LLO Calibration function for AS_Q during S1

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1 Introduction

This note describes the calibration functions to be applied to LLO-S1 data. We used information from DTT swept sines; and minute trends of amplitude in calibration lines as calculated by Sergey's Klimenko's DMT monitor LineMon.

2 Calibration Information

The differential length excitation produced by the calibration lines, by gravitational waves and by any other sources is detected in AS_Q and DARM_CTRL. Using the notation presented in *Notes on LIGO Detectors' Calibration*, Sept 8, 2002, if the differential length excitation is $X_{ext}(f)$, the sensed signal is

$$AS\_Q = X_{ext}\frac{C(f)}{1+H(f)}$$

where $C(F)$ is the AS_Q sensing function (essentially a cavity pole and a DC gain, in counts/m), $A(f)$ is the actuation function (essentially a pendulum transfer function with a DC gain, in m/count), $G(f)$ is the loop filter function (a complicated transfer function, in counts/counts), and $H(f) = A(f)C(f)G(f)$ is the open loop transfer function. All these functions are complex.

We assume that the sensing function can change its gain, since it depends on the fluctuating alignment. If $C(f) \rightarrow \alpha C(f)$, then

$$AS\_Q \rightarrow X_{ext}\frac{\alpha C(f)}{1+\alpha H(f)}.$$

A few times during S1, the gain of the loop filter function $G(f)$ was changed; if we then also consider a possible scaling $G9f) \rightarrow \beta G(f)$, then

$$AS\_Q \rightarrow X_{ext}\frac{\alpha C(f)}{1+\alpha\beta H(f)}.$$

The open loop gain $H(f)$ and several other functions related to the calibration were measured on several occasions during S1. A fairly detailed Matlab/Simulink model for the different components in the DARM loop was used to fit the measured functions. A DMT tool by Sergey Klimenko, LineMon, was used to track the amplitude of calibration lines that were injected to push one of the ETM mirrors. Finally, all this information was put together to obtain a calibration function AS_Q/X_{ext} as a function of time during S1.

3 S1-LLO Science segments

Starting times of “Science segments” were defined by the interferometer being locked and in common-mode (most sensitive) mode, and by the operator in charge having pressed the “science mode” button on a screen, usually after having aligned the interferometer to a satisfactory point. The segments were ended when the interferometer lock status was broken spontaneously, or by any change being performed to the status of the interferometer (when moving a mirror to align, for example). The times of these segments were retrieved by Peter Shawhan’s conlog tool. There were 144 science segments longer than 300 seconds, with start times, stop times and duration is included in Appendix 1. The total time in the science segments adds up to 2010 minutes, slightly more than 170 hours.

4 Calibration Lines

The data taking run S1 started on Aug 23, 15:00 UTC and ended on Sept 09, 15:00 UTC. As state din the LLO e-log, two calibration lines were injected to drive ETMX beginning on August 25, 8:29 UTC, at 36.75 and 972.8 Hz. The lower frequency line was shifted to 51.3 Hz on Aug 25 20:47 UTC (GPS 714343633).

The lines’ amplitudes, SNR, and phases were tracked by a DMT tool written by Sergey Klimenko, LineMon, providing a minute trend. Each estimate is associated with the GPS time indicating the beginning of the minute where the estimate was done.

The lines’ injections were started by a line command typed by the operator in charge, and the process could be stopped if the corresponding “test points” were cleared. This led to some times when the lines were not injected, or were stopped by mistake. Here is a summary information about some special science segments during S1 concerning calibration lines, as detected by the LineMon results:

- Since the lines were started after the beginning date of the run, there are no lines present in the first 16 segments (GPS 714179393-714289919).

- In Segment #17, GPS 714298377-714300184, LineMon for the 972.8 Hz line shows a small amplitude consistent with noise, then a large amplitude of 0.05 counts, then a stable amplitude close to average (0.012 counts), 15min after start of the segment. The amplitude of the line stabilized at amplitudes between 0.010 and 0.015 counts after the minute starting at GPS time 714299280, or Aug 25, 2002 08:27, coinciding with the time stated in the LLO e-log for the start time of the injection of the lines.

- In many segments, the last minute estimate provided by LineMon for the segment is zero. This probably happens because the LineMon estimate if the “BothArmsLocked” condition is satisfied for all seconds of the segment, and this condition is more stringent than the ones used to select the science segments. Since this is probably an indication of rapidly degrading conditions for the interferometer, it seems advisable to disregard the last minute of the segment in these cases. We list here the segments from the list in the Appendix 1 where this last minute should be disregarded:

  17, 19, 24, 32-38, 40-42, 47, 49, 50, 53-55, 57, 60, 67, 78, 80, 82, 84-88, 94, 96, 98, 99, 103-107, 116-121, 123-124, 126-128, 134, 137, 141

- In the first 10 minutes of Segment #20, GPS 714321763-714325909 (starting Aug 25, 2002 14:42 UTC), the 972.8Hz line is absent. After the minute starting on 714322500, LineMon reports a normal amplitude.

- Segment #26, GPS 714345572-714353262, starting Aug 25, 2002 21:19 UTC, is the first science segment where the 51.3 Hz line is detected by LineMon, with an amplitude of 3 counts. This is consistent with the time stated in the e-log when the lower freq line was switched from 36.8Hz to 51.3 Hz. However, in this segment, both lines (972.8Hz and 51.3 Hz) seem to have been switched off sometime in the minute starting at 714345960, then switched back on sometime in the minute starting at 714346620.
- Lines are absent in the segments #70-72, between GPS times 714895157 and 714916227. This was probably due to a forgotten initialization command.
- Both lines disappear in the minute starting at 714955380, in the middle of segment #76 (714954703-714956697). The lines are not present in the segment that follows, #77 (714956883-714957527), and are both present again in segment #78 (714957889-714959107).
- Both lines disappear in the minute starting at 715129560, in segment #98 (715129015-715131001). They are reported with normal amplitude again in the same segment, after the minute starting at 715129740.
- The injected line at 51.3 Hz has 10 times the normal amplitude in segments #107-109, between 715214671 and 715239557. This was due to a mistyped command in the line injection (noted in the elog).

5 Model for S1-LLO DARM loop

We use a Simulink interferometer model, based on Rana's "darm_08.mdl", called "S1darm_08.mdl", shown in Fig1.

The parameters used to load the model are in a file S1darm.m, and are as follow (following the diagram, starting from AS_Q)

- Input matrix constant gain asq2darm=0.014 (from conlog).
- LSC Digital Filters

  We used "Foton" to translate the numbers in the coefficient filters file (/cvs/cds/llo/chans/L1.txt) used during S1, to zeros and poles to use in LTI Matlab models. The filters are digital filters, but we use them in Matlab/Simulink as analog filters (we haven't yet figured out hot make compatible digital and analog filters in the linearization programs). This will introduce errors when getting close to the Nyquist frequency, 8kHz.

  ```
  FM1=zpk([-75.757+i*153.31;-75.757-i*153.31],[0;0],1.99078);

  FM2=zpk([-62.8318;-628.241],[-4389.6-i*4389.6;-4389.6+i*4389.6],976.28);

  FM3=zpk([],[-8485+i*8485;-8485-i*8485],1.43991e+08);

  FM4=zpk([-125.663],[-6.28319],0.996375);

  FM8 = zpk([-6207.83;-6207.84],...

  [-25045.8+i*0.00338783;-25045.8-i*0.00338783],16.2775);

  FM9=zpk([-126.26+i*255.513;-126.26-i*255.513],...

  [-12.6263+i*25.552;-12.6263-i*25.552],1.00005);

  DARMDF= FM1 * FM2 * FM3 * FM8 * FM9;
  ```

