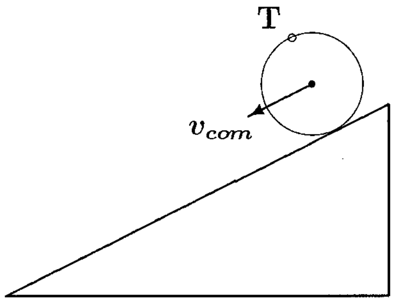
Question 1 (10 points)

A **spherical shell** of mass M and radius R rolls smoothly down an incline. The rotational inertia of the spherical shell about any diameter is $I_{com} = \frac{2}{3}MR^2$. *Circle the correct answer to the questions below.*



(a) (3 pts) The ratio between translational and rotational kinetic energy, $\frac{K_{\text{tran}}}{K_{\text{rot}}}$, is

(i) 1.

(ii) 1/3.

(iii) 2/3.

(iv) 3/2.

(b) (3 pts) During **rolling** the direction of static friction is

(i) up the incline.

(ii) down the incline.

(iii) not enough info to tell.

(c) (2 pts) If v_{com} denotes the center of mass speed, the speed of the **top** of the shell (point **T** directly across from the contact point) is equal to

(i) v_{com},

(ii) $v_{\text{com}}/2$,

(iii) $2v_{\text{com}}$,

(iv) not enough info to tell,

(d) (2 pts) If the spherical shell and a hoop, of the same mass and radius, roll down this incline from the same height, the **hoop** would come to the bottom of the incline

(i) before

(ii) at the same time as

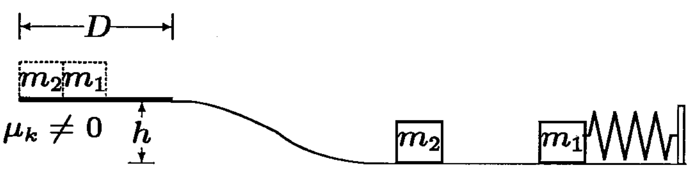
(iii) after

the **spherical shell**.

Problem 1 (24 points)

A spring is initially compressed by 5.0 cm and then released with block $m_1 = 0.20$ kg in front but not attached to it. Block m_1 moves along the horizontal frictionless surface and, shortly after leaving the spring, hits block $m_2 = 0.30$ kg. The two blocks stick together and move until stopping on the horizontal level that is h above the lower surface. The coefficient of kinetic friction of the rough surface is $\mu_k = 0.30$ and the distance they move along the rough surface is $D = 0.80$ m.

(a) (4 pts) Calculate the spring constant if the speed of m_1 at the relaxed spring position is 6.5 m/s.



Before collision E conserved

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$\Rightarrow \boxed{k = \frac{mv^2}{x^2} = 3380\ \frac{kg}{s^2}}$

(b) (6 pts) Calculate the speed of m_1-m_2 after the collision.

Tot Inelastic collision $m_1 v_1 = (m_1 + m_2) V \Rightarrow V = \frac{m_1 v_1}{(m_1 + m_2)} = 2.6\ \frac{m}{s}$

(c) (10 pts) Calculate the height h of the higher level. Neglect the sizes of the blocks relative to the distance D.

$\Delta E_{Mech} = W_{ext} \Rightarrow [\cancel{(m_1+m_2)} g h] - [\frac{1}{2}\cancel{(m_1+m_2)} V^2] = -\mu . DN$

$= -\mu D \cancel{(m_1+m_2)} g$

$\Rightarrow h = \frac{\frac{1}{2}v^2 - \mu D g}{g} \Rightarrow \boxed{h = 0.1049\ m}$

(d) (4 pts) The center of mass of the m_1-m_2 system while on the lower surface (*Circle the right answer.*)

moving to the left   at rest   moving to the right.

Question 2 (10 points)

The figure shows a **stationary** rod of mass m that is held against a wall by a rope and friction between rod and wall. The uniform rod has length L and the angle between the rod and rope is θ. *Circle the correct answer to the questions below.*

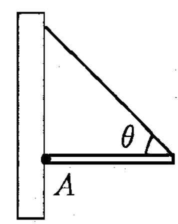
(a) (4 pts) Assume that the rotational axis is perpendicular to the rod and is through point A. Is the magnitude of the torque due to the pull on the rod by the rope

(i) greater than

(ii) less than

(iii) equal to

the magnitude of the torque due to the rod's weight?



