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[9] See, in addition to some of the articles cited above, A. B. Arons, "Phenomenology and logical reasoning in introductory physics courses," Am. J. Phys. **50**, 13–20 (1982).

[10] For examples of the use of laboratory-based interviews in research by the Physics Education Group, see D. E. Trowbridge and L. C. McDermott, "Investigation of student understanding of the concept of velocity in one dimension," Am. J. Phys. **48**, 1020–1028 (1980); D. E. Trowbridge and L. C. McDermott, "Investigation of student understanding of the concept of acceleration in one dimension," Am. J. Phys. **49**, 242–253 (1981); R. A. Lawson and L. C. McDermott, "Student understanding of the work-energy and impulse-momentum theorems," Am. J. Phys. **55**, 811–817 (1987); F. M. Goldberg and L. C. McDermott, "An investigation of student understanding of the real image formed by a converging lens or concave mirror," Am. J. Phys. **55**, 108–119 (1987).

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[16] L. C. McDermott *et al.*, *Tutorials in Physics* (Physics Education Group, University of Washington, Seattle, WA, 1991–1992). This is the tentative title of a set of tutorials currently under development.

Research as a guide for curriculum development: An example from introductory electricity. Part II: Design of instructional strategies

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(Received 2 January 1992; accepted 16 April 1992)

This is the second of two closely related articles that together describe how results from research can be used as a guide for curriculum development. The first article shows how the investigation of student understanding of electric circuits by the Physics Education Group has contributed to the building of a research base. This second article describes how the group has drawn on this resource both in developing a curriculum for laboratory-based instruction and in adapting this curriculum to fit the constraints of a traditional introductory course. Also discussed is how, in turn, development and implementation of the curriculum have enriched the research base.

I. INTRODUCTION

During the past several years, investigations of student understanding in physics have contributed to a steadily growing research base. This is the second of two closely related articles that together describe how this resource can be used to guide the development of curriculum.[1] The subject matter context is dc circuits.

As implemented by the Physics Education Group, the process of using research to guide curriculum development has three parts: (1) conducting systematic investigations of student understanding; (2) applying the results in the development of specific instructional strategies to address specific difficulties; and (3) designing, testing, modifying, and revising the materials in a continuous cycle on the basis of classroom experience with the target population. In the first article, we examine part (1) of this process by summarizing results from our long-term investigation of student difficulties with simple electric circuits. In the present article, we illustrate parts (2) and (3) of the process with examples from two types of curriculum developed by our group. The first is a set of laboratory-based instructional modules, collectively entitled *Physics by Inquiry*.[2] These modules have been specially designed for the preparation of teachers of physics and physical science, but they have also been used successfully with other student populations. The second type of curriculum consists of tutorial materials intended for use in conjunction with the lectures and textbooks that form the core of the typical introductory course.[3]

II. APPLICATION OF RESEARCH TO DEVELOPMENT OF A LABORATORY-BASED CURRICULUM

Curriculum development by the Physics Education Group is based on the premise that meaningful learning will not occur unless students are engaged at a sufficiently deep intellectual level. The instructional approach in *Electric Circuits,* one of the modules in *Physics by Inquiry,* encourages students to make the necessary mental commitment by guiding them through the process of constructing a conceptual model for an electric circuit from direct "hands-on" experience with batteries and bulbs. This basic approach was used by A. Arons in *The Various Language,* a text that served as the point of departure for the development of the module.[4] The use of batteries and bulbs to explore the behavior of simple circuits had been introduced earlier in elementary school science programs.[5]

In addition to the general instructional strategy of model building, *Electric Circuits* incorporates a number of specific strategies designed to address the difficulties identified in the companion article. Since many of these are interdependent and mutually reinforcing, they cannot be isolated from one another and must be addressed together. In the discussion below, experiments and exercises from *Electric Circuits* illustrate how various strategies can be used to address these difficulties. *Italicized statements highlight the intended outcomes of instructional strategies.* The use of italics is meant only to call attention to the difficulty that is being addressed and not to claim that the difficulty has been overcome at that point in the instruction. One utilization of a single strategy is seldom enough to bring about a significant conceptual change.

A. Description of the general instructional strategy

The sequence of instruction in *Electric Circuits* begins with the development of qualitative concepts and reasoning skills. The students perform experiments and draw inferences from their observations to construct the basic concepts of current and resistance. They use both inductive and deductive reasoning to synthesize these concepts into a qualitative model for electric current. This mental picture and set of rules provides the students with a conceptual framework that enables them to predict and explain the behavior of simple circuits. As the students apply the model to circuits of increasing complexity, the need for other concepts, such as potential difference, becomes apparent. (Reasons for the choice of current as the primary concept are presented later in the paper.) The model-building process continues with the development of the semi-quantitative concepts and diagrammatic representations that extend the applicability of the model. Algebraic formalism is introduced only after a sound qualitative foundation has been established.

Below we outline the sequence of activities that trace the logical progression in the development of a conceptual model that is applicable to all resistive circuits. The order in which student difficulties are discussed differs from that in the companion article.

1. Initial development of a conceptual model: qualitative approach

The students begin the process of constructing the model by trying to light a bulb with a battery and a single wire. After finding that only the four arrangements in Fig. 1 will produce a lighted bulb, the students are asked to determine the necessary conditions.

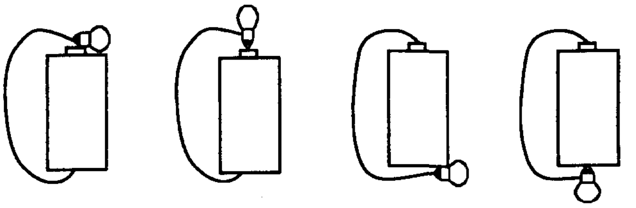


Fig. 1. Four arrangements of a battery, a bulb and a single piece of wire for which the bulb will light.

a. Introduction of the concept of a complete circuit. The students compare the circuits in which the bulb is lit with those in which it is not. By inserting various materials in circuits in which the bulb is lit, they distinguish conductors from insulators. They also examine the internal structure of a light bulb. *The students decide that, for an element to be part of a complete circuit, there must be an internal conducting path between its two terminals, each of which must be connected to a different terminal of the battery through a continuous conducting path.* At this point, circuit diagrams are introduced as a representation for showing electrical connections. The students recognize that the four circuits in which the bulb lights can be represented by the same diagram.

b. Introduction of the concept of current. When the students connect nichrome wire between the terminals of a battery, they note that the wire becomes warm. Their observations provide a basis for the following assumptions: (1) a flow exists in a complete circuit and (2) bulb brightness indicates the amount of flow. The flow is called the electric current, but what is flowing is not identified.

