workshops of the University of Oldenburg with scientific advice of U. Mehrle and G. Rohlfs. I am indebted to Dr. F. Riess (Oldenburg) and Dr. H. O. Sibum (Cambridge, UK) for discussing the ideas contained in this paper.

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[2] After W. F. Magie, *A Source Book in Physics* (McGraw-Hill, New York, 1935), pp. 408–413.

[3] See Ref. 2, p. 412.

[4] See Ref. 2, pp. 413–416.

[5] See Ref. 2, p. 418.

[6] M. H. Shamos, *Great Experiments in Physics* (Henry Holt and Company, New York, 1959), p. 64.

[7] See Ref. 2, p. 409.

[8] See Ref. 1.

[9] C. A. Coulomb, *Vier Abhandlungen über die Elektricität und den Magnetismus,* Ostwald's Klassiker No. 13, edited by W. König (Akademische Verlagsgesellschaft, Leipzig, 1921), p. 9, author's translation.

[10] See C. S. Gillmor, *Coulomb and the Evolution of Physics and Engineering in Eighteenth-Century France* (Princeton U.P., Princeton, NJ, 1971), p. 147.

[11] Unfortunately, this remark is contained neither in Magie's nor in Shamos' translation.

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[13] See Ref. 9, p. 9, author's translation.

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Research as a guide for curriculum development: An example from introductory electricity. Part I: Investigation of student understanding

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This is the first of two closely related articles that together describe how results from research can be used as a guide for curriculum development. This first article shows how the investigation of student understanding of electric circuits by the Physics Education Group has contributed to the building of a research base. The second article describes how the group has drawn on this resource both in developing a curriculum for laboratory-based instruction and in adapting this curriculum to fit the constraints of a traditional introductory course. Also discussed is how, in turn, development and implementation of the curriculum have enriched the research base.

I. INTRODUCTION

The Physics Education Group at the University of Washington has for many years been engaged in a coordinated program of research, curriculum development, and instruction. We have examined student difficulties in various domains of physics and have used the results from this research to design instructional strategies that address these difficulties. This is the first of two closely related articles that together demonstrate how research and curriculum development are conducted by our group as interactive components of a single iterative process.[1] In this article, we show how our investigation of student understanding has contributed to the building of a research base that can be used to guide the development of curriculum that matches the needs and abilities of students.[2] In the second article, we describe how we have drawn on this resource both in developing a curriculum for laboratory-based instruction and in adapting this curriculum to fit the constraints of a traditional introductory course. We also discuss how, in turn, development and implementation of the curriculum have enriched the research base. The subject matter context is simple electric circuits that consist only of batteries and resistive elements.

There is ample evidence from our own research and from other investigations that some serious misconceptions about electric circuits are common among students who have had formal instruction in the relevant material.[3–6] Results from cross-cultural studies indicate that similar incorrect ideas flourish in countries with very different systems of education.[7] It is also becoming increasingly apparent that success in solving quantitative problems is not a reliable measure of conceptual understanding. Instructors at secondary and university levels corroborate our experience that students who can solve standard quantitative problems often cannot answer simple qualitative questions based on the same physical concepts.[8,9] This fact suggests the presence of underlying difficulties that apparently are not adequately addressed by traditional physics instruction.

II. METHODS OF INVESTIGATION

The design of instructional materials that match the needs and abilities of students requires detailed knowledge about the conceptual and reasoning difficulties encountered in the study of a particular topic. To obtain such information, the Physics Education Group conducts systematic investigations of student understanding of specific subject matter. The methods used in this research range from individual demonstration interviews conducted in a formal setting to descriptive studies carried out during instruction in the classroom.

In an individual demonstration interview, a simple demonstration serves as the basis of a dialogue between an investigator and a student. The demonstration usually involves real equipment, but sometimes a computer simulation may be used.[10,11] Through careful analysis of the audiotaped transcripts, we try to identify and characterize specific conceptual and reasoning difficulties. We supplement this information by monitoring student participation in laboratory activities and class discussions. Homework assignments and examinations provide additional data.