(b) (3 pts) The direction of the horizontal component of the force by the wall on the rod at A is

(i) toward right.

(ii) toward left.

(iii) not enough information to decide.

(c) (3 pts) If the rope is suddenly cut, the **direction** of the torque of gravitational force relative to the point A is

(i) into the page.

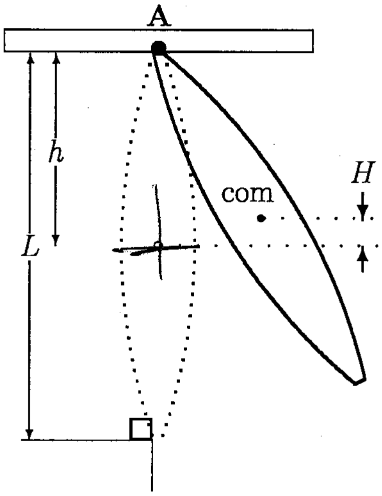
(ii) out of the page

(iii) impossible to determine.

Problem 2 (24 points)

An object has length L, mass M, and its rotational inertia through the center of mass and axis orthogonal to the page is I_{com}. The object is suspended from one end as indicated in the figure. The the object is pulled to the side so that its center of mass is $H = L/5$ higher and then allowed to swing as a pendulum about point **A**. The distance between the center of mass of the object and point **A** is $h = L/2$. At its lowest point the object hits the ball of putty of mass m which becomes attached to it.

(a) (9 pts) Calculate the magnitude and direction of the **object's** angular velocity just **before** it hits the putty.



E is conserved before collision

$\cancel{K_I} + U_I = K_F + \cancel{U_F}$

$\frac{1}{2} I^* \omega^2 = MgH$

$\therefore \omega = \sqrt{\frac{2MgH}{I^*}}$

Direction into page

$I^* = I_{com} + M\left(\frac{L}{2}\right)^2$

$I^* = I_{com} + M\frac{L^2}{4}$

(b) (6 pts) What is the rotational inertia of the **object-putty** system about point **A**?

$\tilde{I}_{tot} = I^* + I_{putty} = I^* + mL^2$

(c) (9 pts) Calculate the magnitude and direction of the angular velocity of the object-putty system just **after** the collision takes place.

L_{TOT} is conserved. $I^* \omega_{before} = \tilde{I}\, \omega_{after}$

$\boxed{\text{Direct into page}}$

$$\boxed{\omega_{AFTER} = \frac{I^* \omega_{before}}{\tilde{I}} = \frac{\sqrt{\left(I_{com} + \frac{ML^2}{4}\right)} \cdot \sqrt{\frac{2}{5} ML}}{\left(I_{com} + \frac{ML^2}{4} + mL^2\right)}}$$

Question 3 (10 points)

Below are the equations for three waves traveling on separate strings.

Wave 1: $y(x,t) = (2.0\,\mathrm{mm})\sin[(2.0\,\mathrm{m}^{-1})x - (2.0\,\mathrm{s}^{-1})t]$

Wave 2: $y(x,t) = (3.0\,\mathrm{mm})\sin[(8.0\,\mathrm{m}^{-1})x - (4.0\,\mathrm{s}^{-1})t]$

Wave 3: $y(x,t) = (1.0\,\mathrm{mm})\sin[(4.0\,\mathrm{m}^{-1})x - (8.0\,\mathrm{s}^{-1})t]$

(a) (2.5 pts) Which wave displaces the string the most?

(i) Wave 1.

(ii) Wave 2.

(iii) Wave 3.

(b) (2.5 pts) Which wave has the maximum wave speed?

(i) Wave 1.

(ii) Wave 2.

(iii) Wave 3.

(c) (2.5 pts) Which wave has the shortest wavelength?

(i) Wave 1.

(ii) Wave 2.

(iii) Wave 3.

(d) (2.5 pts) Which wave has the longest period?

(i) Wave 1.