The students compare the brightness of a single bulb with that of two bulbs connected in series. They observe that the two bulbs are equally bright but dimmer than a single bulb. They are asked to use their assumptions and observations to decide whether the current in the single-bulb circuit is greater than, less than, or equal to that in the series circuit. Since the single bulb is brighter than either of the two in series, the students infer that the current in the series circuit is less. *The students recognize that the equal brightness of identical bulbs connected in series with a battery implies that the current is not "used up."*

Because of manufacturing variability, there are instances in almost every class in which there is a perceptible difference in the brightness of identical bulbs connected in series. This situation can be exploited to help deepen conceptual understanding. By reversing, in turn, the connections to the battery and the order of the bulbs, it is possible to determine if the difference in brightness is due to an inherent characteristic of the bulbs or if it depends on the direction of the current or the order of the bulbs in the circuit. *The students determine that neither the direction of the current nor the order of the elements affects bulb brightness.*

The students connect two bulbs in parallel with a battery and find that the brightness of each is the same as that of a single bulb. They infer that the current through each of the parallel branches is equal to the current through the single bulb and deduce that the current through the battery must be greater. *The students recognize that the equal* *brightness of a single bulb and of all identical bulbs connected in parallel across an ideal battery implies that the current through the battery is not constant but depends on the configuration of the circuit.* The students also note that changes made in one branch connected across the battery do not affect other branches connected in parallel with it, i.e., parallel branches connected across an ideal battery are independent of one another.

Note that the misconceptions that current is "used up" and that the battery is a constant current source have both been addressed at this point. However, as the companion article demonstrates, students do not relinquish these ideas easily. Although they may respond properly to direct questions, they often maintain these incorrect beliefs throughout the course. This deficiency in conceptual understanding becomes especially obvious when students attempt to analyze circuits qualitatively. Examples of specific strategies that have proved useful in addressing these persistent misconceptions appear in an earlier article.[6]

c. Introduction of the concepts of resistance and equivalent resistance. The observation that a bulb becomes dimmer when another bulb is added in series suggests that a bulb presents an obstacle, or resistance, to the current. Most students readily accept the idea that a combination of bulbs in series presents a greater equivalent resistance to the current than a single bulb, but many have difficulty associating a smaller equivalent resistance with a parallel combination of two or more bulbs.

The students connect a bulb to a battery and then add a second bulb in parallel with the first. From the observation that both bulbs are equally bright, the students infer that the current through the battery in this circuit is greater than in a single-bulb circuit. They then replace the second bulb by a series combination of two bulbs and decide that, although the current through the battery is less than before, it is still greater than in a single-bulb circuit. They also recognize that the current through the two parallel branches is not the same.

After completing these experiments, the students are asked to interpret the concept of resistance for a network of parallel branches. They conclude that each additional parallel branch results in a smaller equivalent resistance for the circuit as a whole. The association of a decrease in resistance with an increase in the number of paths for the current makes it seem plausible that the addition of elements can result in a decrease in the resistance in the circuit. Additional experiments confirm that the relative currents through parallel branches vary inversely with the resistances of the branches. *The students recognize that the equivalent resistance depends on the configuration and not merely the number of elements or branches.*

d. Application of the model. The students can now account for the relative brightness of the five bulbs in Fig. 2, a task that we know from research has about a 15% success rate in a standard calculus-based course.[1] Without any calculations, the students are able to conclude that $A=D=E>B=C$.

The students investigate the behavior of circuits of different configurations. For example, they connect two bulbs in series and observe what happens to the first bulb (Bulb A) as a third bulb (Bulb C) is added in parallel with the second (Bulb B), as shown in Fig. 3. The observation that Bulb A brightens when Bulb C is added to the circuit serves as a check that the equivalent resistance of the parallel network of bulbs is less than that of a single bulb. The experiment also demonstrates that a change made in a network has effects beyond that network and suggests that Bulb A can be used as an indicator of changes in current. *The students realize that local reasoning is inadequate for analyzing the behavior of an electric circuit when they note that changes at one point often result in changes at other points.*

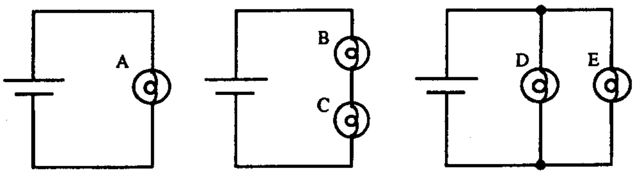


Fig. 2. Students are asked to rank by brightness the five identical bulbs in the circuits shown and to explain their reasoning. They are told to assume that the batteries are ideal. The correct response is $A=D=E>B=C$.

The observation that the brightness of Bulb B changes, when Bulb C is added or removed, indicates that parallel branches not connected across a battery are not independent. The students note that the behavior of Bulb B in this circuit is in marked contrast to that of a bulb in a parallel combination connected directly across an ideal battery. In the latter case, the brightness of the bulb is unaffected by the presence of another bulb in parallel with it. *The students conclude that branches connected in parallel between the terminals of an ideal battery are independent, while those connected in parallel elsewhere in the circuit are not.*

The model developed thus far allows the students to rank bulb brightness in a variety of circuits, but not in all situations. For the circuit in Fig. 3, the students can readily decide that the current through Bulb A will increase when Bulb C is added in parallel with Bulb B. They cannot, however, predict from the model how the brightness of Bulb B changes. Even if the bulbs were ohmic resistors, the students would not be able (except perhaps by a limiting case argument) to determine how the current through Bulb B changes. It is necessary to know whether the current through Bulb A becomes greater than, less than, or equal to, twice its original value. If a parallel network is not connected directly across a battery, an additional concept must be incorporated in the model in order to predict the effect of a change in one branch on other branches within that same network. A need for the concept of potential difference has been established.