By extending our research to include descriptive studies in the classroom, we can examine student progress on a continuous basis over an extended period of time.[12] Insights gained from these sources guide us in formulating written questions that we administer on tests and course examinations to large groups: before, during, and after instruction. Analysis of the responses has not only deepened our understanding of known difficulties but has also extended the scope of our investigation to other aspects of the subject matter. In addition, large-scale testing allows us to estimate the prevalence and persistence of specific difficulties.

In conducting the research reported in this paper, our methods of investigation followed the pattern outlined above. Our early studies consisted primarily of individual interviews in which we asked students to predict the relative brightness of bulbs in real circuits. At first the configurations were somewhat complicated, but it soon became apparent that many students had difficulty with tasks based on very simple circuits. Consequently, we decided to concentrate initially on circuits with only a single ideal battery and two or three bulbs. Later we included more complex resistive circuits. As our knowledge of student difficulties grew, our mode of questioning shifted from interviews toward written tests.

The students involved in this ongoing, long-term investigation have had a wide variation in physics background. At one end of the spectrum are students who have had no prior formal study in physics. At the other end are students who have completed a major or minor in the subject. By far, the largest single population consists of students enrolled in the calculus-based introductory course. Students taking the algebra-based course comprise the second largest group. Other participants in our research have been prospective and practicing teachers in the special physics courses we conduct for teachers of all grade levels.

III. IDENTIFICATION OF SPECIFIC STUDENT DIFFICULTIES

The conceptual and reasoning difficulties encountered by students in analyzing simple circuits vary in severity and frequency. Some difficulties tend to disappear as instruction progresses, but others may persist indefinitely and interfere with the learning of more advanced material. From a detailed examination of the errors made by students during the study of dc circuits, we have identified a number of underlying specific difficulties. We have grouped these into three general categories: an inability to apply formal concepts to an electric circuit, an inability to use and interpret formal representations of an electric circuit, and an inability to reason qualitatively about the behavior of an electric circuit. Although this classification is convenient as an organizational framework, the categories are not mutually exclusive. A concept cannot be isolated from the reasoning process inherent in its definition and application, nor is the ability to use and interpret formal representations (e.g., diagrams, graphs and equations) devoid of conceptual elements. Furthermore, there is inevitably some degree of ambiguity in categorizing errors made by individual students.

In all interviews and tests, we require that students explain their answers. The design of effective instructional strategies for addressing specific difficulties requires knowledge of the reasoning used. An error may be a symptom of a conceptual or a reasoning difficulty or of a combination of both. If faulty reasoning is at the core of difficulty with a concept, focusing attention on the concept alone does not provide students with the assistance they need. Moreover, students may make the same apparent error for different reasons. Although short-answer responses can give an indication of how pervasive a particular error may be, they do not provide sufficiently detailed information to be helpful in curriculum development. Attention must be directed to the underlying cause, not merely to the symptoms.

A. Inability to apply formal concepts to electric circuits

The meaning that students associate with a formal concept in physics is often very different from that which a physicist ascribes to that same concept. In this section, we examine student understanding of the basic concepts used to characterize simple dc circuits. Before discussing specific difficulties with these concepts, we consider three of a more general nature.

1. *Difficulties of a general nature*

Operational definitions insure that technical terms have a precise, unambiguous meaning shared by all physicists. In contrast, the students involved in our investigation often referred to current, voltage, energy, and power inappropriately and sometimes interchangeably. Although many students were able to state definitions for the concepts and manipulate the algebraic symbols used to represent them, they often could not relate the concepts to one another or apply them to a real circuit.

a. Failure to distinguish among related concepts. When an actual circuit containing two identical bulbs in series was shown to students enrolled in the calculus-based physics course, many predicted that one bulb would be brighter than the other. The following excerpts have been taken from interviews with different students. [The letter "I" refers to the investigator and the letter "S" to the student.]

I: ...How would the bulbs within the series circuit compare?

S_1: You have "X" current coming through here and you think this bulb uses up some and so there's not enough here...there's not as much current left for Bulb No. 2.

S_2: Bulb No. 2 would be less bright because you always lose some energy as you go through the wires.