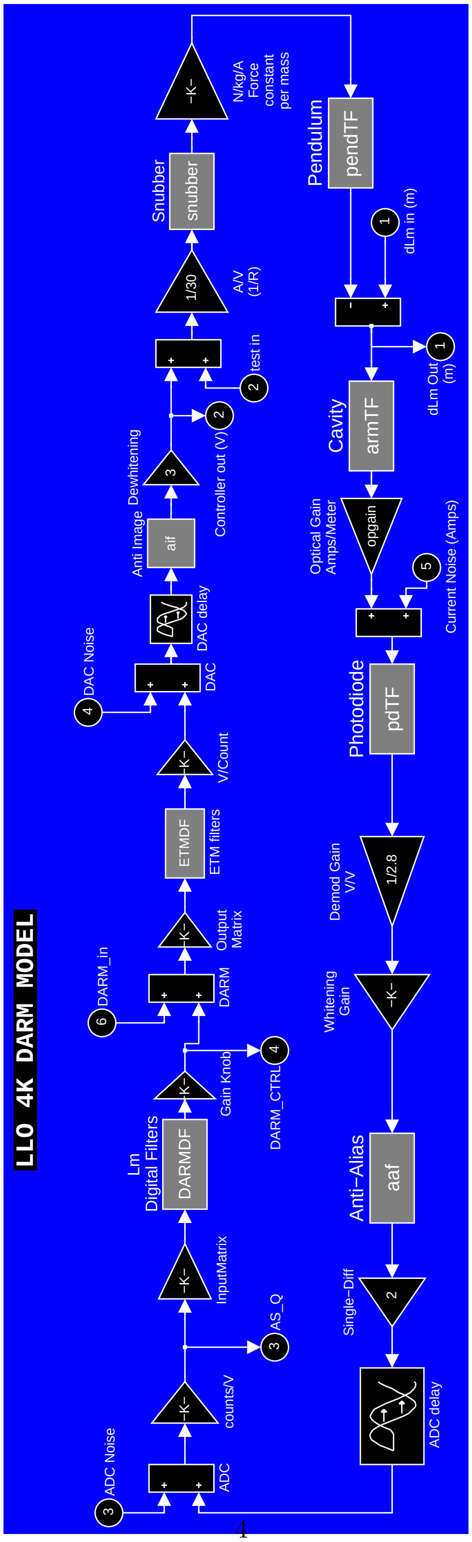

Figure 1: Simulink model S1darm_08.mdl

- Gain knob: GW_K. This gain is negative in the LSC code, but we need a negative sign somewhere to make the Simulink loop stable, as the real loop was (this was probably a minus sign in some whitening/dewhitening board).

  This gain was changed a few times during S1. From conlog queries, L1:LSC-DARM_ERR_K in science segments had the following values:

  ```
  714179393-714212083 -0.187838
  Aug 23, 2002  23:09:40 UTC - Aug 24, 2002  08:14:30 UTC

  714215476-714250468 -0.249838
  Aug 24, 2002  09:11:03 UTC - Aug 24, 2002  18:54:15 UTC

  714256623-714340429 -0.178133
  Aug 24, 2002  20:36:50 UTC - Aug 25, 2002  19:53:36 UTC

  714345572-715395019 -0.231601
  Aug 25, 2002  21:19:19 UTC - Sep 07, 2002  00:50:06 UTC

  715406173-715406478 -0.468361
  Sep 07, 2002  03:56:00 UTC - Sep 07, 2002  04:01:05 UTC

  715408880-715497528 -0.327985
  Sep 07, 2002  04:41:07 UTC - Sep 08, 2002  05:18:35 UTC

  715498597-715527400 -0.231601
  Sep 08, 2002  05:36:24 UTC- Sep 08, 2002  13:36:27 UTC

  715564704-715580548 -0.327985
  Sep 08, 2002  23:58:11 UTC - Sep 09, 2002  04:22:15 UTC
  ```

- Output matrix darm2lm=-0.33 (from output matrix to ETMs, also in conlog).
- ETM filters: there were steep stopbands near the internal modes (near 7 kHz) and a violin mode notch (near 345 Hz) used for the ETM drive. We could not include the stopband filters without running into what looked like Simulink numerical problems. Since they are outside the gw band, we left them out. Thus, we use just the violin notch for the test mass filter:

  ```
  violin = zpk([0+i*2152.47;0-i*2152.47;0+i*2156.2;0-i*2156.2], [-10.7623+i*2152.44;-10.7623-i*2152.4
  -10.781-i*2156.18],1);
  ```

  `ETMDF=violin;`
- DAC gain= (5/32768) Volt/count
- DAC delay: 75μsec. This delay was found to get a good fit to the loop gains.
- Anti-imaging filter: `[z,p,k] = ellip(4,4,60,2*pi*7570,'s'); aif = zpk(z,p,k*`$10^{4/20}$`);`

- Dewhitening gain=3
- Voltage to current gain: 1/30 Amp/Volt
- Snubber =0.63 (this factor is fitted to get the measured DC drive of 1.72nm/ct for DARM_CTRL, it probably represents the effective gain in the output matrix of the controller)
- Coil/pendulum gain: 0.064/10.3 N/kg/Amp
- Pendulum transfer function (m/N/kg)

  ```
  Q = 10;
  w0 = 2*pi*0.76;
  pendTF = 2 * tf(1,[1 w0/Q w0$^2$]);
  ```

  The factor of 2 accounts for Ly-Lx.

- Arm Cavity

  `fc = 87.3; % Hz`

  `armTF = 2*pi*fc * tf(1,[1 2*pi*fc]);`

- Optical gain: 0.47e7 Amps/meter. This gain is multiplied by a fitted factor depending on alignment.
- LSC Photodiode gain: pdTF=400*10 V/Amp.
- Demodulation gain=1/2.8
- Whitening gain: $10^{18/20}$
- Anti-Aliasing filter:

  `[z,p,k] = ellip(8,0.035,80,2*pi*7570,'s');`

  `aaf = zpk(z,p,k);`

- Single-to-differential gain: 2
- ADC delay: 50μsec.
- ADC gain: 32768/10 counts/Volt

The measured DC calibration for DARM_CTRL into effective mirror motion (for (Ly-Lx)/2?) was 1.72nm/count. In this model, that gain is made up of the output matrix (0.33), times the DC gain of the ETM filters (1.0), times the DAC gain (5/32768 Volts/counts), past the DAC delay (unity gain), the antiimaging filter (unity gain), times the dewhitening gain (3.0), times the voltage-to-current gain (1/30 Amp/Volt), times the force constant (0.064/10.3 N/m/Amp), times the DC pendulum gain (1/w0^2=0.044 m/N/kg). This product is:

x= 0.33*(5/32768 V/ct)*3*(1/30 A/V)*(0.064/10.3 N/kg/A)*(0.044m/M/kg)*DARM[ct] =2.74 (nm/ct)* DARM[ct]

Since the measured value is 1.72nm/count, we use the snubber "fudge factor" of 1.72/2.74=0.63. As mentioned before, this factor is probably due to the coefficients in the output matrix for the length to each coil drive, which average 85% of the full value (for minimizing length to angle drive at DC).

6 Calibrations done during the run

Several times during the run, a measurement was done of the loop gain in the DARM loop. We will use one of these measurements, done on September 6, as our "standard". We describe in the next subsection the results of this measurement in gory detail, and then we'll briefly report on the other measurements.

6.1 LLO calibration for Sept 6, 2002 23:00 UTC

We compare the results of this model to the measured swept sine on Sept 06, 2002 at 23:02 UTC, saved in `darm_loop_020906.xml`. We fit a factor to multiply the optical gain so that the loop gains measured and modeled agree. At the calibration time, the loop gain used was GW_K=0.231601.

The swept sine was done injecting a signal in DARM_ERR_EXC. The response of MICH_CTRL, AS_I, AS_Q, REFL_I, REFL_Q, DARM_ERR, CARM_CTRL and DARM_CTRL were measured in DTT, with 7 "A" channels. The measurement was done for frequencies between and 36.7 Hz and 7.4kHz. The open loop gain is the transfer function of AS_Q times the darm2asq gain (0.014) to DARM_ERR. In DTT, this can be plotted using a Yslope=0.014 in the "Units" menu, and is a fast way to check the loop gain, especially near the unity gain frequency. In this measurement, the unity gain is at about 250 Hz. From DTT, we exported the transfer function AS_Q/DARM_ERR, and compared it with our Matlab/Simulink model.

To use the model, we first linearize it using the "linmod" function, and create a state-space representation. All the parameters of the Simulink model from the previous section are collected in the "S1darm.m" file, then the Simulink model is called, and the linearization is done in Matlab as follows:

```
S1darm
S1darm_08
[A,B,C,D] = linmod('S1darm_08');
darmsys = ss(A,B,C,D);
```

The open and closed loop gains can then be calculate in Matlab using bode plot functions (the function "mybodesys" is similar to "bode", but uses frequency in Hertz):

```
f=logspace(log10(f1),log10(f2),1e4);
F=mybodesys(darmsys(2,2),f);
CLG=mybodesys(darmsys(1,1),f);
OLG=-F./CLG;
```

The comparison between measured and modeled open loop gains is shown in Fig2. The best agreement is found with an alignment gain (multiplying the optical gain) of 1.2. The unity gain found in the model with a finely space frequency vector is 248 Hz. The ratio of the magnitudes is within ten percent between 40 Hz and 1 KHz. We believe the growing discrepancy at higher frequencies is due to the analog treatment of the digital filters. The comparison of the measured and modeled phase is within 5 degrees up to 2 kHz.