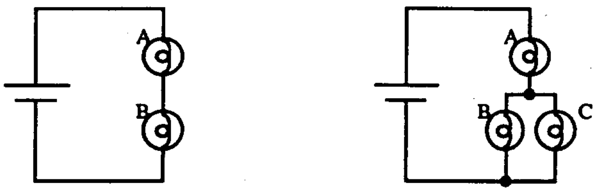


Fig. 3. The resistance in series with Bulb A decreases when Bulb C is added as shown. Thus the total resistance of the circuit decreases, and the current through Bulb A increases.

2. *Extension of the model: Semi-quantitative approach*

Until now, the development of the model has been strictly qualitative. At this point, the approach becomes semiquantitative.

a. Introduction of the ammeter. A bulb is a useful visual aid for the initial development of a qualitative model. However, its utility is limited for a variety of reasons: brightness is not easily quantifiable; bulb resistance depends on current; and the addition of a bulb to a circuit affects the current significantly. Thus, unless it is already in the circuit, a bulb cannot be used to indicate a change in current.

Introduction of the ammeter facilitates development of the model. Without making numerical measurements, the students compare the behavior of an ammeter with that of an indicator bulb, a device with which they are already familiar. They connect the ammeter in series with a battery, bulb, and variable resistor (a long piece of nichrome wire that can be clipped at different points.) They observe that the bulb lights and the meter needle deflects. As the resistance of the wire is increased or decreased, the bulb becomes dimmer or brighter and there is a corresponding deflection of the needle. The students recognize that an ammeter in series with an element behaves like an indicator bulb.

b. Development of the concept of equivalent resistance. As the companion article demonstrates, students sometimes mistakenly use the equivalent resistance of a network instead of the resistance of an individual element. The students perform a series of semiquantitative experiments that are designed to help them distinguish between these concepts. They place two ammeters in separate branches across the same battery. Each ammeter is connected in series with a "black box." In one, the students place a variable resistor made of nichrome wire; in the other, a network of resistors. The students find the length of wire that yields the same deflection for the two ammeter needles. They repeat this procedure with various networks of resistors, noting only the deflection of the needles without being distracted by numerical values. Their observations help them associate with a network a single resistance that is different from that of the individual elements and that is equivalent to the resistance of a length of nichrome wire.

Subsequent experiments help the students begin to quantify their observations. By using an ammeter and segments of nichrome wire, they determine rules for finding the equivalent resistance of elements in series and in parallel. For example, the students observe that two identical resistors in parallel have an equivalent resistance that is one-half of the resistance of one of them. When they substitute bulbs for the wire, they discover that these rules do not hold. Thus they learn to distinguish between ohmic and nonohmic resistances.

c. Introduction of the voltmeter and the concept of potential difference. The need for a new concept to describe the behavior of an electric circuit becomes clear to the students through a series of experiments in which they explore the role of the battery. As they add batteries in series in a single-bulb circuit, they note that the brightness of the bulb increases with each addition. The voltmeter is introduced and the experiment repeated with the meter substituted in the circuit for the bulb. The students observe that the meter needle deflects when the leads are connected across a single battery and that the deflection increases as additional batteries are added in series. The students find that the meter behaves like the bulb (i.e., the brighter the bulb, the greater the deflection.)

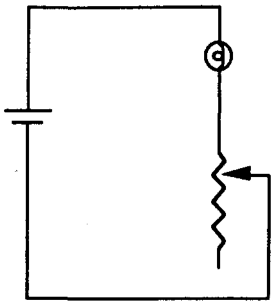


Fig. 4. As the length of wire in the circuit increases, the voltage across the bulb decreases and the bulb dims.

The brightening of the bulb lays the foundation for the interpretation of the battery as the agent that "drives" current through the circuit. The similarity in behavior between the meter needle and a bulb suggests that the voltmeter may also be used as an indicator of the ability of the battery to "drive" current through a circuit.[7] The term *voltage* is introduced and associated with the voltmeter reading. (The term *potential difference* is used after the concept of *potential* has been developed.) In a series of exploratory experiments, the students extend the concept of potential difference to parallel branches and to circuit elements. They find that all branches connected in parallel produce the same meter reading. They note that the needle deflects when the voltmeter is connected across an individual circuit element and relate the potential difference across an element to the ability of the battery to "drive" current through it.

The students investigate how the voltmeter reading across a bulb varies when changes are made in the other circuit elements. For example, they find that the potential difference across the bulb in Fig. 4 increases as the resistance in series with its decreases and that there is a corresponding decrease in the potential difference across that resistance. The students infer the rule that the potential difference across an element increases or decreases as its resistance increases or decreases with respect to the other resistances in series with it.

The introduction of potential difference evokes the tendency to confuse that concept with current. An additional complication is that all observations are consistent with the assumption that bulb brightness is an indicator of potential difference as well as current. In a later section, we describe a series of experiments that help students sharpen the distinction between current and potential difference and between the ammeter and voltmeter.

d. Application of the extended model. The model is now sufficient for determining how the brightness of Bulb B in Fig. 3 will change when Bulb C is added in parallel with it. The resistance of the network containing Bulb B decreases when Bulb C is added. Thus the potential difference across Bulb B decreases and it becomes dimmer. Using the extended model, the students can also predict the relative brightness of bulbs in more complicated circuits. For example, for the circuit in Fig. 5, they are able to determine that $A > D = E > B = C$.

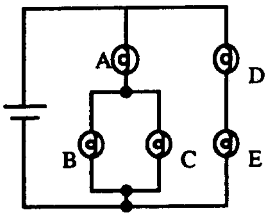


Fig. 5. Students can use the model to predict the relative brightness of bulbs in relatively complicated circuits such as the one shown. The correct ranking of bulbs is A > D = E > B = C.