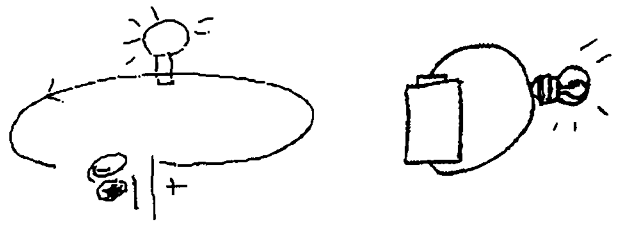
S_3: Maybe this bulb in the first position is gobbling up the power.

As the quotes above illustrate, different students identified different concepts as the quantity "used up" in an electric circuit. Some students named more than one concept. The indiscriminate use of language reflected the fragmentary nature of their understanding. The lack of consistency often made it impossible to interpret unambiguously the meaning a student ascribed to a particular technical term.

b. Lack of concrete experience with real circuits. Many students have no observational or experiential base that they can use as a foundation for constructing the formal concepts of introductory electricity. This deficiency in background can be a serious handicap when students attempt to relate electrical concepts to real circuits. In a survey of a large calculus-based physics class, we found that 60% of the students lacked previous experience with simple circuits. Only about 15% indicated that they had some familiarity with batteries and bulbs.

c. Failure to understand and apply the concept of a complete circuit. To most physics instructors, the concept of a complete circuit is so simple that it is usually given only perfunctory attention in an introductory course. Most students quickly learn the definition but do not necessarily develop the ability to apply the concept.

During an interview, a student from the algebra-based course drew the diagram shown in Fig. 1(a) to show a light bulb connected to a battery. Although she remarked that a complete circuit was necessary, the wire in her sketch touched only the cylindrical base of the bulb. In a test administered in the calculus-based course after the students had studied the topic of dc circuits, approximately 55% of the students did not draw complete circuits when asked to illustrate how to light a bulb with a single wire and a battery. A common error was to show the bottom tip of the bulb in contact with a wire connected between the terminals of the battery, as shown in Fig. 1(b). Such mistakes indicate a failure to recognize that for a bulb to be part of a complete circuit, its two terminals must be connected externally to different terminals of the battery and internally to each other through the filament.



(a) (b)

Fig. 1. Diagrams drawn by students to show how to light a bulb given a battery, a bulb, and a single piece of wire.

2. *Difficulties with concepts related to electric current*

Many difficulties that students have with electric circuits seem to be due to an inadequate understanding of the concept of current. We examine one of these difficulties below in the context of a single question, in which students are asked to rank the five identical bulbs in Fig. 2 according to relative brightness and to explain their reasoning. They are to assume that all the batteries are ideal (i.e., that the internal resistance is negligible).

This question has been administered to more than 500 university students and has proved fruitful for eliciting some common misconceptions. Almost every possible bulb order has appeared. When the question has been asked on course examinations approximately 10% of the students in algebra-based courses, and 15% of the students in calculus-based courses are able to correctly rank the bulbs. Whether the question is administered before or after instruction does not seem to affect the outcome. We have found the same success rate among university graduates. A recent administration of this question to more than 100 science and science education faculty yielded similar results.

The problem may be solved either quantitatively or qualitatively. Although not strictly appropriate, use of Ohm's law yields the proper ranking. However, to compare relative brightness, no calculations are required. It is sufficient to reason from a simple qualitative model, in which bulb brightness is related to current or potential difference. In the companion article, we outline the development of such a model and show how it can be used to determine that Bulb A, Bulb D, and Bulb E will be equally bright and

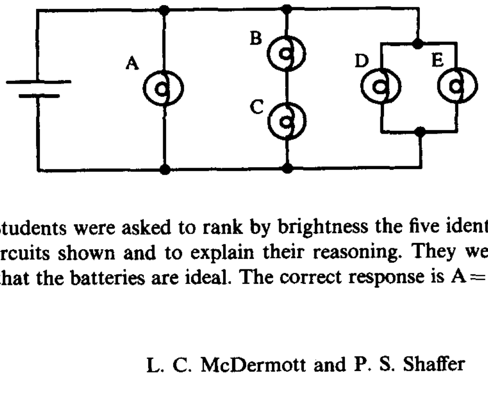


Fig. 2. Students were asked to rank by brightness the five identical bulbs in the circuits shown and to explain their reasoning. They were told to assume that the batteries are ideal. The correct response is $A = D = E > B = C$.

