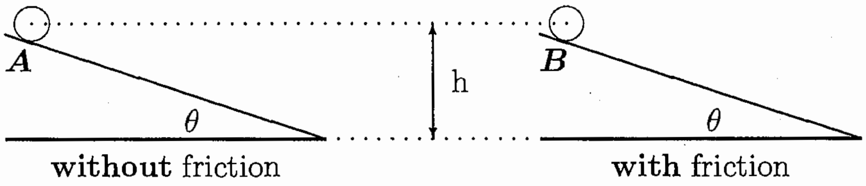
Question 1 (10 points)

Identical disks are released from rest from two identical inclines, as illustrated in the figure. In the first slide there is no friction while there is friction between the slide and the disk in the second case causing the disk B to roll down smoothly. *Circle the correct answer to the questions below.*



(a) (2.5 pts) The angular velocity of disk A is

(i) zero.

(ii) impossible to determine.

(iii) positive.

(iv) negative.

No friction ⇒ no torque for rolling

All pot. energy ⇒ translational Kinetic Energy

(b) (2.5 pts) At any moment, the total kinetic energy of disk A is

(i) equal to

(ii) larger than

(iii) smaller than

that of disk B.

$U + K = \text{const}$ at same height

$K_{TOT} = K_{T_A} = K_{T_B} + K_{R_B}$

(c) (2.5 pts) The velocity of the center of mass, at the bottom of the incline, of disk A is

(i) equal to

(ii) larger than

(iii) smaller than

that of disk B.

(d) (2.5 pts) The acceleration down the incline of disk A is

(i) equal to

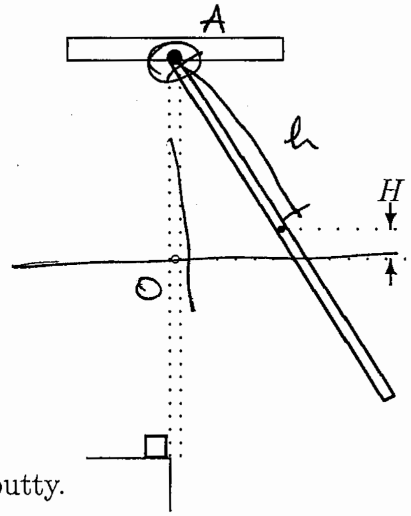
(ii) larger than

(iii) smaller than

that of disk B.

Problem 1 (20 points)

A thin rod of length 1.2 m and mass 3.8 kg is suspended from one end as indicated in the figure. The rod is pulled to the side so that its center of mass is $H = 20$ cm higher and then allowed to swing as a pendulum. At its lowest point the rod hits the ball of putty of mass 100 g and becomes attached to it.



(a) (4 pts) What is the rotational inertia of the rod about the rotational axis at one end of the rod?

$I_{COM} = \frac{1}{12} M L^2 \qquad I_A = I_{COM} + M h^2 \qquad h = L/2$

$\hookrightarrow I_A = \frac{1}{12} M L^2 + M \frac{L^2}{4} = \frac{1}{3} M L^2$

thus $\boxed{I_A = \frac{1}{3}(3.8\,\text{kg}) \cdot (1.2\,\text{m})^2 = 1.824\ \text{kg m}^2}$

(b) (6 pts) Calculate the rod's kinetic energy just before it hits the putty.

Conserv. of Energy $\quad U_i = U_f \qquad E_i = MgH$

$E_f = \frac{1}{2} I_A \omega^2$ OR $\frac{1}{2} I_{COM} \omega^2 + \frac{1}{2} v_{COM}^2$

$v_{COM} = \omega L/2$

$\cancel{U_f}^{0} + K_f = E_f$

thus $\boxed{K_f = MgH = 3.8\,\text{kg}\ 9.8\,\frac{\text{m}}{\text{s}^2} \cdot 0.2\,\text{m} = 7.448\ \text{J}}$

Also, since $K_f = \frac{1}{2} I_A \omega^2 \Rightarrow \omega = \sqrt{\frac{2 K_f}{I_A}} = 2.857 \frac{r}{s}$

(c) (10 pts) What is the magnitude and direction of the angular velocity of the rod-putty system just after the collision takes place.