Finally, we present a calibration, valid for Sept 06, 22:52 UTC (when a clean locked section started) to 23:02 (when the swept sine started). Notice that this is NOT a "science" segment, since calibration procedures were being made. However, the alignment and sensitivity during the time of observation for noise spectra were typical.

1. **Calibration function Cb(f) for AS_Q, in strain/counts.**

   This is obtained from the Simulink model, for a frequency vector evenly spaced from 0Hz to 2048Hz, with 1/64 Hz spacing:

   ```
   S1darm
   ```

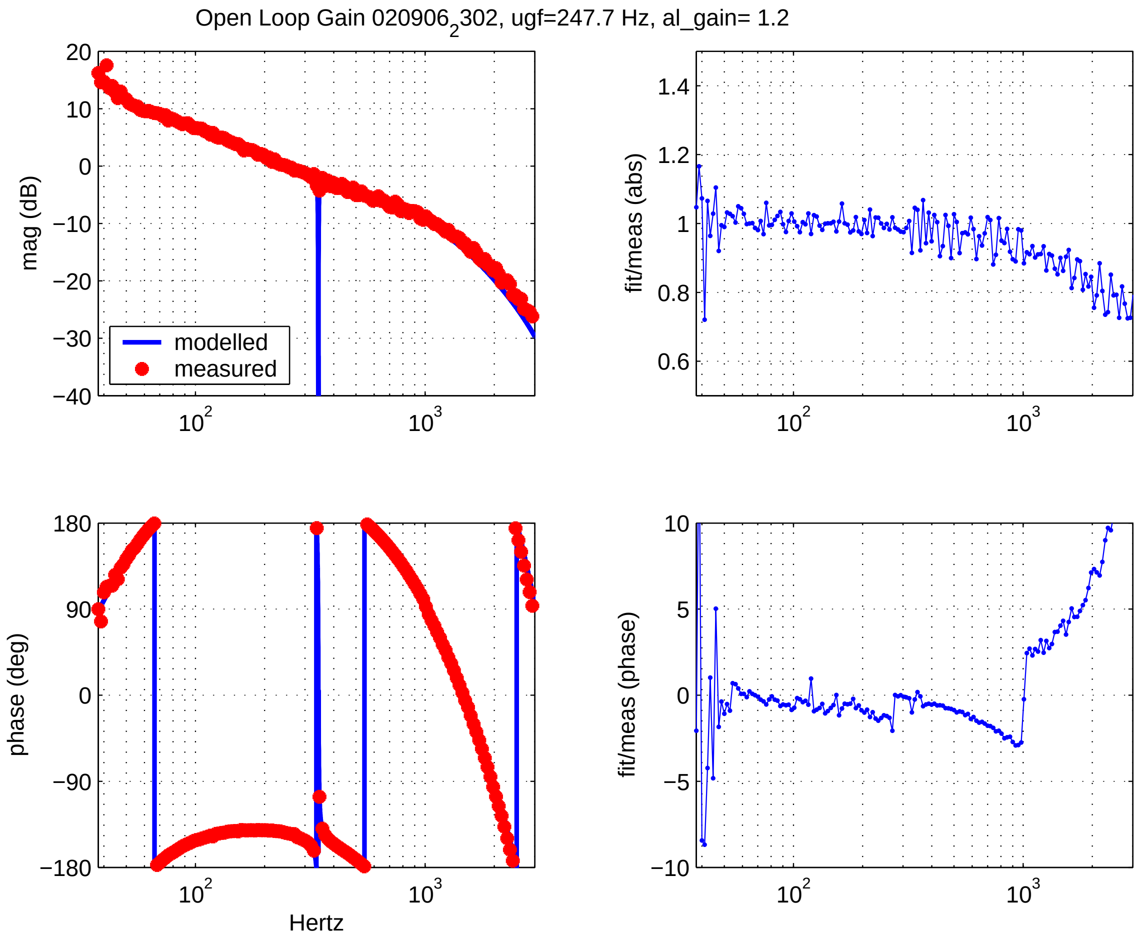

Figure 2: Comparison between swept sine measurement on Sept 06 and modelled gain.

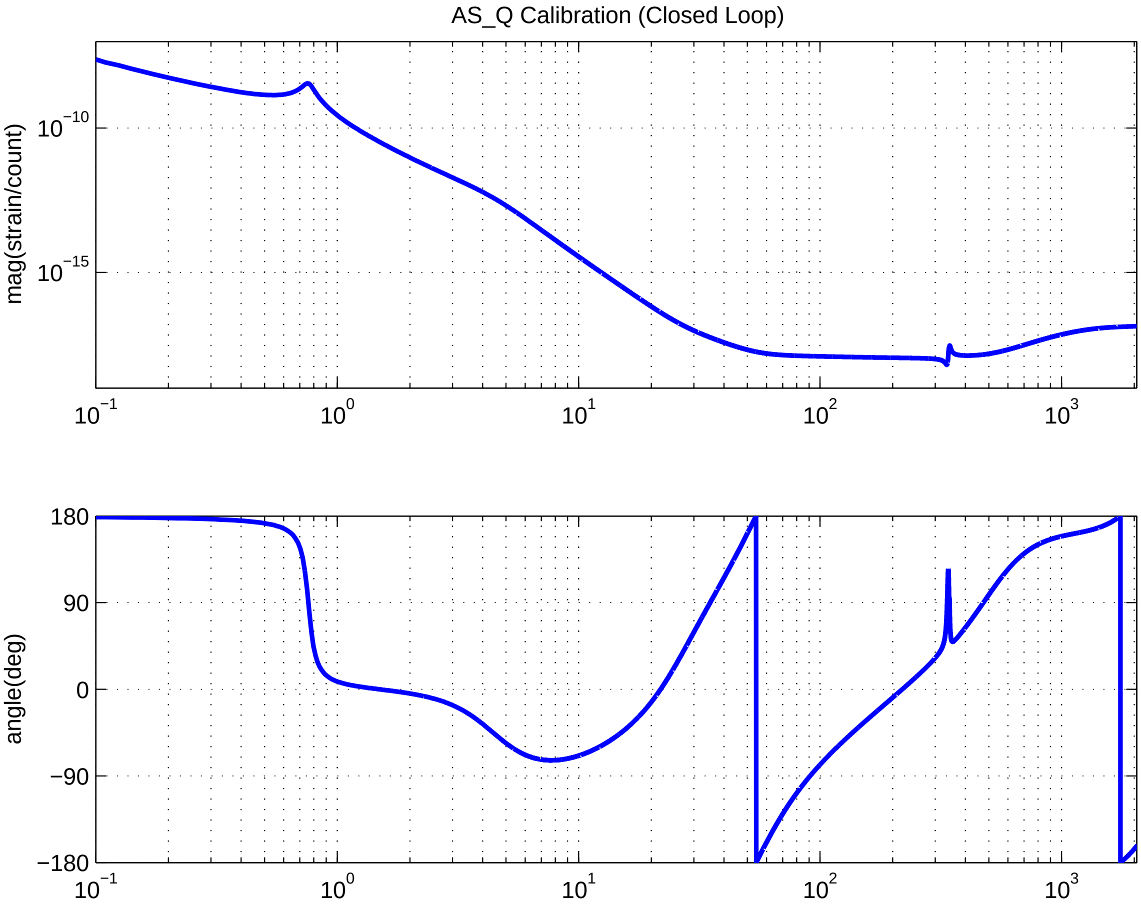

Figure 3: Calibration function for ASQ.

```
S1darm_08
[A,B,C,D] = linmod('S1darm_08');
darmsys = ss(A,B,C,D);
fd=[0:1/64:2048];
calib=1./mybodesys(darmsys(3,1),fd);
calib=calib/4e3;
```

The output is plotted in Figure3, and the frequency triplets (frequency, magnitude and phase) written in the file ASQCalibration.txt.