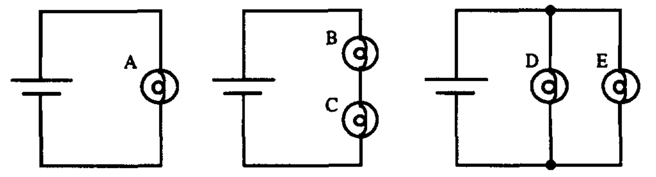
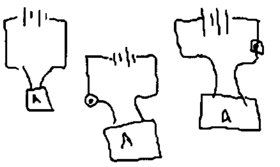


Fig. 3. Variant of the circuit diagrams shown in Fig. 2. In this case, the relative ranking of bulbs by brightness does not depend on whether or not the battery is ideal.



Student: ...I just don't see how I'd see that much difference whether the light bulb is there or not - isn't the current the same all of the time?

Interviewer: ...What would you predict?

Student: ...That the ammeter would read the same in all three circuits.

Fig. 4. The student in the dialogue above expresses the belief that the current through the battery is independent of the circuit to which it is attached.

brighter than the other two bulbs, which will be equal in brightness to each other (i.e., A=D=E>B=C).

For real batteries the relative ranking of the bulbs would be different (i.e., A>D=E>B=C). To determine whether a significant number of incorrect responses may have been due to a failure to notice that the batteries are ideal, we asked a similar question for the circuit in Fig. 3. Although in Fig. 2, the relative ranking of the bulbs depends on whether the batteries are ideal or real, this is not the case in Fig. 3. If the battery in Fig. 3 were real, all the bulbs would be dimmer, but the relative ranking would be the same as for an ideal battery (i.e., A=D=E>B=C). If students had responded incorrectly to the question based on Fig. 2 because they treated the batteries as real, an increase in the percentage of correct responses might have been expected for Fig. 3. However, this percentage was the same for the two cases.

d. Belief that direction of current and order of elements matter. In ranking the bulbs for the circuits shown in Fig. 2, some students indicated a belief that the direction of the current and the order of the elements make a difference. One student explained that Bulbs A, B, D, and E are equally bright "because current is free to flow through these bulbs without having to flow through any other [bulb] first." This student reasoned that "Bulb C is less bright than Bulb B because the current will meet some resistance as it flows through Bulb B." Others used similar reasoning to predict that Bulb C would be brighter than Bulb B. The choice for the brighter bulb depended on whether the student was thinking of conventional or electron current.

e. Belief that current is "used up" in a circuit. The belief that current is "used up" is a common misconception. As mentioned earlier, it is not necessarily a sharply differentiated concept of current that students have in mind. The language many use, however, strongly suggests that they think of current as constantly being produced by the battery and "used up" by the elements in a circuit.

In predicting the relative brightness of the bulbs in Fig. 2, many students claimed that one bulb in the series circuit would be brighter than the other. The most common incorrect explanation was that "Bulb B is brighter than Bulb C because Bulb B 'uses up' the current first and Bulb C gets 'left over' current." To solve quantitative problems, these students may assume that the current is the same at all points in a circuit or branch when elements are connected in series. However, we have found that for many students the statement that current is conserved remains an abstraction that they cannot apply to qualitative questions.

f. Belief that the battery is a constant current source. Perhaps the most pervasive and persistent difficulty that students have with dc circuits is the belief that the battery is a constant current source (i.e., the current through a battery always has the same value). They often overlook the critical role played by the resistance in determining the current. The following ranking by a student in the calculus-based course for the bulbs in Fig. 2 reflects the belief that the current in all three circuits is the same.

> "A, B and C [are] all equal [in brightness] and brighter than D and E, which are equal to each other. The same current *i* goes through A, but in the third circuit the current is divided between D and E."

The belief that the current through the battery is independent of the rest of the circuit often emerges when changes are proposed. The student who drew the circuits in Fig. 4 made the comments below about how the presence or absence of a bulb would affect the ammeter.

S: ...I just don't see how I'd see that much difference whether the light bulb is there or not—isn't the current the same all of the time?

I: What would you predict?

S: That the ammeter would read the same in all three circuits.