3. Further extension of the model: quantitative approach

The concepts of current, resistance and potential difference are now quantified. Since the emphasis in this article is on the development of conceptual understanding, we describe only briefly the portion of the curriculum in which the approach is quantitative.

a. Quantification of basic concepts. With the aid of an ammeter and voltmeter, the students develop operational definitions for the concepts of current, potential, potential difference and resistance. Having operationally defined the basic electrical concepts, the students extend the model to include algebraic relationships. They make measurements that lead to the formulation of Kirchhoff's first and second rules and determine the relationship between current and potential difference for ohmic materials (Ohm's law). They also confirm the formulas that they have determined experimentally for the equivalent resistance of elements connected in series and in parallel.

b. Introduction of the concepts of power and energy. The students place two nonidentical bulbs in a circuit, first in series and then in parallel. They note that when the bulbs are in series, the current through them is the same; and that when they are in parallel, the potential difference across them is the same. Each bulb is brighter in one of the two cases. The students conclude that neither current, nor resistance, nor potential difference alone is sufficient to determine brightness. Although they can use the model to rank identical bulbs, they cannot predict the brightness of nonidentical bulbs. When the need to extend the model is recognized, the concept of power is introduced.

A single bulb, two bulbs in parallel, and two bulbs in series are connected in separate circuits to a battery and left for an extended period of time. The students note that the battery with the parallel connections runs down first, followed by the one in the single-bulb circuit, and finally the one in the series circuit. Thus they realize that a battery has a finite lifetime and that the shortest lifetime is associated with the one in which the current was initially the greatest. These experiments pave the way for identifying the energy as the quantity that is dissipated. The students can now reconcile their intuitive belief that something is "used up" in the circuit with their formal knowledge that current is conserved.

c. Modification of the model to include real batteries. Until this point, the experiments upon which model development has been based have been designed so that the approximation of an ideal battery is valid. This restriction is now relaxed by calling attention to small effects that the students may have observed but cannot explain with their model. For example, they may have noticed that bulb brightness does not stay constant as additional bulbs are added in parallel between the battery terminals.

Using an ammeter and a voltmeter, the students examine how the current and terminal voltage change as parallel branches are added across a fresh battery. They note that the total current increases with each branch but that the terminal voltage does not change appreciably as long as the total current is small enough. The value of the potential difference across a fresh battery is contrasted with that across an older battery with the same number of parallel branches. These experiments lead to an operational definition for an ideal battery as one that maintains a constant potential difference in the circuit under consideration.

The students observe the similarity in behavior between the two bulbs in the parallel network in Fig. 3 (in which the battery is ideal but the bulbs are not connected directly across it) and two bulbs in a parallel network across a real battery. They can now understand why it is customary to represent a real battery by an ideal battery in series with a resistance and why the model must be extended to include the internal resistance when it is not negligible compared to the resistance of the rest of the circuit. *The students understand that an ideal battery maintains a constant potential difference between its terminals (not a constant current) and can recognize the circumstances under which a real battery can be considered ideal.*

B. Choice of current as the basis for initial model building

In the model-building process that has been described, the initial model development is based on the concept of current. Potential difference or energy are other possibilities.[8] However, these alternatives have disadvantages that we believe outweigh the benefits.

Starting with potential difference introduces a need for both a "pressure" and a flow that results from that pressure. Thus very early in the development of the model, the concepts of both potential difference and current are present. It has been our experience that the concept of a flow is more intuitive to students, especially to those who are not mechanically inclined, than are ideas related to pressure. The choice of current as the initial concept permits model development to begin with a single concept. The introduction of potential difference can be delayed until students are familiar with current. Although lack of a causal agent may be disturbing to a physicist, it does not seem to be a serious problem for most students. When it becomes necessary in the development of the model, most students readily identify the battery as the agent that drives current through the circuit. Regardless of which concept forms the basis for model building, once the other is introduced, difficulty in discriminating between the two will arise and needs to be expressly addressed.

Almost invariably during the process of developing a model, students will try to base their explanations on energy considerations. We have found that the introduction of the concept of energy early in the model development leads to complications. Students often fail to distinguish current from energy. When they attempt to reason on the basis of energy, they have difficulty in reconciling the dissipation of energy with the conservation of current. We think that it is easier for students to develop a consistent conceptual framework from a single primary concept instead of from two concepts that they may not have fully separated.

An instructional strategy that seems well matched to a model based on current and that often appears attractive to instructors is the use of water analogies. It has been our experience, however, that lack of familiarity with the concept of pressure and the phenomena of fluid flow can make it difficult for students to use such analogies in reasoning about circuits. Consequently, we have chosen not to have the development of a conceptual model for electric circuits depend on an intuitive understanding of the behavior of fluids.

C. Treatment of specific difficulties

The process of constructing a conceptual model helps students synthesize the basic electric concepts into a coherent framework. However, we have found that the use of model building as an instructional strategy is usually not sufficient. Many students cannot develop a functional understanding of the material unless certain difficulties are explicitly addressed.

1. Difficulties with reasoning

Many of the difficulties identified in the companion article are at least partly due to incorrect reasoning. To address them, we have found it necessary to insist that students articulate their reasoning, not only in constructing the model but in applying it. It has been our experience that this practice helps develop both reasoning ability and conceptual clarity.

The construction of the model requires inductive as well as deductive thinking. Concepts are first introduced in their simplest form and later elaborated, as needed, to account for more complex phenomena. The process of refining concepts through a series of successive approximations, which reflects the way physics has developed, helps improve the match between the intellectual demands of the material and the cognitive development of students. Repeated practice in applying the model and in justifying predictions helps reinforce the correct interpretation of concepts. As students recognize the need for a holistic perspective in analyzing circuits, they learn to resist the tendency to think locally and sequentially. *The students develop a powerful conceptual model that they can apply to predict and explain the behavior of simple dc circuits.* Students who have a conceptual understanding of Kirchhoff's rules, the role of the battery and the conservation of current do not have to depend on mathematical manipulation to solve qualitative problems.