2. **Sensing function C(f) for AS_Q, in counts/strain.**

   This function is the inverse of the calibration function calculated above, except that now the loop is open. This function involves essentially just the cavity pole transfer function. To keep consistency and

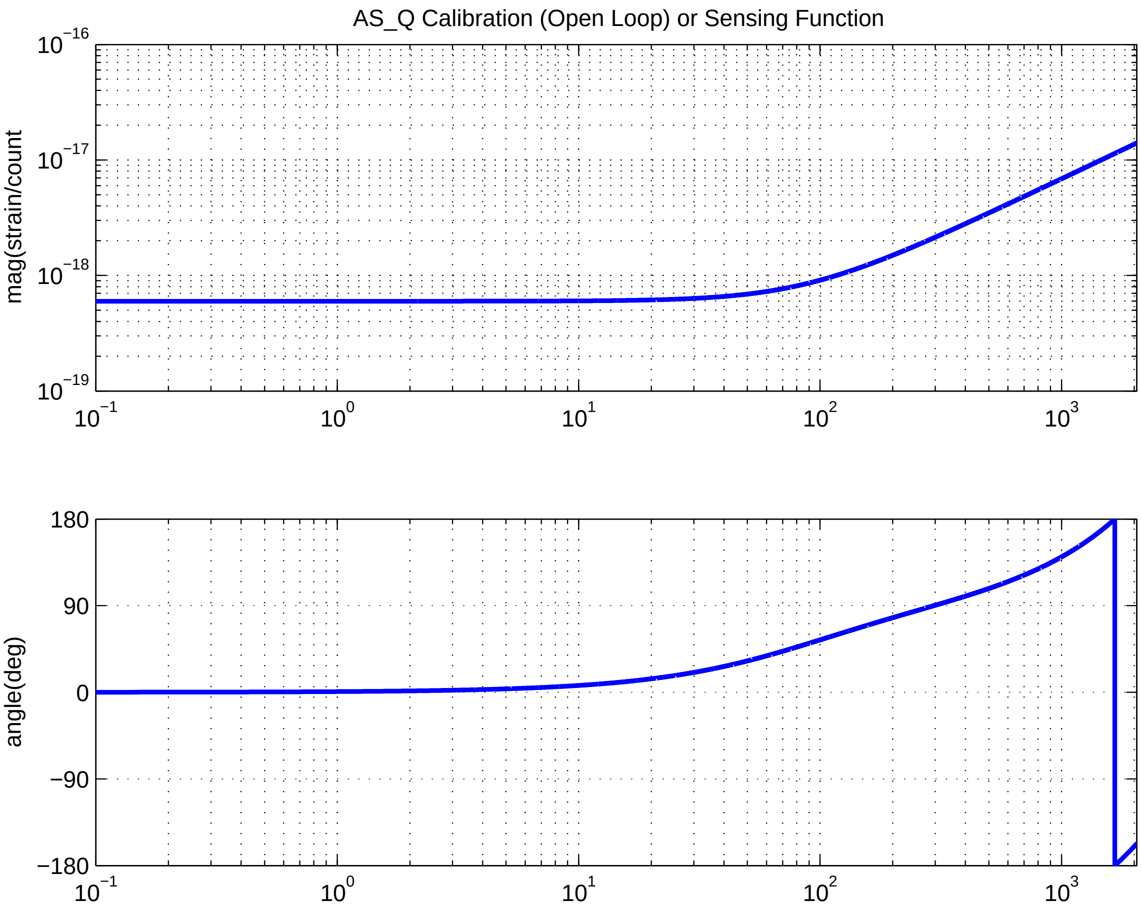

Figure 4: Calibration function for ASQ.

include the effects of the ADC time delay included in the model, we obtain the sensing function with another Simulink model that has the servo links broken. The function is plotted in Figure4, and the frequency triplets written in a file ASQSensing.txt

The sensing function was

$$C(f) = \frac{1}{4\text{km}} \frac{4.9 \times 10^{14}\text{counts/m}}{1 + s/(2\pi 87.3\text{Hz})} \times AAF(f),$$

where $AAF(f)$ is an 8th order, 80dB attenuation analog elliptic antialiasing filter at 7.57 kHz.

3. Open Loop Gain H(f)

We calculate the open loop gain, again from the Simulink model, that is related to the calibration and sensing functions through $Cb(f) = (1 + H(f))/C(f)$. This function is calculated as the forward loop

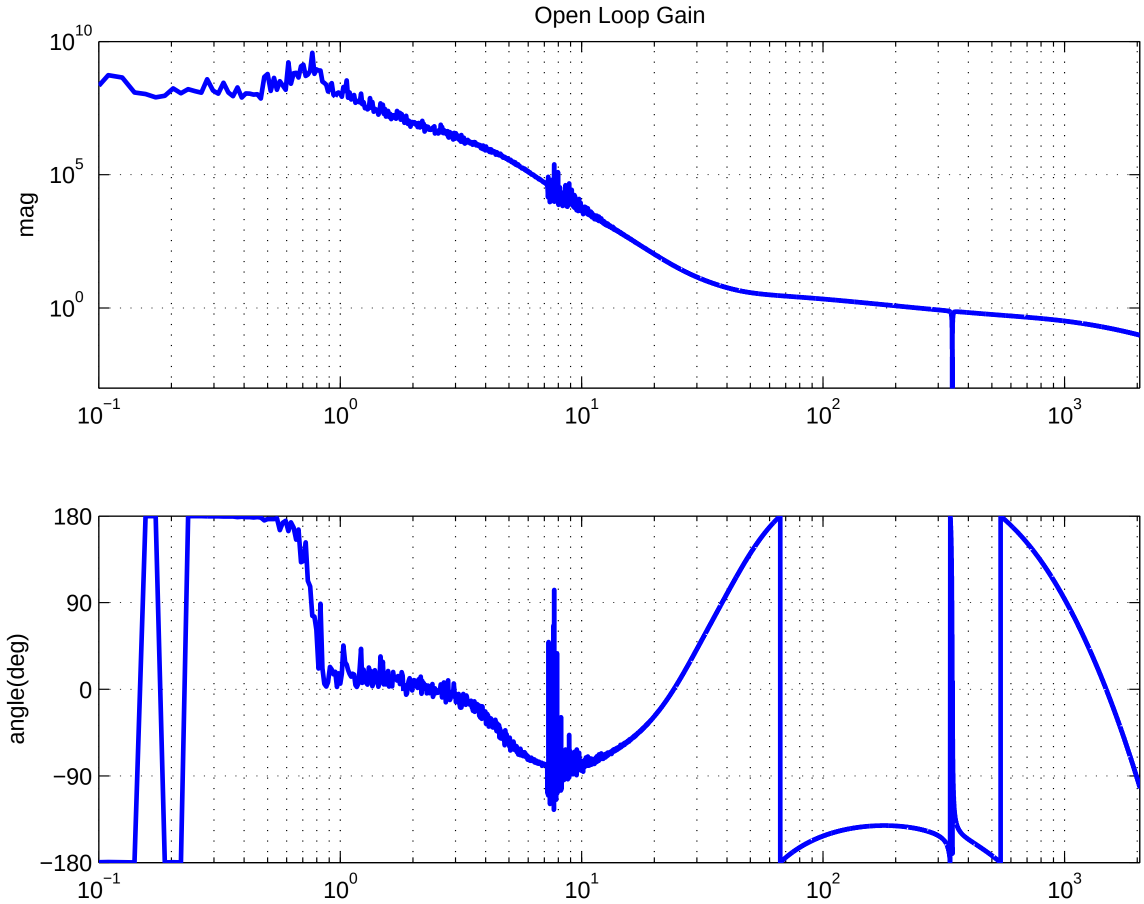

Figure 5: Open Loop Gain from S1darm_08.

function darmsys(2,2) divided by the closed loop function darmsys(1,1). Since this function has a very large dynamic range, there are numerical errors visible at low frequencies. We can better estimate the function from the model by using the analytical expressions known for each block, but for consistency we use the same method as used when fitting the alignment gain to the loop gains. The (noisy) curve is plotted in Figure5, and the triplets (always for the same frequency vector) saved in the file OpenLoopGain.txt.

The unity gain frequency of the DARM loop was 248 Hz (with 35 degrees of phase margin). The maximum phase margin (39 degrees) is obtained when the unity gain frequency is 179 Hz, which would need the gain to be 25% lower. We see that the gain can only be increased by 38% before the loop goes unstable at the frequency of 336Hz. The gain can be decreased by a factor of 0.35 before the loop becomes unstable at a frequency of 66 Hz.

We will see later that the fluctuations in the calibration depend mostly on the levels of buildup power in the arms, and of the sideband in the recycling cavity. Useful indicators of these alignment-dependent powers are the channels L1:LSC-LA-PTRT_NORM, L1:LSC-LA-PTRT_NORM, and L1:LSC-LA-SPOB_NORM. We expect the calibration to follow the fluctuations in the parameter calculated as the square root of (L1:LSC-LA-PTRT_NORM +L1:LSC-LA-PTRT_NORM)*L1:LSC-LA-SPOB_NORM.