According to this student, who received a grade of 3.8 out of a possible 4.0 in the algebra-based course, the bulb would have no effect. Although she indicated that she had heard that a meter could be damaged if connected incorrectly, she did not seem to recognize the relevance of the warning. Even for good students, traditional instruction appears to be ineffective in altering the apparently intuitive idea that a battery produces a constant current.

The idea of a constant current source was not evoked for the single battery in Fig. 3. The specific difficulties elicited in a particular instance depend on the details. Two situations that appear identical to physicists may seem very different to students. Thus the reasoning that students bring to bear in the two situations may be different. Of course, the converse is also true; students may treat situations as similar that physicists consider quite different.

3. Difficulties with concepts related to potential difference

Students often do not distinguish sharply between current and potential difference. Usually underlying this lack of conceptual clarity is uncertainty about the role of the battery.

a. Failure to recognize that an ideal battery maintains a constant potential difference between its terminals. Many students do not think of a battery as a device that, if ideal, maintains a constant potential difference between its terminals that is independent of the network to which it is attached. (The term "network" is used here to denote a portion of a circuit with two identifiable terminals that connect it to the rest of the circuit.) Instead, they think of

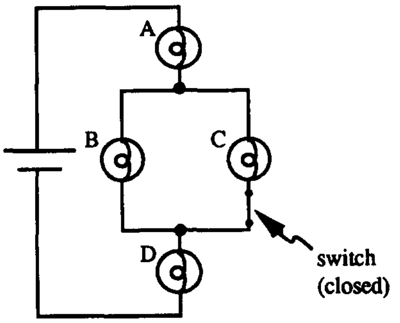


Fig. 5. For the circuit shown, students were asked to predict: (1) the relative brightness of the identical bulbs while the switch is closed, and (2) how opening the switch will affect the brightness of each bulb.

the battery primarily as a constant current source. In response to the question on the relative brightness of the bulbs in Fig. 2, a student in the calculus-based course stated that "Bulbs D and E are equal [in] brightness but dimmer than A...[the] potential across each bulb is equal but not as much as A." This student did not associate the rated battery voltage with the potential difference across the branches connected to the terminals of that (ideal) battery.

b. Failure to distinguish between branches connected in parallel across a battery and connected in parallel elsewhere. In a written examination, students were asked to predict how opening the switch in Fig. 5 would affect the brightness of Bulb B. Because the potential difference across Bulb B would increase, it would get brighter. However, many students claimed that the brightness would remain the same since Bulb B was part of a parallel connection. These students failed to recognize the conditions under which parallel branches are independent of one another.

If two parallel branches are connected directly across an ideal battery, a change made in one has no effect on the other. However, if the two parallel branches are not connected directly across the battery, a change in one branch affects the other. The ability to distinguish between these two types of parallel connections becomes important when real, instead of ideal, batteries are introduced. It is customary to represent a real battery by an ideal battery in series with a resistance. The portion of the circuit diagram in Fig. 5 that contains the battery, Bulb A and Bulb D is equivalent to this approximation. Thus Bulb B and Bulb C can be thought of as a parallel network connected between the terminals of a real battery. To understand the effect of replacing an ideal battery by a real battery, students must understand how a resistance in series with a network affects that network.

c. Failure to distinguish between potential and potential difference. On a course examination in calculus-based physics, students were shown the circuit diagram in Fig. 5 with the switch in the closed position and asked to rank the bulbs according to brightness. The correct ranking ($A=D>B=C$) can be arrived at either by recognizing that the current through Bulbs A and D is greater than through either Bulb B or C, or by reasoning that the potential difference across both Bulbs A and D is greater than across the parallel combination of Bulbs B and C.

More than half of the students in the class did not rank the bulbs correctly. One of the most frequent incorrect answers, given by 35% of the students, was $A>B=C>D$. Usually this prediction was supported by one of two incorrect explanations. Some of the students claimed that since current was "used up" by A, B, and C, there was less left for D. This difficulty has already been discussed. About an equal number of students used an argument based on the faulty reasoning given below.

> "Bulb A is the brightest because the potential is highest. Bulb B and Bulb C are next because they're on the same potential. Bulb D is the dullest due to the lowest potential."

The explanation offered by this student indicates a failure to discriminate between the concepts of potential and potential difference. The student did not realize that the brightness of identical bulbs depends on how they are connected in the circuit, not on where they are connected. He mistakenly associated the brightness of a bulb with the value of the potential at one of its terminals, rather than with the potential difference between the terminals.