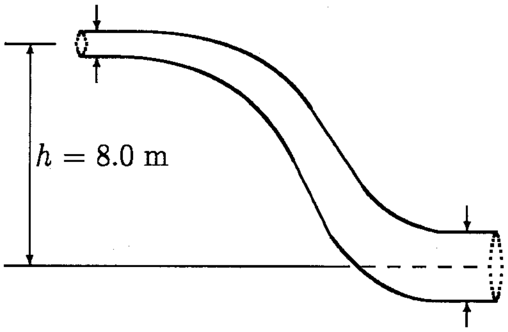
(ii) Wave 2.

(iii) Wave 3.

Problem 3 (24 points)

Water is flowing with speed of 10.0 m/s through a pipe of diameter 0.2 m. The water gradually descends 8.0 m as the diameter of the pipe increases to 0.4 m. The insert of the pipe is shown below.

(a) (8 pts) What is the speed of water at the lower level?



Continuity eqn.

$A_1 v_1 = A_2 v_2$

$A = \pi \left(\frac{D}{2}\right)^2$

$A = \frac{\pi}{4} D^2$

$D_1^2 v_1 = D_2^2 v_2$

$$\boxed{v_2 = \frac{(0.1)^2}{(0.2)^2} 10\,\text{m/s} = 2.5\,\frac{\text{m}}{\text{s}}}$$

(b) (16 pts) If the pressure at the upper level is 1.2×10^5 Pa, what is the pressure at the lower level?

Bernoulli's pple.

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g \cancel{h_2}^{\,0} + \frac{1}{2}\rho v_2^2$$

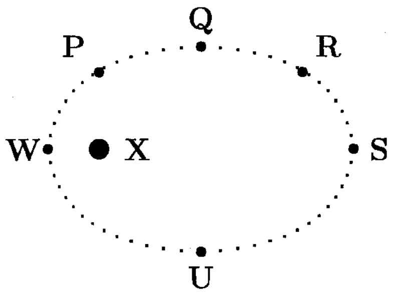
$$P_2 = P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

$$P_2 = (1.2 \cdot 10^5\,\text{Pa}) + \left(1000\,\frac{\text{kg}}{\text{m}^3}\right) 9.8\,\frac{\text{m}}{\text{s}^2} \cdot 8\,\text{m} + \frac{1}{2}\left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left[\left(10\,\frac{\text{m}}{\text{s}}\right)^2 - \left(2.5\,\frac{\text{m}}{\text{s}}\right)^2\right]$$

$$\boxed{P_2 \simeq 2.45\ 10^5\,\text{Pa}}$$

Question 4 (10 points)

A planet travels in an elliptical orbit about a star **X** as shown.



(a) (3 pts) The magnitude of the acceleration of the planet is

(i) greatest at point **U**.

(ii) greatest at point **S**.

(iii) greatest at point **W**.

(iv) the same at all points.

(b) (4 pts) At what pair of points is the speed of the planet the same?

(i) **P** and **R**.

(ii) **W** and **S**.

(iii) **Q** and **U**.

(c) (3 pts) At what point is the speed of the planet minimum?

(i) Point **Q**.

(ii) Point **S**.

(iii) Point **W**.

Question 5 (10 points)

The work that a Carnot engine delivers is one tenth of the energy it absorbs to be able to operate.

(a) (2.5 pts) What is the efficiency of this engine?

10   1   1/10   *impossible to determine*

(b) (2.5 pts) What is the fraction of the heat that goes to "waste" to the cold reservoir?

9/10   9   1/9   *impossible to determine*

(c) (2.5 pts) If this engine operates such that its cold reservoir is at $200K$, the temperature of the hot reservoir is roughly,

422 K   322 K   222 K   *impossible to determine*

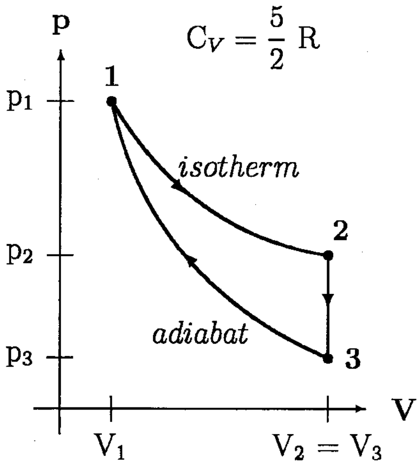
(d) (2.5 pts) The magnitude of total change of entropy during the isothermal expansion phase is

equal   *larger*   *smaller*   *impossible to determine*

than the magnitude of total change of entropy during the isothermal compression phase.