Between the times 715387980 (22:53 UTC) and 715389120 (23:11 UTC), when the swept sime for the calibration described was being done and when we will assume it is valid, the average power level in PTRT_NORM (from a minute trend in that period) was 1587 ± 7, in PTRR_NORM was 1403 ± 7 and SPOB_NORM was 123 ± 2. The minute trends of LineMon for the AS_Q lines in the same period were 3.0 ± 0.1 counts for the 51.3 Hz line (A151) and $(16.6 \pm 0.3) \times 10^{-3}$ counts for the 972.8 Hz lines (A1972).

6.2 Other LLO calibrations

Several other measurements of loop gains were made during the S1 run. The actual measurements are plotted in Fig6. As expected, the open loop gains change by a constant scaling factor, leaving the phase unchanged.

Each measurement was fitted to the Simulink model, using the corresponding GW_K gain, and fitting a factor, al_gain, to multiply the nominal optical gain. The results for these fitted gains, including the standard Sept 06 measurement described in the previous sections, are as follow:

Meast. Date (UTC)	GW_K	al_gain
Aug 24 08:18	0.249838	0.80
Aug 25 04:52	0.178133	1.00
Aug 25 10:18	0.178133	0.90
Aug 26 23:49	0.231601	0.95
Sept 06 23:02	0.231601	1.20
Sept 09 02:17	0.327985	0.75

7 Calibration versus Time

We expect the optical gain parameter to change with alignment, in particular with fluctuations of the square root of the mean arm power, times the sideband amplitude in the recycling cavity. These fluctuations can be tracked with the 16Hz channels PRTR_NORM, PTRR_NORM (measuring arm power), and SPOB_NORM (measuring sideband amplitude). There are observed fluctuations of 10-30% or more in a given science segment, especially if it is long, and if alignment degrades at the end. What follows is the details on how we used Sergey Klimenko's LineMon results on estimation of calibration lines to track the changes in optical gains.

We have two numbers for each minute during S1, the amplitude of the two calibration lines in AS_Q. We will assume the amplitude of the motion produced with the injected lines is the same as in the reference time, and take ratios between the amplitudes at any given time and the amplitudes at the reference time. These ratios will be:

$$R_i(t) = \frac{AS\_Q(f_i,t)}{AS\_Q(f_i,t_0)} = \frac{C(f_i,t)}{C_0(f_i,t)} \frac{1+H_0(f_i,t)}{1+H(f_i,t)}$$

We can begin assuming the simplest case, where the only change is due to alignment, and the sensing function $C(f)$ differs from $C_0(f)$ by a constant $C(f,t) = \alpha(t) C_0(f,t_0)$. Then,

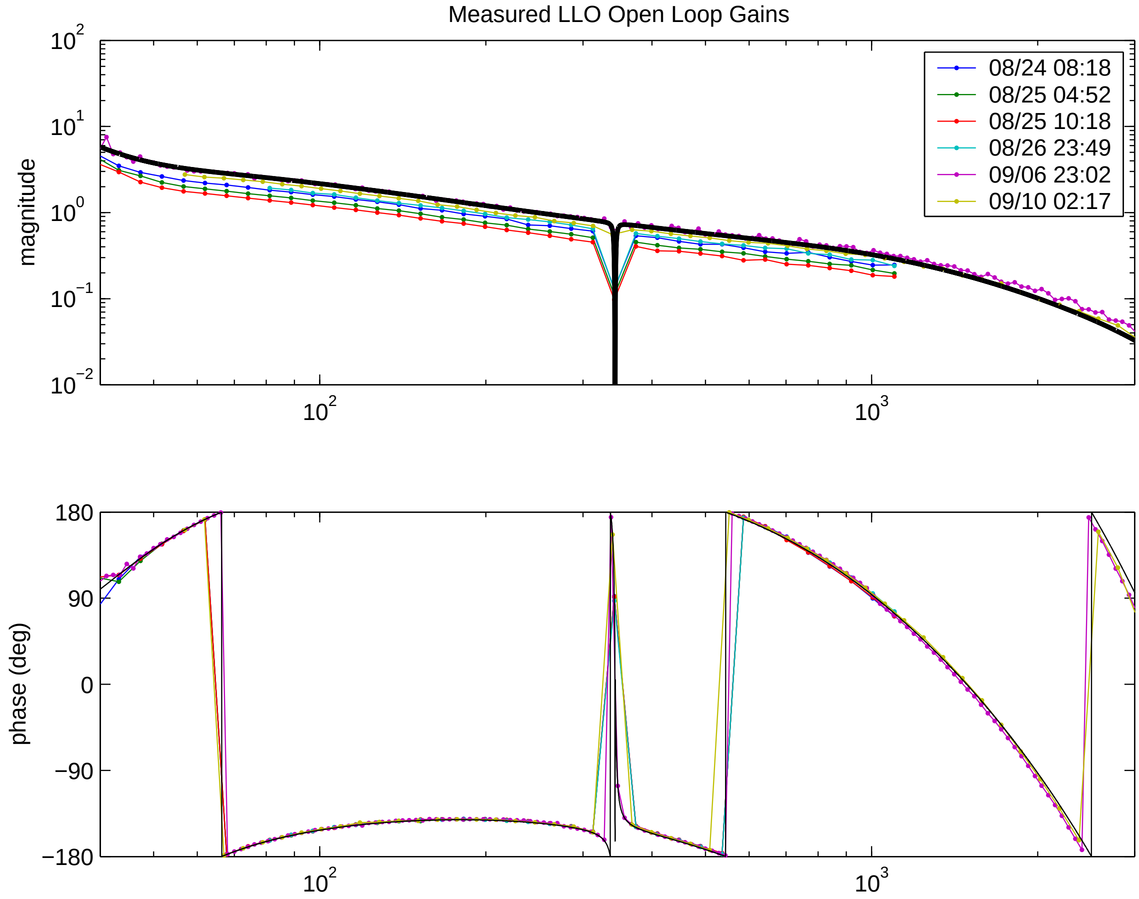

Figure 6: Open Loop Gain measurements for the LLO DARM loop.

$$R_i(t) = \alpha(t)\frac{1 + H_0(f_i)}{1 + \alpha(t)H_0(f_i)}$$

If we assume that there is just a change by a constant factor in the open loop gain, the ratio of line amplitudes in AS_Q and DARM_CTRL should be exactly the same ones R_i, given by the formula above.

The reference open loop gain (measured on Sept 06) at 51.3 Hz is H_0(52.3Hz)=-3.0147 + 2.0605i, and at 972.8 Hz is H_0(972.8Hz)=-0.0536 + 0.3315i. When measuring power spectra, we only measure amplitudes, so

$$R_i(t) = \alpha(t)\frac{|1 + H_i|}{|1 + \alpha(t)H_i|}$$

$$R_i^2(t) = \alpha^2(t)\frac{|1 + H_i|^2}{(1 + \alpha(t)\Re H_i)^2 + \alpha^2 \Im H_i{}^2}$$

$$R_1^2 = \alpha^2 \frac{8.30}{(1 - 3.01\alpha)^2 + \alpha^2 4.25}$$

$$R_2^2 = \alpha^2 \frac{1.01}{(1 - 0.05\alpha)^2 + \alpha^2 0.11}$$

From each ratio, we have a quadratic equation for α: $a_i\alpha^2 + b_i\alpha + c_i = 0$, with $a_i = R_i^2|H_i|^2 - |1 + H_i|^2$, $b_i = 2R_i^2\Re H_i$ and $c = R_i^2$. For our line frequencies, these functions of the ratios are:

$$a_1 = 13.33R_1^2 - 8.30 \quad a_2 = 0.12R_2^2 - 1.01 \tag{1}$$

$$b_1 = -6.03R_1^2 \qquad b_2 = -0.11R_2^2 \tag{2}$$

$$c_1 = R_1^2 \qquad c_2 = R_2^2 \tag{3}$$

We have then two solutions derived from each ratio, and two ratios:

$$\alpha_\pm(R_i) = R_i \frac{R_i\Re H_i \pm \sqrt{|1 + H_i|^2 - R_i^2 \Im H_i^2}}{|1 + H_i|^2 - R_i^2|H_i|^2}$$

If R=1, we should get back $\alpha = 1$:

$$\alpha_\pm|_{R_i=1} = \frac{\Re H_i \pm |1 + \Re H_i|}{1 + 2\Re H_i} = 1, \quad \frac{-1}{1 + 2\Re H_i}$$

So, we see that depending on whether the sign of $1 + \Re H_i$ is positive (for 972.8 Hz) or negative (for 51.3Hz), we need to consider α_+ or α_-, respectively. We can also see that for some ratios R_i the solutions may become complex, or tend to infinity: these turn out to be approximately the ratios for R_1 where the loop becomes unstable. So, for all realizable ratios, we should obtain a real solution to the equations.

