,\cancel{m}R^2}} = 63 \text{ rad/sec}$$

(b) 4 pts What is the horizontal component of the translational velocity of the cylinder at the instant before impact?

Horizontal velocity is constant in projectile motion.

When cylinder leaves the roof, speed is $V = \omega R$

Horizontal component is $V_x = V\cos\theta = \omega R\cos\theta$

$= 5.4$ m/s

(c) 3 pts What is the angular speed of the cylinder at the instant before impact?

There is no torque produced by gravity,

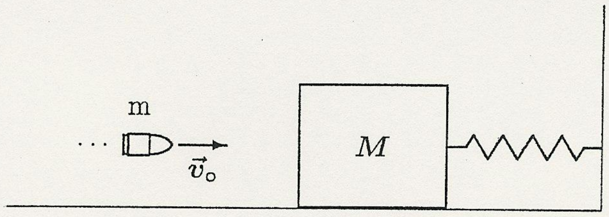
so there is no angular acceleration:

angular velocity will be constant during the fall.

$\omega_f = \omega_0 = 63$ rad/s (from part a)

## Problem 4

A bullet of mass m strikes the surface of a block with mass M, as is illustrated below. The bullet is traveling to the right with speed $v_0$ at the instant of impact, but it comes to rest within the block instantaneously. The rightmost surface of the block is attached to a spring with known spring constant k, with the rightmost end of the spring being permanently attached to the wall, as shown. There is no friction between the floor and the bottom surface of the block.




(a) 5 pts Calculate the speed of the block plus bullet at the instant immediately following impact.

Momentum is conserved in the collision:

$$m v_0 = (m + M) v \Rightarrow v = \frac{m}{m+M} v_0$$

(b) 12 pts After the impact the spring compresses a distance L before the block and bullet come instantaneously to rest. Derive an expression for L in terms of numerical constants, m, M, k, and $v_0$ as you might find necessary. You may use the back of this page for calculation if you want.

Energy is conserved in spring motion (but not in the collision!):

$$\frac{1}{2}(m+M) v^2 = \frac{1}{2} k L^2$$

$$\Rightarrow L = \sqrt{\frac{m+M}{k}}\, v = \frac{m v_0}{\sqrt{k(m+M)}}$$