Problem 4 (24 points)

One mole of an ideal diatomic gas goes through the thermodynamic cycle shown in the $p-V$ diagram below. Assume $V_1 = 0.1\,\mathrm{m}^3$, $V_2 = 0.3\,\mathrm{m}^3$, and $p_1 = 1\,\mathrm{atm}$.



(a) (6 pts) Calculate pressures p_2 and p_3.

Along isotherm $P_1V_1 = P_2V_2 \Rightarrow P_2 = \frac{P_1V_1}{V_2} = 0.33\,\text{atm}$

Along adiabat $P_1V_1^\gamma = P_3V_3^\gamma \Rightarrow P_3 = P_1\left(\frac{V_1}{V_3}\right)^\gamma = 0.214\,\text{atm}$

(using $\gamma = \frac{C_P}{C_V} = \frac{7/2 R}{5/2 R} = 7/5$)

(b) (6 pts) Calculate the temperature T_1.

$PV = nRT \Rightarrow T_1 = \frac{P_1V_1}{nR} = 1219\,K$

(c) (6 pts) Calculate the amount of heat absorbed (Q_A) and released (Q_R) during one cycle

$Q_{12} = W_{12} = nRT\ln(V_2/V_1) = (1\,\text{mol})(8.31\,\frac{J}{\text{mol}\,K})(1219\,K)\ln\left(\frac{0.3\,m^3}{0.1\,m^3}\right) = 11129\,J$

$Q_{23} = \Delta E_{int} = nC_v\,\Delta T = (1\,\text{mol})(\frac{5}{2}\,8.31\,\frac{J}{\text{mol}\,K})(-436\,K) = -9058\,J$

(using $T_3 = \frac{P_3V_3}{nR} = 783\,K$)

$Q_{31} = 0$

$\boxed{Q_A = 11129\,J;\quad Q_R = 9058\,J}$

(d) (6 pts) Find the efficiency of this cycle.

$\boxed{\varepsilon = 1 - \frac{|Q_L|}{|Q_H|} = 18.6\,\%}$

Problem 5 (24 points)

(a) (10 pts) What mass of water at 99° C must be mixed with 150 g of ice at 0° C to produce liquid water at 25° C. The heat of fusion of water is 333 kJ/kg and its specific heat is 4190 J/(kg K). Assume that the system is thermally isolated and closed.

$99°C \to 372\,K$

$25°C \to 298\,K.$

$Q_{WATER} = m_w\, c\,(298-372)$ (in °C would be OK as well)

ICE: TRANSF + WARMING UP.

$Q_{ICE} = m_I L + m_I C_w (298-273) = 65.66\,J$

$\therefore\ Q_{WATER} + Q_{ICE} = 0$

$\therefore\ Q_w = -Q_{ICE} \Rightarrow m_w = -\dfrac{65.66\,J}{C_{WATER}\,(298-372)}$

$\boxed{m_w = 212\,g}$

(b) (10 pts) What is the change of entropy of the mass initially as ice when taken from ice at 0° C to water at 25° C?

$\Delta S = \Delta S_{transf} + \Delta S_{WARMUP} = \dfrac{m_I L}{T} + m_I C_w \cdot \ln\left(\dfrac{T_f}{T_i}\right)$

$\boxed{\Delta S_{TOTAL} = 238\,\dfrac{J}{K}}$

(c) (4 pts) What happens to the entropy of this closed system during the process described above? *Circle the right answer.*

Increases *Decreases* *Stays the same* *Impossible to determine*

Question 6 (10 points)

Consider one mole of an ideal gas

(a) (2.5 pts) The temperature – – – – – – – –– during an adiabatic compression.

INCREASES DECREASES REMAINS THE SAME

(b) (2.5 pts) The temperature – – – – – – – –– during a decrease in pressure at constant volume.

INCREASES DECREASES REMAINS THE SAME

(c) (2.5 pts) The internal energy – – – – – – – –– during an isothermal compression.

INCREASES DECREASES REMAINS THE SAME

(d) (2.5 pts) The entropy – – – – – – – –– during an isothermal expansion.

INCREASES DECREASES REMAINS THE SAME