We also need to assume that the filter function $G(f)$ can change by a factor β, to include the known changes in the loop parameter GW_K. Considering both parameters α and β, we see that $C \to \alpha C$ and $H \to \alpha\beta H$, so the ratio of the calibration lines amplitudes is

$$R_i(t) = \alpha(t)\frac{|1 + H_i|}{|1 + \alpha(t)\beta(t)H_i|}$$

In principle, we could derive both parameters α and β from the two calibration lines. However, since the parameter β is known, we prefer to use the calibration lines to solve for α and have a consistency check. The quadratic equation to solve for can be written as an equation for $alpha\beta$:

$$R_i^2(t) = \alpha^2(t)\frac{|1+H_i|^2}{(1+\alpha\beta\Re H_i)^2 + \alpha^2\beta^2\Im H_i{}^2}$$

$$(1+2\alpha\beta\Re H_i + \alpha^2\beta^2\Re H_i{}^2 + \alpha^2\beta^2\Im H_i{}^2)R_i^2 = \alpha^2(t)|1+H_i|^2$$

$$(\alpha\beta)^2(R_i^2|H|^2 - |1+H_i|^2/\beta^2) + 2(\alpha\beta)R_i^2\Re H_i + R_i^2 = 0$$

So, we have a quadratic equation for the loop gain parameter $\alpha\beta$ with coefficients depending on the lines' ratios and the gain parameter β: $a_i = R_i^2|H|^2 - |1+H_i|^2/\beta^2$, $b_i = 2R_i^2\Re H_i$ and $c_i = R_i^2$.

From the known changes in DARM_ERR_K values noted in previous sections, we have 6 different possible values of β: 0.77, 0.81, 1, 1.11, 1.42 and 2.02. For each of these values, we prepare a table of solutions for $\alpha\beta$ as a function of line amplitudes ratios R_i. We plot in Fig7 the solutions obtained for $\alpha\beta$ for each of the possible values of β, as a function of the ratio of the amplitude of the calibration lines to the standard one on Sept 06. We see that the ratio of the high frequency line is very linear in the gain change. The ratio of the low frequency line varies in a non-linear way, making it less useful for deriving gain parameters.

8 Examples

8.1 Longest segment in S1

We plot in Figure8 the solutions obtained for each ratio for the longest S1 segment, starting on GPS 714787127. We see that the results agree, but the solutions obtained from the low frequency line are noisier than the ones from the high frequency one. This is probably due to the fact that the estimate of the low frequency line is noisier, and to the the non-linear formula for the gain ratio derived from this line, as shown in Fig.7. The changes in gain are also seen to qualitatively track the changes in arm power, presumably due to alignment.

8.2 From one calibration to another

We can use the calibration line tracking to see if we can recover some of the other actual LLO calibrations from the September 06 one. As described previously, the calibration done on Aug 25 at 10:18 uses GW_K=0.178133 and is best fitted with a alignment factor 0.88 multiplying the optical gain. This means we have factor $\beta = 0.178133/0.231601 = 0.77$, a factor $\alpha = 0.88/1.2 = 0.73$, and a gain factor $alpha\beta = 0.73 \times 0.77 = 0.56$.

The calibration lines at 972.8 Hz has an amplitude reported by LineMon equal to 0.0120 ± 0.0001 between GPS times 714305880 (Aug 25, 2002 10:17:47 UTC) and 714306060 (10:20:47 UTC). (The lower frequency line was being injected at a different frequency at this time, so we only have the estimate for one line.) As a ratio to the standard line amplitude in September 6, $(16.6 \pm 0.3) \times 10^{-3}$, we have $R_2 = 12.0/16.6 = 0.723$. From the data used in the plots in Fig.7 for $\beta = 0.77$, we get that this amplitude ratio corresponds to a gain factor $\alpha\beta = 0.55$. This is then (amazingly) consistent with the measured loop gain factor, within 2%.

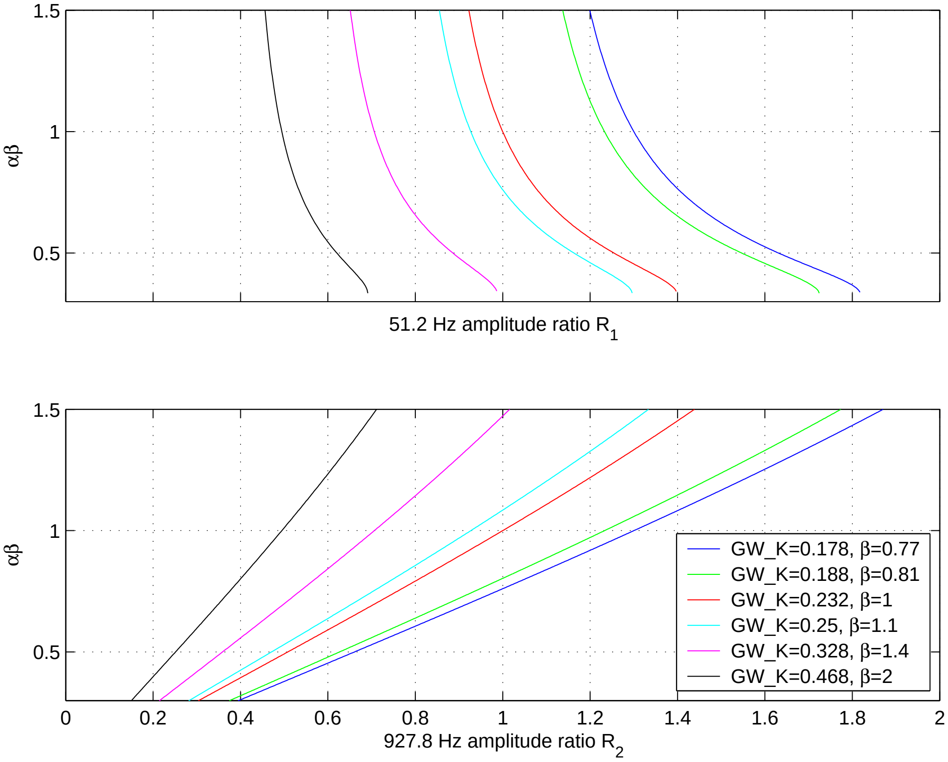

Figure 7: Solutions obtained for $\alpha\beta$ for each of the possible values of β, as a function of the ratio of the amplitude of the calibration lines to the standard one on Sept 06.

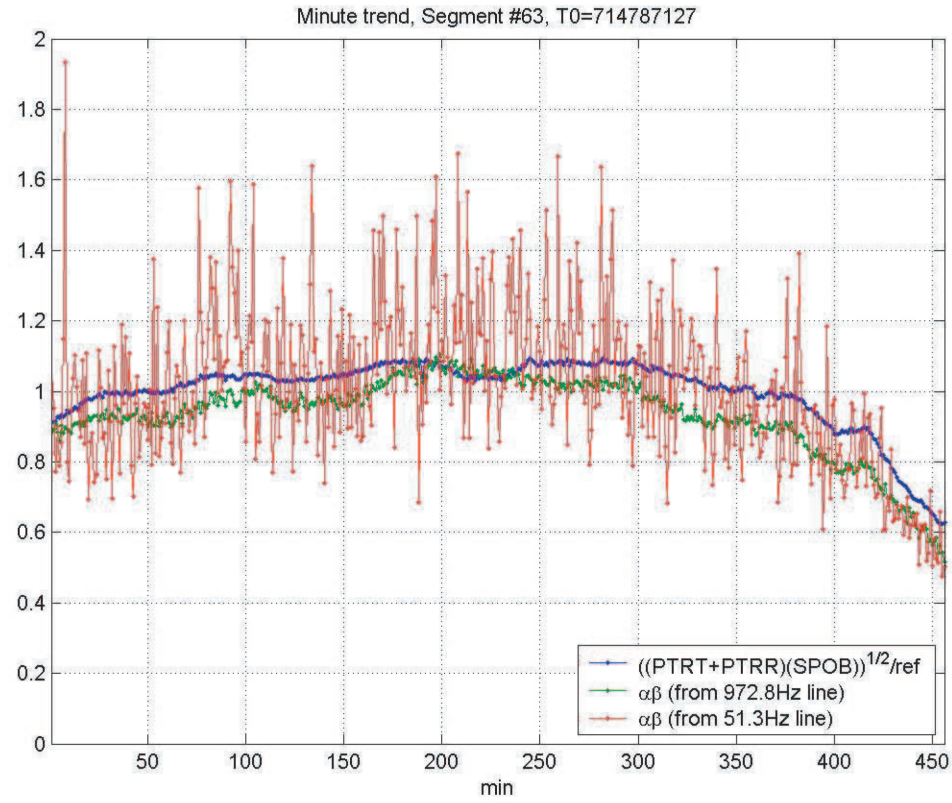

Figure 8: Gain ratio α obtained from the calibration line amplitudes, compared with the expected alignment function determining the optical gain.