2. Difficulties with diagrammatic representations

As is discussed in the companion article, many students have difficulty in interpreting circuit diagrams. They often cannot decide, even for relatively simple circuits, whether elements or networks are connected in series or in parallel. Exercises in *Electric Circuits* provide practice in recognizing these connections in circuits of increasing complexity. *The students learn to identify the key features that define series and parallel connections.*

Another common difficulty concerns the relationship between real circuits and their diagrammatic representations. There are many exercises in the module that focus on the equivalence between various circuit diagrams of the same circuit and the relationship between a real circuit and its diagrammatic representation. Through repeated practice in analyzing diagrams, such as those shown in Fig. 6, the students extend their understanding of the features that are represented on a circuit diagram and those that are not. They overcome their initial tendency to make a direct correspondence between the lines and the arrangement of elements on a circuit diagram and the wires and the placement of elements in the physical system. *The students learn to interpret a circuit diagram as a representation of the electrical elements and connections and not of physical or spatial relationships.*

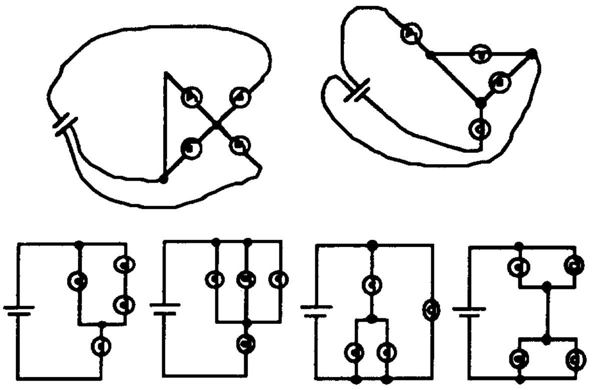


Fig. 6. For each of the sketches of real circuits shown, students are asked to identify the corresponding standard circuit diagram. Both circuits can be represented by the circuit diagram (b).

3. Difficulties with concepts

In the course of constructing the model, students often correct, on their own, many mistaken ideas that they have about electric circuits. However, there is ample evidence that certain conceptual difficulties tend to persist unless specifically treated. Helping students overcome deeply-rooted ideas is not simply a matter of telling them which mistakes to avoid. Often they are unaware of the discrepancy between the instructor's words and their own thoughts. The error described may appear trivial and thus be ignored. Students who recognize a predisposition toward making a particular error may momentarily suppress the tendency. The underlying difficulty may remain latent, only to emerge in another context.

For a significant conceptual change to occur, students must be actively engaged in the learning process. We have found that an effective means for obtaining the necessary degree of mental involvement is to generate a conceptual conflict and to require students to resolve it. We make frequent use of an instructional strategy in which the tendency to make a particular error is deliberately exposed. The underlying difficulty is then explicitly addressed. The procedure, which has general applicability, may be summarized as a sequence of steps that can be characterized as: *elicit, confront, resolve.* These steps do not define a single strategy but a continuum. Inherent in the instructional approach is the recognition that one encounter is almost never enough to overcome a deep-seated conceptual difficulty. Repeated challenges are necessary. In addition, students must be given multiple opportunities to apply the

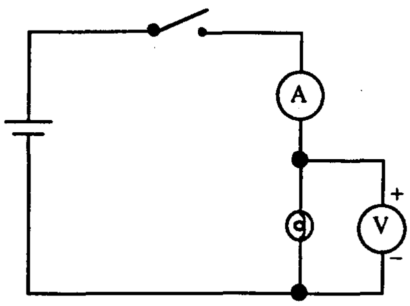


Fig. 7. Students alternately remove the voltmeter and the ammeter from the circuit and observe the changes in both bulb brightness and in the reading on the remaining meter. The observations permit students to determine the relative magnitudes of the resistances of the meters.

same concepts in different contexts, to reflect upon these experiences, and to generalize from them.

Certain misconceptions are so pervasive that they seem to be intuitive. Perhaps the two most common are the belief that current is "used up" and that the battery is a constant current source. Although both misconceptions are addressed in the course of building the model, we have found that many students need additional help. Examples of the use of the *elicit, confront, resolve* strategy to treat these difficulties appear in an earlier article.[6]

A tendency to confuse related concepts underlies many of the errors described in the companion article. In *Electric Circuits,* variants of the strategy summarized above are used to help students sharpen the distinction between concepts. *The students draw on their own observations and reasoning to formulate, refine, and differentiate the basic concepts used to characterize electric circuits.* Three examples are given below.

a. Current vs potential difference. One symptom of a latent difficulty in separating the concepts of current and potential difference is the ineptitude many students display with ammeters and voltmeters, both in the laboratory and on paper. As discussed in the companion article, students who have used these meters in the laboratory may not understand the basic operating principles. They often do not regard the instruments as circuit elements and ignore the critical condition that all measuring devices must satisfy, i.e., they must not appreciably affect the rest of the system.

In a series of experiments, the students confront the differences in the function of ammeters and voltmeters and identify the nature of the connections necessary for the ammeter to measure current and the voltmeter to measure potential difference. The students compare the behavior of a circuit in which a voltmeter is connected in series with a bulb and a variable resistor with a similar circuit containing an ammeter. They observe that when the voltmeter is in series with the bulb, the bulb does not light, but the voltmeter does not read zero. They also observe that as the resistance of the wire increases or decreases, the bulb remains unlit and the voltmeter reading does not change. The students recognize that, unlike an ammeter, a voltmeter in series with an element does not indicate current.

The students also do other experiments in which they determine the relative internal resistances of the two instruments. They set up the circuit in Fig. 7, a typical arrangement for finding an unknown resistance with an ammeter and a voltmeter. They remove first one and then the other meter, examine the effect on bulb brightness and on the reading on the remaining meter, and infer how the resistance of each meter compares to the resistance of the bulb. Instead of being told that the internal resistance of the ammeter is small and that of the voltmeter large, they conclude that this must be the case.