This same difficulty has also appeared when some students rank the bulbs in the series branch in Fig. 2. They explain that Bulb B is brighter than Bulb C because Bulb B is at a higher potential. Students often do not understand that the value assigned to the potential at a point in a circuit is merely a number that is determined by the convention of taking the negative terminal of the battery as a reference. The numerical value has no other significance.

4. Difficulties with concepts related to resistance

In addition to current and potential difference, the concept of resistance is essential for the analysis of an electric circuit. We have found that students often do not understand the meaning of this concept sufficiently well to apply it in other than a rote fashion.

a. Tendency to focus on number of elements or branches. A very basic error that we found even among good students early in the calculus-based course was an initial tendency to focus on the number of circuit elements rather than on the configuration. One example is provided by the following explanation given by a student who was a prospective teacher. After stating that Bulbs B, C, D, and E in Fig. 2 would be equally bright but dimmer than Bulb A, the student explained that these four bulbs "would each be half as bright [as Bulb A] because the same strength battery [is being] used on two bulbs." In other words, the student reasoned that if two identical bulbs are connected in a circuit with a battery, the results should be the same regardless of how they are connected.

In determining how the current divides through parallel branches, some students considered only the number and not the relative resistances of the branches. The following prediction and explanation for the relative ranking of the bulbs in Fig. 3 illustrates this error.

> "$A=B=C>D=E$. The current...is equally divided among the [three] paths. B and C are equal to A because the current travels through each bulb one at a time. Bulb D and Bulb E are less because the current splits between them."

The failure to take into account the relative resistances of parallel branches was also evident when students were asked to rank the bulbs in the three circuits in Fig. 2. In that case, however, the ranking $A=B=C>D=E$ can also be attributed to a belief that the current through the battery is a constant, independent of the circuit to which it is attached. Whether the students reasoned that the current through the three batteries was the same (as they did for Fig. 2) or that the current in all three branches was the same (as they did for Fig. 3), it was possible for them to arrive at the identical incorrect ranking. The fact that different kinds of faulty reasoning may produce similar answers to a question demonstrates the difficulty of trying to classify unambiguously the errors made by students.

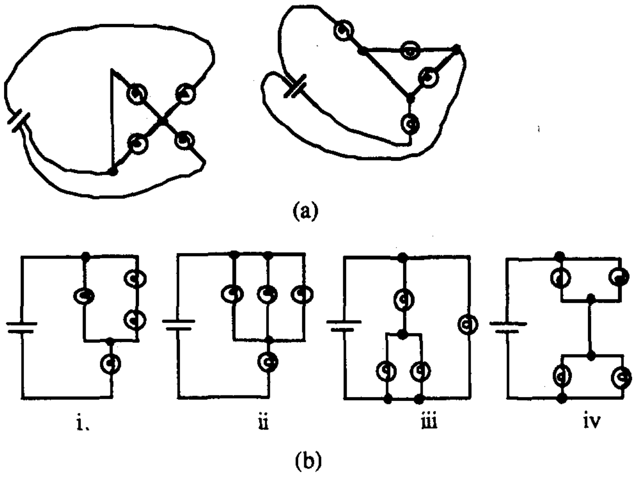
b. Failure to distinguish between the equivalent resistance of a network and the resistance of an individual element. Students who concentrate on the number of elements and ignore the configuration often think of the resistance of a network as an increasing function of the number of elements. Although they can use a formula to calculate the effect on the resistance of adding an element in parallel to a network, these students often have not explicitly confronted the fact that the equivalent resistance decreases with the addition of the element.

At a more sophisticated level, we have found that a difficulty in distinguishing between the equivalent resistance of a network and the resistance of an individual element is quite common. When asked to predict the relative brightness of the five bulbs in Fig. 2 on a course examination, virtually none of the students in the calculus-based class tried to solve the problem by using the concept of the equivalent resistance qualitatively. About 40% of the students immediately began to use algebra to find the equivalent resistance of the series and parallel circuits. They then substituted their answers (R, $2R$, or $R/2$) into the formula for the power and associated the result they obtained with the brightness of each of the bulbs in the network. Thus their ranking depended on which form of the formula they had used ($P=I^2R$ or V^2/R). For example, when values for the equivalent resistance are substituted in $P=V^2/R$, the resulting bulb order is $\mathrm{D}=\mathrm{E}>\mathrm{A}>\mathrm{B}=\mathrm{C}$.