9 Final Result

Finally, we can then report on a LLO calibration versus time for science segments in S1. The elements of this calibration are as follow:

- **A standard open loop gain $H(f)$ and a sensing function $C(f)$.** These are the functions derived from the September 06 calibration, described in previous sections.

- **Functions $\alpha(t)$ and $\alpha\beta(t)$**. These are derived from the LineMon estimates of the 972.8Hz calibration line, and the know values of β. These are ratios to the standard values on Sept 06.

  A plot of these values is shown in Fig9. The data itself is found in the file `www.phys.lsu.edu/faculty/gonzalez/S1cali` We include in Appendix 2 a mean value and the standard deviation for the $\alpha\beta$ values found in each science segment.

- **A scaling law**. The calibration function for AS_Q in strain/counts is then obtained as follows:

$$Cb(f) = \frac{1 + \alpha\beta(t)H(f)}{\alpha(t)C(f)}$$

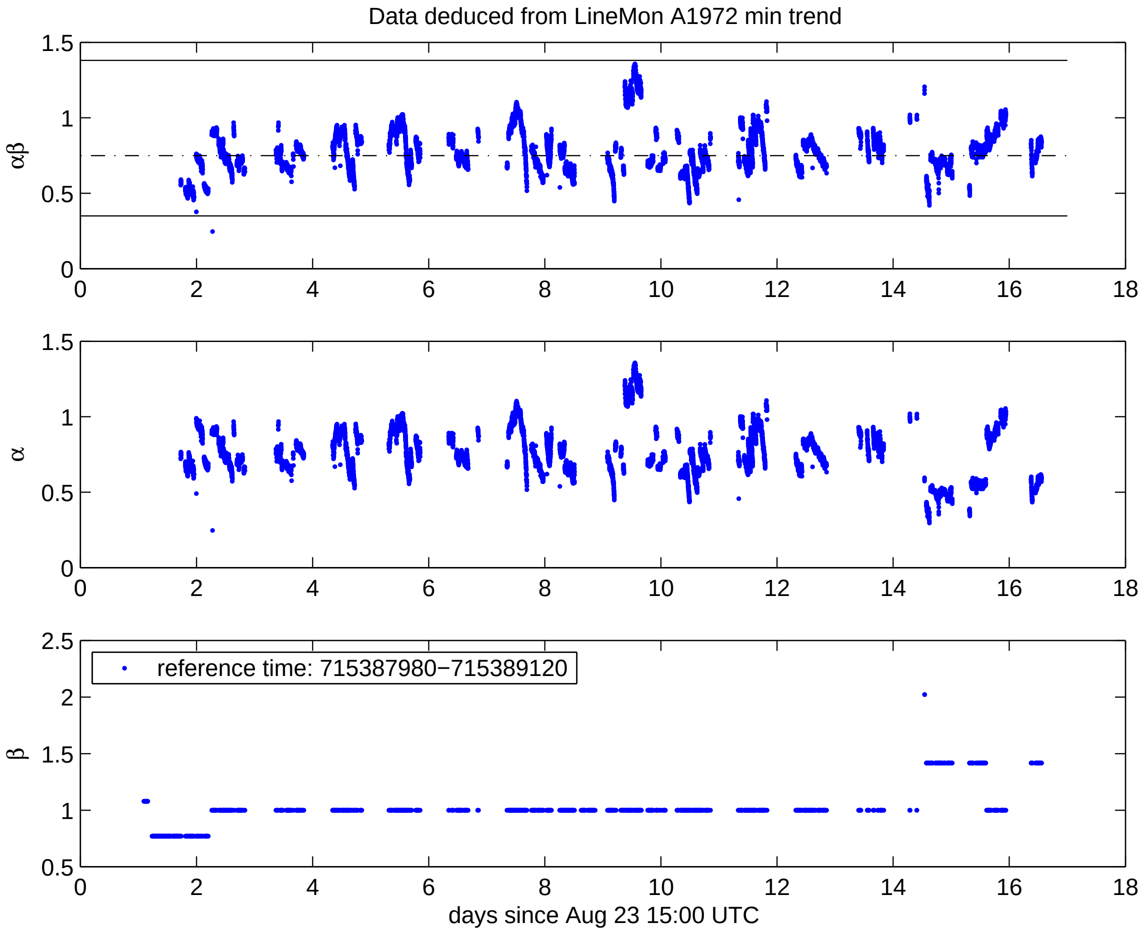

Figure 9: Calibration parameters $\alpha(t)$, $\beta(t)$ and $\alpha\beta(t)$ as obtained from Sergey's Klimenko Line Mon estimates of the amplitude of the calibration line at 972.8Hz. The data is shown only for the science segments in S1 at LLO longer than 300 seconds. The solid lines in the top graph represent the realizable limits of the loop gain measured on Sept 6. The dash-dotted line is the gain with optimal phase margin.

Appendix 1: L1 Science Segments longer than 300 seconds

#	Start GPS	Stop GPS	duration(sec)
1	714179393	714187750	8357
2	714188658	714192005	3347
3	714192730	714193314	584
4	714193938	714195263	1325
5	714207263	714208867	1604
6	714211194	714212083	889
7	714215476	714218723	3247
8	714219236	714225406	6170
9	714226434	714230304	3870
10	714232692	714237164	4472
11	714244366	714245897	1531
12	714246841	714250468	3627
13	714256623	714274679	18056
14	714275221	714285051	9830
15	714286652	714289919	3267
16	714290079	714297963	7884
17	714298377	714300184	1807
18	714306152	714310177	4025
19	714311211	714319144	7933
20	714321763	714325909	4146
21	714326294	714327654	1360
22	714328776	714329836	1060
23	714330776	714331120	344
24	714331276	714332040	764
25	714334071	714340429	6358
26	714345572	714353262	7690
27	714355756	714362596	6840
28	714362634	714376302	13668
29	714378032	714379015	983
30	714382916	714391606	8690
31	714393094	714394824	1730
32	714441127	714441603	476
33	714442178	714442516	338
34	714443043	714443408	365
35	714443434	714443897	463
36	714444489	714444933	444
37	714446973	714449202	2229
38	714449851	714451667	1816
39	714456000	714464277	8277
40	714466667	714467177	510

41	714470947	714479555	8608
42	714480883	714482666	1783
43	714525339	714528413	3074
44	714528493	714537106	8613
45	714538413	714544251	5838
46	714544666	714551387	6721
47	714551840	714553443	1603
48	714555195	714558278	3083
49	714559456	714562801	3345
50	714566102	714568861	2759
51	714609074	714614759	5685
52	714616437	714640434	23997
53	714641394	714643154	1760
54	714648991	714654215	5224
55	714655031	714655782	751
56	714698133	714699112	979
57	714702645	714705076	2431
58	714709322	714719141	9819
59	714720615	714724435	3820
60	714725029	714727095	2066
61	714741350	714742650	1300
62	714784782	714785685	903
63	714787127	714814599	27472
64	714821114	714827968	6854
65	714829599	714835288	5689
66	714836136	714839132	2996
67	714843437	714851172	7735
68	714863311	714870517	7206
69	714870794	714885458	14664
70	714895157	714901722	6565
71	714904857	714913881	9024
72	714914348	714916227	1879
73	714934241	714944755	10514
74	714945950	714947379	1429
75	714954068	714954560	492
76	714954703	714956697	1994
77	714956883	714957527	644
78	714957889	714959107	1218
79	714960246	714972003	11757
80	714972968	714984863	11895
81	714994230	714995586	1356
82	714995611	715002501	6890
83	715006003	715007573	1570
84	715007654	715008184	530
85	715009095	715009817	722
86	715012493	715013600	1107

