Using Calibration Lines in S1/LLO

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1 Introduction

A note on a way to use information in the calibration lines to produce an evolving interferometer calibration.

2 Calibration Lines

The lines were injected to drive ETMX beginning on August 25, 8:29 UTC, at 36.75 and 972.8 Hz, and the low freq line was shifted to 51.3 Hz on Aug 25 20:47 UTC (GPS 714343633).

The differential length excitation produced by the calibration lines, by gravitational waves and by any other sources is detected in AS_Q and DARM_CTRL. Using the notation presented in *Notes on LIGO Detectors' Calibration*, Sept 8, 2002, if the differential length excitation is $X_{ext}(f)$, the sensed signals are

$$AS\_Q = X_{ext} \frac{C(f)}{1 + H(f)}$$

$$DARM\_CTRL = X_{ext} \frac{G(f)C(f)}{1 + H(f)} = X_{exc} \frac{1}{A(f)} \frac{H(f)}{1 + H(f)}$$

where $C(F)$ is the AS_Q sensing function (essentially a cavity pole and a DC gain, in counts/m), $A(f)$ is the actuation function (essentially a pendulum transfer function with a DC gain, in m/count), $G(f)$ is the loop filter function (a complicated transfer function, in counts/counts), and $H(f) = A(f)C(f)G(f)$ is the open loop transfer function. All these functions are complex. We assume that the sensing function can change its gain, since it depends on the fluctuating alignment. The idea is to use the information in the calibration lines appearing in DARM_CTRL and AS_Q to change the calibration function $AS\_Q/X(f)$, which is used in the gravitational wave search algorithms.

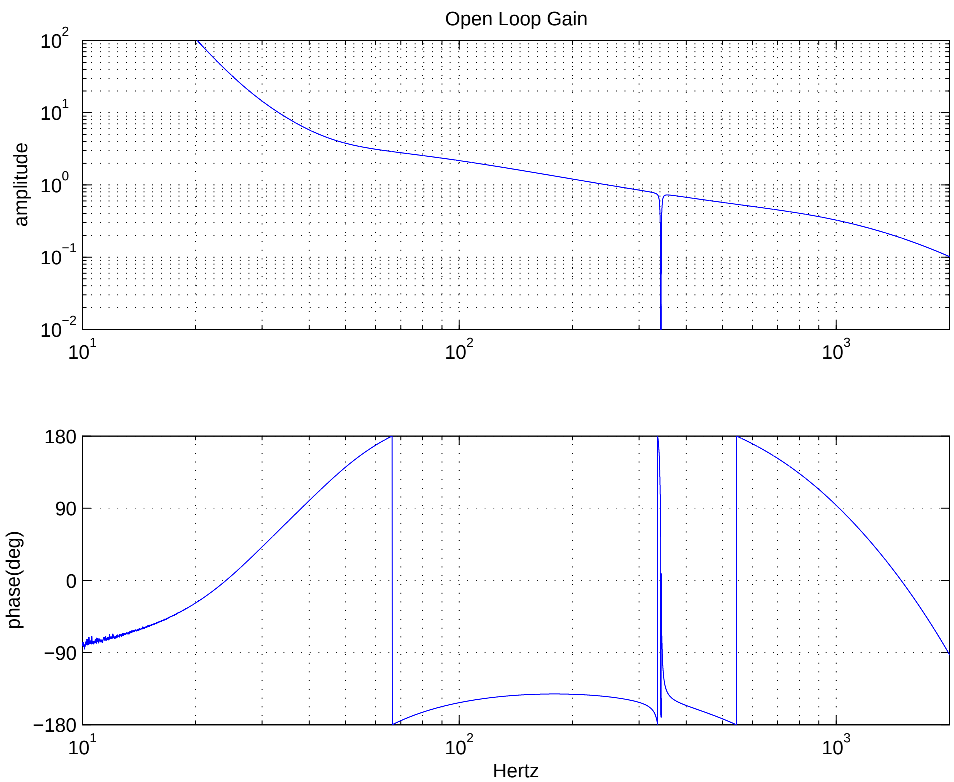

Figure 1: Open Loop gain $H(f)$ measured on Sept 6, and taken as reference.

3 LLO calibration for Sept 6, 2002 23:00 UTC

The function $H(f)$ was measured on Sept 6, 2002, at 23:02 UTC, and a model was fitted to provide the individual functions $A(f)$, $G(f)$ and $C(f)$. The open loop gain is plotted in Fig1. The unity gain frequency of the DARM loop was 248 Hz (with 35 degrees of phase margin). The maximum phase margin (39 degrees) is obtained when the unity gain frequency is 179 Hz, which would need the gain to be 25% lower. We see that the gain can only be increased by 38% before the loop goes unstable at the frequency of 336Hz. The gain can be decreased by a factor of 0.35 before the loop becomes unstable at a frequency of 66 Hz.