We have found that students often do not seem to distinguish between the resistance of a single element and the equivalent resistance of a network containing that element. Many students did not realize that to determine the brightness of an individual bulb, the values of current, potential difference, and resistance used must pertain directly to that bulb. They did not seem to regard the equivalent resistance as an abstraction that is primarily useful for finding the total current or potential difference in a branch, network, or circuit. Often it appeared that the students were thinking of the equivalent resistance in the circuit as if it were a property of an individual bulb within the circuit. It should be noted that many of the students who were unable to arrive at a correct qualitative solution had earlier successfully solved more complicated circuit problems by using Ohm's law and Kirchhoff's rules.

c. Difficulty in identifying series and parallel connections. To find the equivalent resistance of a network, students must be able to identify series and parallel combinations of elements. They usually can recognize these connections between two elements but often cannot do so when several are involved. For example, when a single element is in series with two elements connected in parallel, students often claim that the single element is in series with one of the elements in the parallel combination.

Many students cannot decompose circuits with four or five resistances into series and parallel connections, especially when the circuits are drawn in an unconventional manner, as are the two in Fig. 6(a). When we asked students to determine which standard circuit diagram in Fig. 6(b) corresponds to each of the circuits in Fig. 6(a), we found that they tended to focus on the physical lines connecting the elements rather than on the electrical connections represented by the lines. Lacking an adequate procedure for determining the types of connections between the bulbs, the students would often fail to recognize that the second circuit in Fig. 6(b) is the correct diagram for both circuits in Fig. 6(a). Some students would obtain a different answer each time they attempted to solve the problem.


(a)

(b)

Fig. 6. Students were asked to identify the corresponding standard circuit diagram for each of the sketches of a real circuit shown in (a). The correct answer is that both circuits correspond to circuit (ii).

Students often fail to extract the critical features of a series or parallel connection that would enable them to identify such connections in complicated circuits. The term *series* often evokes the idea of sequentiality, rather than a specific type of connection. The term *parallel* often retains a geometrical rather than electrical interpretation.

B. Inability to relate formal representations and numerical measurements to electrical circuits

As has already been illustrated, students often manipulate formulas without relating the algebraic symbols to concepts. They also have difficulty in interpreting diagrammatic representations of a circuit and numerical measurements of electrical quantities.

a. Failure to recognize that a circuit diagram represents only electrical elements and connections, not physical or spatial relationships. A real circuit is often represented by a diagram that bears little resemblance to the circuit itself. Students may not be able to recognize the relationship between the features of a diagram and the electrical elements and wires that comprise the physical system. In the diagram, the layout of elements may differ greatly from their actual physical arrangement, and the electrical connections may seem very different from the connecting wires in the physical system. In such cases, many students are unable to make the proper correspondence between the circuit diagram and a real circuit represented by the diagram. Moreover, a given circuit can be represented by several diagrams that appear to be superficially quite different but that are, in reality, identical. Students often fail to recognize that all

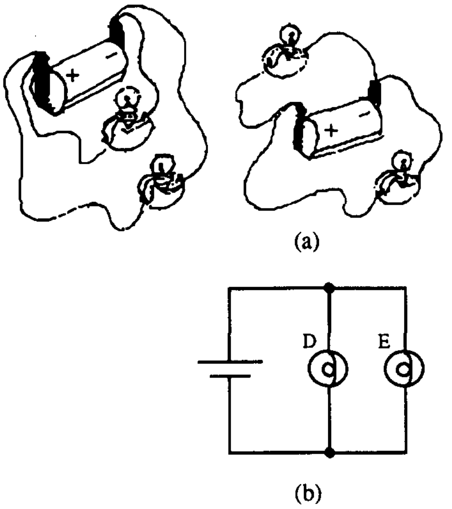


Fig. 7. Not all students recognize that the same circuit diagram (b) can represent each of the circuits shown in (a).

diagrams that indicate the same electrical connections represent equivalent real circuits that are electrically identical to one another.