87	715016891	715017675	784
88	715018546	715020672	2126
89	715037875	715040770	2895
90	715042854	715049750	6896
91	715050018	715056682	6664
92	715057043	715059944	2901
93	715060994	715068081	7087
94	715068363	715071751	3388
95	715072231	715076987	4756
96	715078945	715085707	6762
97	715087221	715087870	649
98	715129015	715131001	1986
99	715131968	715136058	4090
100	715137729	715143694	5965
101	715143761	715147329	3568
102	715147401	715150628	3227
103	715150901	715155666	4765
104	715158189	715169292	11103
105	715170362	715171172	810
106	715171505	715172000	495
107	715214671	715224062	9391
108	715225450	715230938	5488
109	715231053	715239557	8504
110	715239653	715243907	4254
111	715246903	715252724	5821
112	715253857	715260539	6682
113	715308140	715311758	3618
114	715320749	715324011	3262
115	715329739	715331455	1716
116	715336570	715338080	1510
117	715339728	715343230	3502
118	715344555	715345509	954
119	715384241	715384811	570
120	715394660	715395019	359
121	715406173	715406478	305
122	715408880	715413700	4820
123	715415136	715415481	345
124	715416922	715418104	1182
125	715422021	715427503	5482
126	715428525	715432632	4107
127	715435264	715436829	1565
128	715439989	715443021	3032
129	715444237	715446278	2041
130	715447010	715447667	657
131	715472932	715473775	843
132	715475497	715478498	3001

133	715482436	715483544	1108
134	715485964	715490954	4990
135	715491574	715497528	5954
136	715498597	715505575	6978
137	715508997	715513575	4578
138	715514277	715515939	1662
139	715519422	715524275	4853
140	715525219	715527400	2181
141	715564704	715566855	2151
142	715571160	715571579	419
143	715572524	715573716	1192
144	715574673	715580548	5875

Appendix 2: Calibration parameters versus time

We include here a mean value for the parameter $\alpha\beta$ in each science segment where the line was injected. The columns are:

- **Seg#**, as in Appendix 1.
- **Start GPS** second of the science segment, as in Appendix 1.
- **Nmin** is the number of points used to calculate the mean value of the parameter. Each data point is an estimate for the line amplitude for a given minute. If the parameter value was constant and the error was just statistical error in the estimates, we expect it to scale with $1/\sqrt{N}$.
- **mean(ab)** is the mean value of the parameter $\alpha\beta$ in this segment.
- **std(ab)** is the standard deviation of the data points used to calculate the mean in $\alpha\beta$.
- **mean(ab/b)** is an average value for the parameter α in each segment, obtained dividing the mean value of $\alpha\beta$ by the known value of β.

```
Seg# Start GPS Nmin mean(ab)std(ab) mean(ab/b)

17 714298377 14 0.573 0.012   0.745
18 714306152 66 0.503 0.017 0.654
19 714311211 131 0.517 0.028 0.672
20 714321763 56 0.735 0.014 0.956
21 714326294 21 0.716 0.011 0.931
22 714328776 17 0.694 0.019 0.902
23 714330776 5 0.704 0.013 0.915
24 714331276 11 0.669 0.02 0.87
25 714334071 105 0.527 0.016 0.685
26 714345572 115 0.902 0.012 0.902
```

```
27 714355756 113 0.805 0.028 0.805
28 714362634 227 0.706 0.044 0.706
29 714378032 15 0.913 0.026 0.913
30 714382916 144 0.71 0.018 0.71
31 714393094 28 0.654 0.013 0.654
32 714441127 6 0.765 0.014 0.765
33 714442178 4 0.729 0.021 0.729
34 714443043 3 0.795 0.0072 0.795
35 714443434 6 0.767 0.015 0.767
36 714444489 5 0.944 0.019 0.944
37 714446973 35 0.757 0.032 0.757
38 714449851 28 0.682 0.016 0.682
39 714456000 137 0.661 0.018 0.661
40 714466667 7 0.738 0.029 0.738
41 714470947 141 0.797 0.016 0.797
42 714480883 28 0.751 0.012 0.751
43 714525339 50 0.794 0.028 0.794
44 714528493 142 0.879 0.037 0.879
45 714538413 96 0.908 0.018 0.908
46 714544666 111 0.715 0.069 0.715
47 714551840 25 0.67 0.025 0.67
48 714555195 50 0.613 0.055 0.613
49 714559456 54 0.882 0.037 0.882
50 714566102 44 0.85 0.015 0.85
51 714609074 93 0.884 0.05 0.884
52 714616437 399 0.873 0.13 0.873
53 714641394 28 0.723 0.036 0.723
54 714648991 85 0.795 0.046 0.795
55 714655031 10 0.784 0.029 0.784
56 714698133 15 0.862 0.022 0.862
57 714702645 39 0.842 0.018 0.842
58 714709322 162 0.736 0.02 0.736
59 714720615 62 0.702 0.021 0.702
60 714725029 33 0.712 0.034 0.712
61 714741350 21 0.895 0.021 0.895
62 714784782 14 0.681 0.012 0.681
63 714787127 457 0.936 0.11 0.936
64 714821114 113 0.758 0.033 0.758
65 714829599 94 0.675 0.023 0.675
66 714836136 49 0.6 0.015 0.6
67 714843437 127 0.772 0.067 0.772
68 714863311 119 0.773 0.032 0.773
69 714870794 243 0.642 0.034 0.642
70 714895157 0 NaN NaN NaN
71 714904857 0 NaN NaN NaN
72 714914348 0 NaN NaN NaN
```

```
73 714934241 174 0.643 0.07 0.643
74 714945950 23 0.81 0.016 0.81
75 714954068 7 0.746 0.01 0.746
76 714954703 11 0.782 0.012 0.782
77 714956883 0 NaN NaN NaN
78 714957889 19 0.655 0.017 0.655
79 714960246 195 1.15 0.038 1.15
80 714972968 196 1.24 0.057 1.24
81 714994230 22 0.696 0.0096 0.696
82 714995611 113 0.709 0.016 0.709
83 715006003 25 0.903 0.02 0.903
84 715007654 7 0.879 0.012 0.879
85 715009095 10 0.665 0.0093 0.665
86 715012493 17 0.673 0.012 0.673
87 715016891 11 0.723 0.02 0.723
88 715018546 34 0.739 0.017 0.739
89 715037875 48 0.877 0.02 0.877
90 715042854 114 0.623 0.012 0.623
91 715050018 110 0.592 0.078 0.592
92 715057043 47 0.753 0.026 0.753
93 715060994 117 0.613 0.07 0.613
94 715068363 54 0.633 0.013 0.633
95 715072231 78 0.734 0.035 0.734
96 715078945 111 0.728 0.034 0.728
97 715087221 10 0.869 0.019 0.869
98 715129015 30 0.703 0.051 0.703
99 715131968 66 0.961 0.023 0.961
100 715137729 98 0.682 0.034 0.682
101 715143761 59 0.743 0.066 0.743
102 715147401 53 0.799 0.072 0.799
103 715150901 78 0.937 0.052 0.937
104 715158189 183 0.87 0.09 0.87
105 715170362 10 1.08 0.018 1.08
106 715171505 6 1.04 0.032 1.04
107 715214671 155 0.668 0.03 0.668
108 715225450 90 0.818 0.017 0.818
109 715231053 141 0.842 0.021 0.842
110 715239653 70 0.805 0.019 0.805
111 715246903 96 0.751 0.021 0.751
112 715253857 110 0.692 0.018 0.692
113 715308140 59 0.893 0.025 0.893
114 715320749 53 0.789 0.054 0.789
115 715329739 27 0.864 0.043 0.864
116 715336570 23 0.809 0.044 0.809
117 715339728 57 0.794 0.045 0.794
118 715344555 14 0.799 0.014 0.799
```

```
119 715384241 8 0.993 0.016 0.993
120 715394660 4 1 0.014 1
121 715406173 3 1.18 0.022 0.585
122 715408880 79 0.525 0.042 0.371
123 715415136 4 0.734 0.016 0.519
124 715416922 18 0.75 0.016 0.53
125 715422021 90 0.67 0.036 0.473
126 715428525 67 0.708 0.018 0.5
127 715435264 24 0.716 0.018 0.506
128 715439989 49 0.651 0.016 0.46
129 715444237 33 0.718 0.016 0.507
130 715447010 10 0.623 0.013 0.44
131 715472932 13 0.528 0.019 0.373
132 715475497 49 0.79 0.024 0.558
133 715482436 17 0.797 0.036 0.563
134 715485964 81 0.773 0.019 0.546
135 715491574 98 0.787 0.017 0.556
136 715498597 115 0.854 0.035 0.854
137 715508997 75 0.899 0.028 0.899
138 715514277 27 0.927 0.018 0.927
139 715519422 80 0.993 0.024 0.993
140 715525219 35 1.01 0.02 1.01
141 715564704 34 0.725 0.075 0.512
142 715571160 6 0.714 0.018 0.504
143 715572524 19 0.755 0.016 0.533
144 715574673 97 0.832 0.019 0.587
```