The sensing function was

$$C(f) = \frac{2.29 \times 10^{17} \text{counts/m}}{s + 2\pi 87.3 \text{Hz}} \times AAF(f),$$

where $AAF(f)$ is an 8th order, 80dB attenuation analog elliptic antialiasing filter at 7.57 kHz.

Between the times 715387980(22:53 UTC) and 715389120 (23:11 UTC), the average power level in PTRT_NORM (from a minute trend in that period) was 1587 ± 7. The minute trend of LineMon for the AS_Q line in the same period was 1.50 ± 0.07 counts for the 51.3 Hz line (A151) and $(8.3 \pm 0.3) \times 10^{-3}$ counts for the 972.8 Hz lines (A1972).

This provides a starting point to find out the calibration changes at other times in the S1 run.

4 Longest LLO Science segment: 714787127-714814599

This segment, 7h38min long, shows a significant degradation in alignment in the last 20min, so it maybe a good check of our assumptions on the uses of the calibration lines. The initial time was Aug 30, 23:58:34 UTC. The DARM_ERR_K gain, which was changed a few times during the run, was equal to GW_K=-0.232 in all science segments from GPS 714345572 (8/25 21:19:19 UTC) until GPS 715395019 (9/7 00:50:06 UTC). We used DataViewer to get minute trends of the normalized arm power in the Y-arm (L1:LSC-LA_PTRT), and the LineMon amplitudes for each calibration line (L1:LSC-A151, L1:LSC-A1972). The line amplitudes were obtained from results of Sergey Klimenko's DMT monitor LineMon. Even with minute trends, the LineMon results showed fluctuations that seemed large with respect to expected gain variations over a minute time scale (10-20%), so we smoothed the results with a 5 minutes window. We scaled the power by the power in the reference spectrum (1587 counts), and the LineMon outputs for the values at the same reference time (1.5 for 51.3 Hz and .0083 for the 972.8 Hz). We show these trends in Fig.2.

The trends at the end of the segment show the right correlations: the power goes down, indicating a degrading alignment; the amplitude of the low frequency line goes up (presumably because the loop gain goes down, and therefore there is more residual motion); and the amplitude at the high frequency line goes down (presumably due to a decreasing sensitivity). We want now to see if the numbers derived from these behaviors make sense.

We have two numbers for each time, the amplitude of the two calibration lines in AS_Q. We will assume the amplitude of the motion produced with the injected lines is the same as in the reference time, and take ratios between the amplitudes at any given time and the amplitudes at the reference time. These ratios will be:

$$R_i(t) = \frac{AS\_Q(f_i, t)}{AS\_Q(f_i, t_0)} = \frac{C(f_i, t)}{C_0(f_i, t)} \frac{1 + H_0(f_i, t)}{1 + H(f_i, t)}$$

We can begin assuming the simplest case, where the only change is due to alignment, and the sensing function $C(f)$ differs from $C_0(f)$ by a constant $C(f, t) = \alpha(t) C_0(f, t_0)$. Then,