We have found that, when the initial encounter with the ammeter and voltmeter is through qualitative investigation rather than quantitative measurements, students are more likely to pay attention to differences in function and external connections instead of concentrating only on the process of taking numerical data. By comparing and contrasting the behavior of the meters with that of the bulb, an element with which they are familiar, the students understand how the instruments must be connected not to affect the circuit. *The students recognize that the ammeter and the voltmeter are circuit elements and can make the proper inferences about the magnitude of the meter resistance and the nature of the external connections for each instrument.* The process of interpreting observations and inferring relative magnitudes for the resistance of the meters not only helps students remember how to connect these instruments but also helps them differentiate current from potential difference.

b. Potential vs potential difference. It is evident that students confuse the concepts of potential and potential difference when, to determine bulb brightness, they use the value of the potential at a terminal rather than the potential difference across the bulb. To help students recognize the key differences between these concepts, the module focuses attention on their operational definitions.[9]

The students sharpen the distinction between potential and potential difference by performing the operations that define both concepts. They connect three identical bulbs in series across a battery and attach the negative lead of the voltmeter to the negative terminal of the battery. They place the positive lead at the terminals of each bulb successively and record the meter readings. They find that a unique number is associated with each terminal and identify the procedure they have used as an operational definition for the potential at a point in a circuit. The students calculate the difference between the values of the potential at the terminals of each bulb, a procedure that serves as an operational definition for the magnitude of the potential difference across an element. The students then place the voltmeter across each bulb and note that each reading is approximately equal to the potential difference that was calculated for that bulb. Since the bulbs are of equal brightness, it is clear to the students that potential difference, not potential, is the relevant concept in predicting bulb brightness.

c. Resistance vs equivalent resistance. Even after students become accustomed to the idea that a network can be considered to have an equivalent resistance, they may not recognize the circumstances in which the concept is useful. As the companion article demonstrates, they may not understand that a network of elements can be considered to have a single resistance for some purposes, such as finding the total current in a network, but not for others, such as calculating the power dissipated in an individual element. For example, when asked to rank the brightness of the bulbs in Fig. 2, many students calculated the equivalent resistance in each circuit and substituted this value into the formula for the power dissipated in an individual bulb.[1]

We have found that a research task that elicits a difficulty can often be modified and incorporated into an effective instructional strategy to address that difficulty. Our experience with the circuit in Fig. 2 in research suggested how quantitative problems could be used to help differentiate the concepts of resistance and equivalent resistance.

The students are asked to solve a variety of circuit problems and to compare their results with predictions based on the model. In trying to resolve discrepancies, they must reflect on the interpretation of the algebraic symbols. This process helps make explicit for the students the fact that the power dissipated in a circuit element depends on its own resistance, not the equivalent resistance of the network. *The students learn to differentiate between the equivalent resistance of a network and the resistance of an individual element and to recognize the circumstances in which each of these concepts should be applied.*

4. Lack of real world experience

Many of the difficulties described in the companion article seem to stem from an inability to relate physics formalism to real circuits. As they work through the experiments in *Electric Circuits,* the lack of familiarity with actual circuits that is frequently a handicap for many students is gradually replaced by a well-founded physical intuition. *The students gain concrete experience with real circuits through inquiry-oriented instruction in a laboratory setting.* The connections between physical concepts, their formal representations and the real world are made explicit in exercises in the module.

Laboratory-based model development offers other benefits. Students can gain a perspective of physics as a process of raising questions about the real world and of searching for answers through direct investigation. They are often surprised they can develop a model with powerful predictive capability without introducing ideas for which they lack evidence, such as the polarity of the battery or the existence of electrons. Since there is a strong tendency to teach as one has been taught, it is especially critical for prospective teachers to experience science as a process of inquiry, not as a compendium of facts to be memorized.

III. ADAPTATION OF LABORATORY-BASED CURRICULUM FOR USE IN A STANDARD COURSE

We recognize that widespread adoption of a laboratory-based curriculum is not feasible in courses with large enrollments. A more pragmatic approach is necessary. As a means to this end, we are developing tutorial materials for use in conjunction with lectures and textbooks, the primary mode of instruction in introductory physics.[3] A major objective is to counter the passive learning environment of the typical course and to secure a mental commitment from the students. The purpose is not to deliver additional information but to help students deepen their conceptual understanding and develop skill in scientific reasoning.

We have found that in some instances portions of the laboratory-based curriculum can be successfully modified for use in large lecture-based courses. However, adapting these materials to fit the constraints of a standard introductory course requires compromise. Hands-on experience and open-ended investigations must be replaced by more narrowly focused demonstrations and tasks. Dialogues that provide guidance through questioning to help individual students arrive at their own answers give way to group discussions. Inevitably, there is a shift in emphasis from the development of concepts to their application.

A. Nature of the tutorial materials

The tutorial materials consist of units of related activities that focus on important elements of the standard curriculum. Carefully structured worksheets guide students through prescribed tasks that require explanations of reasoning. Students predict the effect of specified changes in a system, observe and analyze demonstrations, construct interpretive graphs and diagrams, and solve problems that emphasize qualitative understanding.

The preferred learning environment for the type of intellectual activity characterized above is a small group. We have tried to approximate this learning situation in a large class through tutorial sessions, in which 20–25 students can work together in groups of 3 or 4. The term *tutorial* is meant to emphasize the difference in content and teaching mode between these weekly sessions and the more traditional recitation sessions that focus on problem-solving. In a modified form, the tutorial materials can also serve as the basis of an interactive lecture.[10] In either case, tutorial session or interactive lecture, the instructor is expected to act more like a facilitator of discussion than a dispenser of knowledge.

The tutorials have been designed for use with a system of pretests and course examinations. The tests inform the instructor about the level of student understanding and help identify for the students what they are expected to learn from the tutorial. They also serve to set the stage for the tutorial so that the limited time in a 50-min session can be used efficiently. There is also a strong link between the course examinations and the tutorials. Every examination includes questions from material covered in the tutorials. Strict adherence to this practice helps focus the attention of students on important issues. The grading policy in the course supports the tutorials, not only through the content on the examinations but also through requirements of attendance and homework submission.