E is ~~not~~ conserved but L is! thus $L_f = L_i$

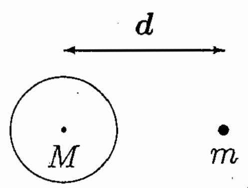
L_i (Just before it hits the putty) $= I_A \omega = 1.824\ \text{kg m}^2 \times 2.857\ \frac{1}{s}$

$L_f = I_f \omega_f = (I_A + M_{putty} \cdot L^2)\, \omega_f = (1.968\ \text{kg m}^2) \cdot \omega_f$

$\Rightarrow \boxed{\omega_f = \frac{I_A \omega}{I_f} = 2.64\ \frac{1}{s}}$

Question 2 (10 points)

A satellite of mass m orbits a planet of mass M at a distance d from the planet's center. The satellite mass is much smaller than the mass of the planet.
Circle the correct answer to the questions below.



(a) (4 pts) The total mechanical energy of the system is

$U = -\frac{GMm}{r}$

$E = -\frac{1}{2}\frac{GMm}{r}$

(i) twice the potential energy of the system.

(ii) ~~half the~~ potential energy of the system.

(iii) the same as the potential energy of the system.

(iv) impossible to determine.

(b) (3 pts) If the distance between the two objects is tripled, the force between them is

(i) 1/3 of

(ii) 1/2 of

$F \sim \frac{1}{r^2} m_1 m_2$

(iii) 1/9 of

(iv) the same as

the original force.

(c) (3 pts) If the satellite is taken to a different distance of $4d$, the period of this new orbit is

(i) 4 times

Keppler's 3rd law

(ii) 6 times

$T^2 \sim R^3$

(iii) 8 times

(iv) the same as

that of the original orbit.

so $T_1^2 \sim d^3 \Rightarrow T_1 = d^{3/2}$

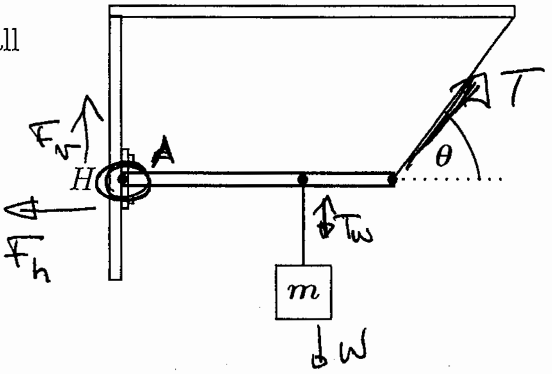
$T_2^2 \sim (4d)^3 = 4^3 d^3$

$\Rightarrow T_2 = 4^{3/2} d^{3/2} = 8 d^{3/2}$

Problem 2 (20 points)

A massless bar is employed to support a block of mass $m = 300\,\text{kg}$ aided by a wire with the tension of $T = 5000\,\text{N}$. The block hangs at the distance that is 2/3 of the length of the bar, from the hinge.

(a) (6 pts) Draw a free-body diagram for the bar. Shaw all the forces, important angles, and a pivot point.



Equilibrium $\vec{F} = 0$; $\vec{\tau} = 0$

take pivot point at A

(F_x) $\Rightarrow$ $-F_h + \cos\theta\, T = 0$

(F_y) $\Rightarrow$ $F_v + \sin\theta\, T - T_W = 0$ ($T_W = mg$)

(Torque) $\Rightarrow$ $-\frac{2}{3}\ell\, T_W + \ell \sin\theta\, T = 0$

(b) (6 pts) Calculate the value of $\boldsymbol{\theta}$.

From Torque equ $\Rightarrow$ $\sin\theta\, T = \frac{2}{3}(mg) \Rightarrow \sin\theta = \frac{1}{T}\left(\frac{2}{3}mg\right)$

$\sin\theta = 0.392$

$\theta = \sin^{-1}(0.392)$

$\boxed{\theta \simeq 23^\circ}$

(c) (4 pts) Calculate the magnitude of the horizontal force on the bar from the hinge.

From (X) $\boxed{F_h = \cos\theta\, T = 4602.5\ \text{N}}$

(d) (4 pts) Calculate the magnitude of the vertical force on the bar from the hinge.

(From y) $\boxed{F_v = -\sin\theta\, T + mg = -1953\text{N} + 2940\text{N} = 987\text{N}}$

Question 3 (10 points)

The figure below shows an open tank filled with water and five points centered at the respective surfaces. Points A and C are on vertical surfaces and points B and D are on horizontal surfaces, as indicated.

Circle the correct answer to the questions below.