$$R_i(t) = \alpha(t) \frac{1 + H_0(f_i)}{1 + \alpha(t) H_0(f_i)}$$

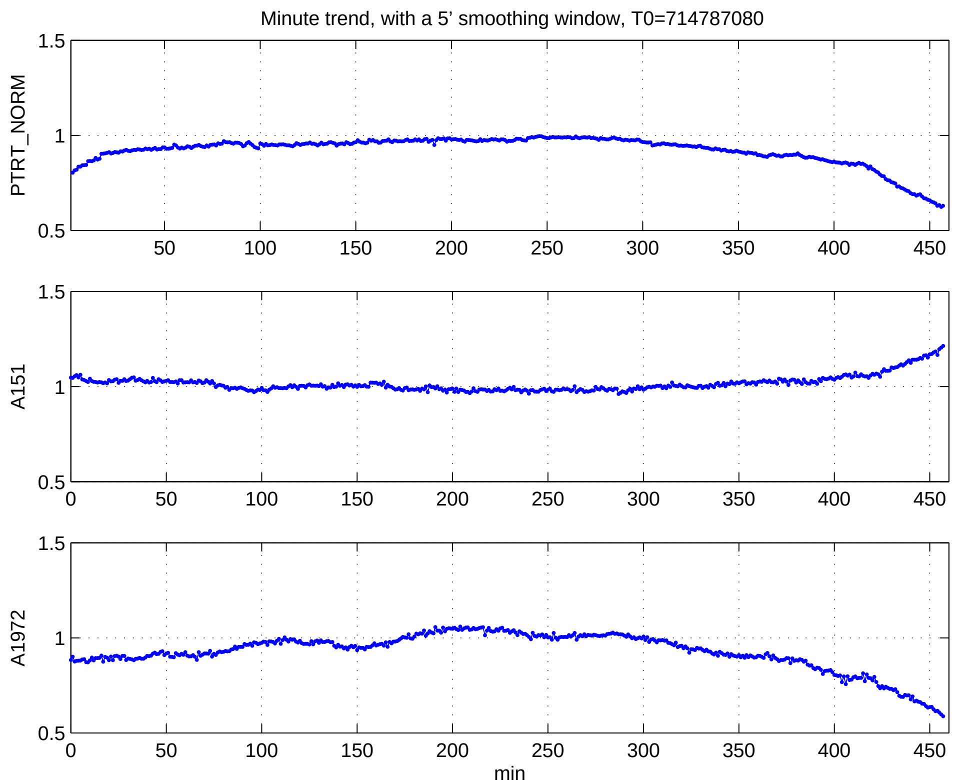

Figure 2: Trends observed on power in the Y-arm, line amplitude in AS_Q at 51.3 Hz and 972.8 Hz, as measured by LineMon.

If we assume that there is just a change by a constant factor in the open loop gain, the ratio of line amplitudes in AS_Q and DARM_CTRL should be exactly the same ones R_i, given by the formula above.

The reference open loop gain at 51.3 Hz is H_0(52.3Hz)=-3.0147 + 2.0605i, and at 972.8 Hz is H_0(972.8Hz)=-0.0536 + 0.3315i. When measuring power spectra, we only measure amplitudes, so

$$R_i(t) = \alpha(t)\frac{|1+H_i|}{|1+\alpha(t)H_i|}$$

$$R_i^2(t) = \alpha^2(t)\frac{|1+H_i|^2}{(1+\alpha(t)\Re H_i)^2 + \Im H_i{}^2}$$

$$R_1^2 = \alpha^2\frac{8.30}{(1-3.01\alpha)^2 + 4.25}$$

$$R_2^2 = \alpha^2\frac{1.01}{(1-0.05\alpha)^2 + 0.11}$$

We have seen that the gain ratio α can only vary between 0.35 and 1.38. We plot in Fig3 the ratios R_1 and R_2 obtained for these gain changes in each calibration line. We see that the ratio of the high frequency line is very linear in the gain change. The ratio of the low frequency line varies in a non-linear way, between 1.4 (for the lowest possible gain) to 0.94 (for the highest possible gain).

From each ratio, we have a quadratic equation for α: $a_i\alpha^2 + b_i\alpha + c_i = 0$, with $a_i = R_i^2|H_i|^2 - |1+H_i|^2$, $b_i = 2R_i^2\Re H_i$ and $c = R_i^2$. For our line frequencies, these functions of the ratios are:

$$a_1 = 13.33R_1^2 - 8.30 \quad a_2 = 0.12R_2^2 - 1.01 \tag{1}$$

$$b_1 = -6.03R_1^2 \qquad b_2 = -0.11R_2^2 \tag{2}$$

$$c_1 = R_1^2 \qquad c_2 = R_2^2 \tag{3}$$

We have then two solutions derived from each ratio, and two ratios:

$$\alpha_\pm(R_i) = R_i\frac{R_i\Re H_i \pm \sqrt{|1+H_i|^2 - R_i^2\Im H_i^2}}{|1+H_i|^2 - R_i^2|H_i|^2}$$

If R=1, we should get back $\alpha = 1$:

$$\alpha_\pm|_{R_i=1} = \frac{\Re H_i \pm |1+\Re H_i|}{1+2\Re H_i} = 1, \quad \frac{-1}{1+2\Re H_i}$$

So, we see that depending on whether the sign of $1 + \Re H_i$ is positive (for 972.8 Hz) or negative (for 51.3Hz), we need to consider α_+ or α_-, respectively. We can also see that for some ratios R_i the solutions may become complex, or tend to infinity: these turn out to be approximately the ratios for R_1 where the loop becomes unstable.