B. Description of a tutorial

The inability of students in the calculus-based course to solve simple qualitative problems prompted us to try to adapt a portion of *Electric Circuits* to the constraints of a lecture-based course. We realized that limited time and space precluded giving students the opportunity to construct a model from their own experience. Therefore, we decided to include as demonstrations abbreviated versions of the key experiments that provide the basis for the logical development of the model. For specificity, we describe below an instructional format in which the tutorial materials are used in an interactive lecture and in a subsequent tutorial session.

The sequence of activities is essentially the same as in the laboratory-based curriculum. In the interactive lecture, demonstrations with a large bulb and a power supply replace "hands-on" activities with batteries and bulbs. Worksheets and questions posed by the instructor guide students in making the critical observations and in doing the reasoning necessary for formulating the model. In the tutorial session that follows the interactive lecture, small groups of students work together in analyzing circuits of various configurations. As they try to apply the model, the students must confront and resolve some common conceptual and reasoning difficulties. For example, the tutorial materials explicitly address the misconception that current is "used up."

The process of adapting a laboratory-based curriculum to a lecture-based course has contributed significantly to our knowledge of how students learn physics. Data collected from classroom observations, tests, and course examinations have yielded detailed information about the nature, prevalence, and tenacity of specific student difficulties. The enriched research base has guided ongoing modification of both the tutorials and the lectures. The laboratory-based curriculum has also benefitted from these improvements.

IV. EFFECTIVENESS OF INSTRUCTIONAL APPROACH

We have been continuously engaged in formative evaluation and modification of both the laboratory-based curriculum and the tutorial curriculum. Classroom observations, tests, and course examinations have been used to monitor student progress. We also conduct summative evaluation through final examinations and through post-tests administered one or more quarters after the relevant course has been completed.

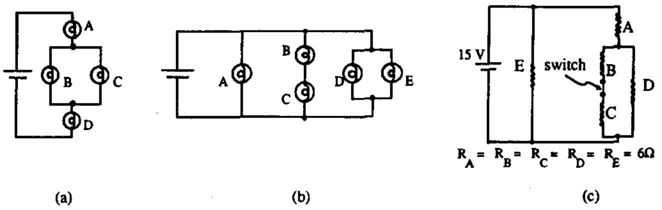
A. Effectiveness of the laboratory-based curriculum

The instructional environment in which *Electric Circuits* has been developed and modified provides many opportunities for judging the effectiveness of the curriculum. The instructors do not lecture but circulate throughout the room while the students work through experiments and exercises. A high instructor-to-student ratio allows the staff to engage students in dialogues that permit in-depth questioning. In addition, the students interpret their laboratory experience on homework assignments, term papers and course examinations. The quality of both oral and written explanations provides evidence of a sounder conceptual understanding of dc circuits than most students develop in the typical calculus-based course.

It is difficult in a laboratory-based course to make a totally objective determination of the effectiveness of instructional materials. However, there is some hard evidence that our curriculum on electric circuits is more successful than the traditional approach. Although many of our students have had considerably less preparation than those in the standard courses, their performance on qualitative questions has been consistently better. Below, we discuss results from using the tutorial materials in a lecture-based course, in which the large number of students and more controlled environment permit a more rigorous assessment.

B. Effectiveness of the tutorial materials

We compared the performance of students with whom the tutorial materials were used with that of students taught in a more traditional way. Results from tests administered over several years indicate that the level of student conceptual understanding on electric circuits in "standard" courses does not vary much from one class to another, at least at the same institution. It does not seem to matter at what point in the course certain types of qualitative questions are asked. As mentioned earlier, both before or after instruction, approximately 15% of the students in a standard introductory course give a correct ranking for the bulbs in Fig. 2. Analysis of responses to this question and others indicated that all student populations involved in the assessment could be regarded as approximately equivalent.



(a) (b) (c)

Fig. 8. Circuits used in evaluation of tutorial curriculum. The circuit shown in (a) was used to assess short-term student learning. Circuits (b) and (c) were used to assess longer term retention.

1. Short-term assessment

An examination question based on the circuit shown in Fig. 8(a) was administered to about 500 students at three different institutions. Four courses were involved: the lecture-based course with tutorials, a parallel calculus-based course without tutorials, a laboratory-centered calculus-based course at a liberal arts college (in which some of the tutorial materials were used), and an algebra-based course at another university.[11,12] The students were asked to rank the relative brightness of the four bulbs and to explain their reasoning. Fewer than 50% of the students in the two lecture-based courses without tutorials predicted the relative brightness of the bulbs correctly. Although one of the courses was algebra based and the other was calculus based, there was little difference in the performance of the students from the two classes. On the other hand, more than 75% of the students in the calculus-based course who participated in the tutorial sessions gave the proper ranking. The correct response was also given by about 65% of the students in the liberal arts college.

There was a significant difference in the prevalence of certain errors between students at the University of Washington who had and had not participated in tutorial sessions. For example, approximately 40% of the students in the course without tutorials predicted that $A > B = C > D$. Analysis of the explanations indicated that this error could be attributed to one of two specific difficulties. Many of the responses reflected the belief that current was "used up" by the bulbs. In stating that Bulb D would be dimmer because it was at a lower potential, some students revealed confusion between potential and potential difference. In contrast, only 5% of the students in the course with tutorials and only 10% of the students in the liberal-arts college course that incorporated some of the tutorial materials claimed that Bulb A and Bulb D in Fig. 8(a) would differ in brightness.

In the two courses at the University of Washington, the students were also asked to calculate the potential difference across each bulb on the assumption that the bulbs could be treated as ohmic resistors. The students in the course with tutorials did markedly better than those in the parallel course. This result is consistent with results obtained from comparisons in performance on other quantitative questions between students with and without tutorials.[13]

2. *Longer-term assessment*

A post-test was administered to about 100 students in the third quarter of a calculus-based physics course to assess retention of the material on dc circuits covered in the second quarter. Different students in the class had studied this topic at different times with different instructors. About half of the students had worked with the tutorial materials on electric circuits, either one or two quarters previously. The rest had not participated in the tutorials.