(a) (2.5 pts) Which indicated point has the largest pressure due to water?

(i) A.

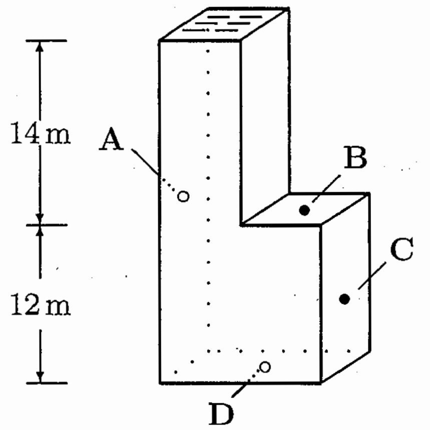
(ii) B.

(iii) C.

(iv) D.

$P = P_{Top} + \rho g h$

D has the largest h.



(b) (2.5 pts) Which indicated point has the least pressure due to water?

(i) A.

(ii) B.

(iii) C.

(iv) D.

(c) (2.5 pts) Whould the answers change if the container would be closed at the top?

(i) Yes.

(ii) No.

(iii) Not enough information to tell.

(d) (2.5 pts) What would the values of the pressure be at the chosen points if the container is filled with sea water that has larger density than water?

(i) The same.

(ii) Larger values.

(iii) Smaller values.

$\rho g h$ would be larger if ρ is increased maintaing g, h const

Problem 3 (20 points)

A hypothetical planet has a mass of $M = 3.0 \times 10^{23}$ kg, a radius of $R = 4.0 \times 10^6$ m and no atmoshere. A 18.0 kg probe is to be launched vertically from its surface.

(a) (6 pts) Calculate the probe's escape speed from the surface of the planet.

$$\frac{1}{2} m v_e^2 - G\frac{Mm}{r} = 0 \Rightarrow \boxed{v_e = \sqrt{\frac{2GM}{r}} = 3163 \frac{m}{s}}$$

(b) (6 pts) If the probe is launched with an initial kinetic energy of 6.0×10^7 J, calculate the probe's kinetic energy when it is 6.0×10^6 m from the center of the planet.

$$E_i = K_i + U_i = 6\,10^7\,J - G\frac{Mm}{r} \qquad (r = 4\,10^6\,m)$$

$$E_f = K_f + U_f = K_f - \frac{GMm}{r'} \qquad (r' = 6\,10^6\,m)$$

E is conserved

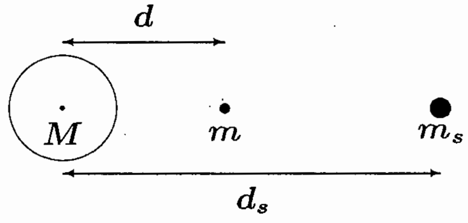
$\Downarrow$

$$K_f = K_i + U_i - U_f$$

$$K_f = 6\,10^7\,J - G\frac{Mm}{r} + G\frac{Mm}{r'}$$

$$\boxed{K_f \cong 6\,10^7\,J - 3\,10^7\,J \simeq 3\,10^7\,J}$$

(c) (8 pts) Suppose now that the planet has a satellite with mass $m_s = 1.6 \times 10^7$ kg, which is orbiting palnet at a distance $d_s = 2.0 \times 10^7$ m from the center of the planet.
Calculate the potential energy of the probe-planet-satellite system when the probe is at $d = 6.0 \times 10^6$ m from the center of the planet and just below the satellite.



$$U = -G\left(\frac{M m_p}{d} + \frac{M m_s}{d_s} + \frac{m_p m_s}{[d_s - d]}\right)$$

$$U = -G\left[\frac{3\,10^{23} \cdot 18\,kg^2}{6 \cdot 10^6\,m} + \frac{3\,10^{23} \cdot 1.6\,10^7\,kg^2}{2\,10^7\,m} + \frac{18 \times 1.6\,10^7\,kg^2}{1.4\,10^7\,m}\right]$$

$$U \cong -G\left[9\,10^{17}\frac{kg^2}{m} + 2.4\,10^{23}\frac{kg^2}{m} + 20.6\frac{kg^2}{m}\right] = -1.6\,10^{13}\frac{m^3}{kg\,s^2} \cdot \frac{kg^2}{m} = -1.6\,10^{13}\frac{kg\,m^2}{s^2}$$

$$\boxed{U \cong -1.6\,10^{13}\,J}$$