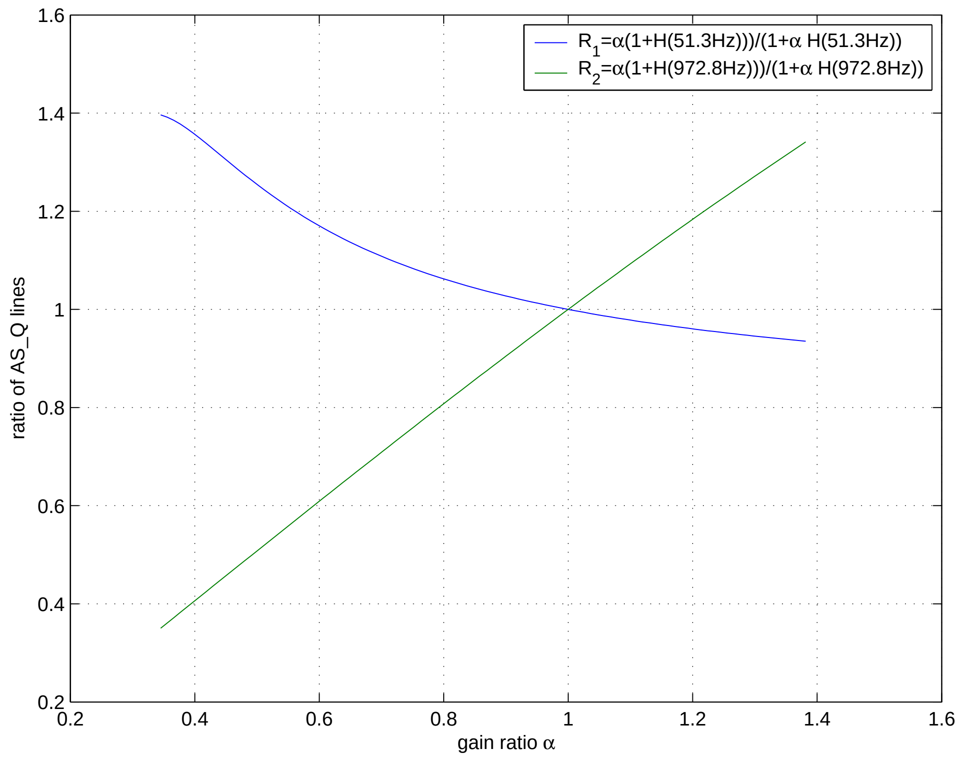

Figure 3: Ratio of calibration lines for a given change α in optical gain, referred to the reference loop gain measured on Sept 6, when $\alpha = R_i = 1$.

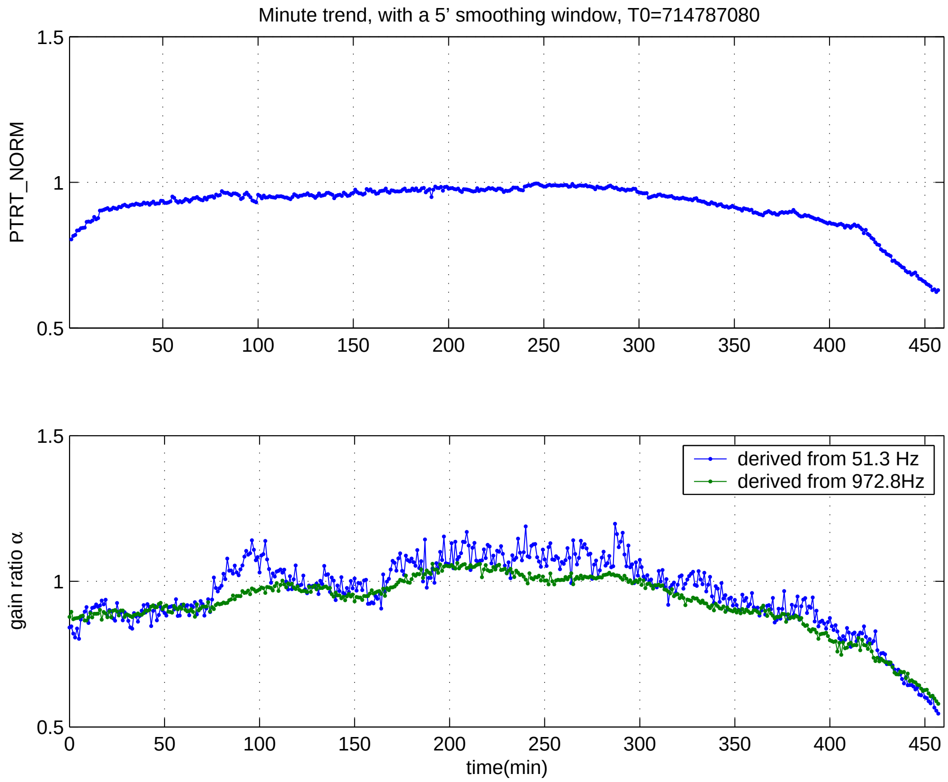

Figure 4: Gain ratio α obtained from the calibration line amplitudes.

We plot in Figure4 the solutions obtained for each ratio. We see that the results agree, but the solutions obtained from the low frequency line are noisier than the ones from the high frequency one. This is probably due to the non-linear formula for the gain ratio derived from this line, as shown in Fig.3. The changes in gain are also seen to qualitatively track the changes in arm power, presumably due to alignment. These calculations seem to confirm the assumption that the gain changes by a single constant factor, related to alignment; and that the changes can be tracked by the high frequency line in AS_Q.