Both qualitative and quantitative questions were asked on the test. The qualitative questions involved ranking the bulbs shown in Fig. 8(b) according to brightness and the resistors shown in Fig. 8(c) according to the currents through them. Of the students who had not used the tutorial materials, approximately 15% obtained the correct ranking for the bulbs in Fig. 8(b) (i.e., $A=D=E>B=C$). On the other hand, of the students with whom the tutorial materials had been used in a previous quarter, 45% gave the proper ranking together with acceptable explanations. A comparison of the explanations given by the two groups revealed that the tutorial students were much less likely to claim that current was "used up" and to use sequential reasoning. It has been our experience that students generally do not recognize the similarity between the circuits in Fig. 2 and Fig. 8(b). Thus the difference in performance between the students who had and had not used the tutorial materials cannot be attributed to experience with the circuits in Fig. 2.

The quantitative part of the test asked for the current through Resistor A in Fig. 8(c) with the switch closed. The students were told to assume that the battery was ideal and to use the values for the resistances and potential difference shown in the figure. A slightly higher percentage of the tutorial students obtained the correct answer (1.5 A). Our experience that an emphasis on concept development does not adversely affect, and may even improve, quantitative problem solving is consistent with experience at other universities.[14]

3. *Summary*

The results from the assessments described above indicate comparable success in solving quantitative circuit problems by students who had used the tutorial materials and by those who had not. Although the emphasis on concept development in the tutorial classes meant that less time could be spent on problem solving, the tutorial students did somewhat better in arriving at numerical solutions under test conditions. In both groups, students who were able to analyze a circuit quantitatively were often unable to analyze that same circuit qualitatively. Apparently, the ability to solve a circuit problem numerically does not necessarily indicate a corresponding level of qualitative understanding.

The difference in the responses of the two groups to questions requiring conceptual understanding was significant. Students from the tutorial classes performed substantially better on all of the qualitative problems. The tutorial materials also seem to have been more successful than the standard approach in addressing some serious misconceptions.

V. CONCLUSION

The majority of students who participated in our investigation took an approach that was almost entirely mathematical. Many were unable to solve simple problems that required only a qualitative understanding of the material. Often the reasoning the students used indicated that they thought of the concepts of current, potential difference and resistance primarily as variables in algebraic formulas. For standard problems, skill in mathematical manipulation may suffice. However, to be able to deal with more complex situations, students must integrate concepts into a consistent framework. Most do not achieve this synthesis on their own. In this paper, we have described how the use of model building as a general instructional strategy, coupled with additional strategies to address specific difficulties, can help students develop a functional understanding of dc circuits.

Although a topic from introductory electricity was used as the context for this discussion, the use of research to guide curriculum development has broad applicability.[15,16] We have found that the cycle of designing, testing, modifying, and revising curriculum has deepened our understanding of how students think about physics. Without research, we could not have developed the laboratory-based curriculum to its present state nor would we have known how to design appropriate tutorial materials for a lecture-based course.

We believe that there is a need for ongoing systematic investigation into the nature of student difficulties in all topics of the standard introductory course. However, contributions to the research base should not be limited to the identification and analysis of difficulties but should also include descriptions of instructional strategies that have been demonstrated to be effective. If experience has shown that certain methods appear not to work, this information should also be reported. By building a research base and using it to guide curriculum development, we can increase the likelihood that we are making progress, not only individually but as a profession, in helping students understand physics.

ACKNOWLEDGMENTS

The work reported in this article has extended over many years and has been a joint effort by many members of the Physics Education Group, past and present. In particular, M. L. Rosenquist was the co-author with L. C. McDermott of the original version of *Electric Circuits.* Special thanks are due S. Lea (University of North Carolina at Greensboro) for pilot testing the module and suggesting improvements. Substantive contributions to the development of the tutorial materials were made by G. Francis and K. Wosilait. The assistance of J. Valles has been invaluable. In addition, the authors would like to thank P. Laws (Dickinson College) and D. Sokoloff (University of Oregon) for their participation in the assessment and A. Arons and E. Jossem (The Ohio State University) for their helpful comments on the manuscript. Partial support was provided by a series of grants from the National Science Foundation, of which the most recent is MDR 8950322.

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Renormalization of a model quantum field theory

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(Received 9 December 1991; accepted 30 April 1992)

Renormalization is the technique used to eliminate infinities that arise in quantum field theory. This paper shows how to renormalize a particularly simple model, in which a single mass counterterm of second order in the coupling constant suffices to cancel all divergences. The model serves as an accessible introduction to Feynman diagrams, covariant perturbation theory, and dimensional regularization, as well as the renormalization procedure itself.

I. INTRODUCTION

Quantum field theory—in particular, quantum electrodynamics—has produced by far the most exacting predictions in all of physics. The magnetic dipole moment of the electron, for example, has been calculated to 14 significant digits, and the result confirmed in the laboratory with exquisite precision.[1] And yet, a straightforward application of the basic rules leads to nonsensical infinities which must be circumvented before intelligible results can be obtained. This process, which is known as "renormalization," stands as one of the greatest triumphs of theoretical physics.[2] Unfortunately, renormalization is generally considered too difficult and sophisticated for most graduate students, let alone undergraduates. Part of the problem is that "realistic" field theories are burdened by distracting features such as spin and gauge invariance, and even artificial "textbook" examples (the so-called ϕ^3 and ϕ^4 theories)[3] involve diverting technical complications.

Our aim in this paper is to present a reasonably complete and self-contained treatment of renormalization[4] for a very simple model: the "ABC theory."[5] We hope that this study will be accessible to advanced undergraduates and to nonspecialists who would like to know (in something more than a merely qualitative sense) what renormalization is all

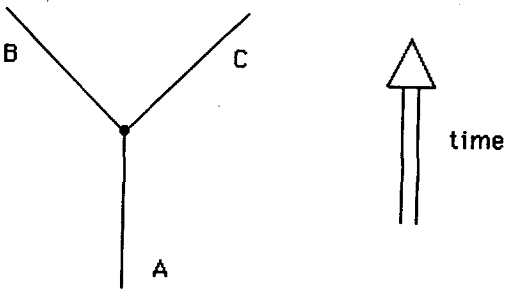

Fig. 1. The basic vertex in ABC theory.