Students have a strong tendency to focus attention on physical characteristics rather than on electrical connections. For example, we have observed that inexperienced students are often unaware that all three circuits in Fig. 7(a) are electrically equivalent to the circuit diagram in Fig. 7(b). We have also found similar difficulty among more experienced students, especially in situations involving more complicated circuits in which either the diagrammatic representation or the actual circuit has extra junction points that have no essential electrical significance. In addition to not being able to make the proper correspondence between a circuit diagram and a real circuit, students often cannot determine the relationships that may exist among circuit diagrams of very different appearance. For example, they may not realize that two diagrams that appear dissimilar, such as those in Fig. 8, are electrically identical and represent the same circuit.[13]

b. Failure to treat meters as circuit elements and to recognize the implications for their construction and external connections. Students in two concurrent calculus-based physics classes with different instructors were given a pretest consisting of two questions on the use of an ammeter and a voltmeter. There is no reason to believe that the student populations in the two classes were different. About half of each class had completed a standard experiment on Ohm's law in the associated laboratory course. The other half of each class was not enrolled in the laboratory course. The pretest was given shortly after the laboratory experiment to the large class (~200 students) but before resistive circuits had been discussed in the lecture. The pretest was also administered to the second, smaller class (~40 students) after the relevant material had been covered in lecture. In the interim, the students taking the laboratory course performed other experiments involving the use of the meters.

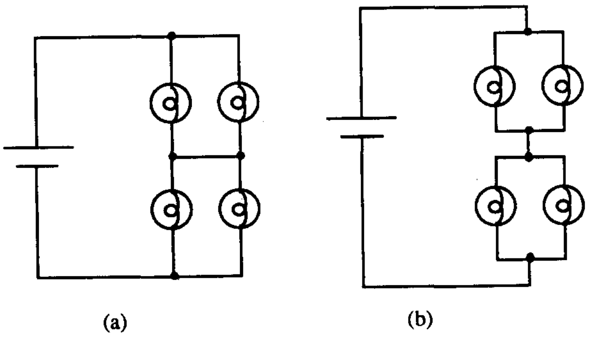


Fig. 8. Many students have difficulty recognizing that the circuit diagrams (a) and (b) are electrically equivalent.

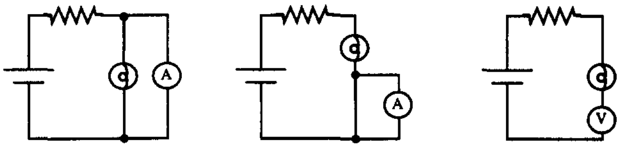


Fig. 9. Students were shown a circuit diagram in which a resistor and bulb are connected in series across a battery. They were asked to draw a circuit diagram to show how to connect an ammeter to measure the current through the bulb and a diagram to show how to connect a voltmeter to measure the potential difference across the bulb. Some common errors are illustrated above.

In the first question, the students were shown a circuit diagram of a light bulb in series with a battery. They were asked to draw a diagram to show how they would connect an ammeter to measure the current through the bulb and a separate diagram to show how they would connect the voltmeter to measure the voltage across the bulb. In the larger class, about 50% of the students were able to draw correct diagrams. In the class in which the question was given after the topic of dc circuits had been completed, the results were the same. Some of the incorrect diagrams drawn by the students are illustrated in Fig. 9. This same question was also asked on a final examination in another large class at the end of a subsequent quarter. Again, only 50% of the students gave a correct response. In this case, all of the students were enrolled in the laboratory course.

The second question on the pretest was assigned to determine if the students understood why the ammeter must be connected in series with an element and the voltmeter in parallel. This question was altered slightly before it was administered to the smaller class. The second version is illustrated in Fig. 10, in which circuit E contains a resistor and a single bulb connected in series to a battery, while each of the other diagrams also includes either an ammeter or a voltmeter. The students were asked to predict the relative brightness of the bulbs in the five circuits and to explain the reasoning they used.

We expected the students to state that the brightness of the bulb in circuits A and D, in which the meter is properly connected to the bulb, would be about the same as in circuit E. For the two circuits B and C, in which the meter is improperly connected, we were willing to accept a variety of answers, depending on the explanation given. The results from the administration of the second question in both classes were the same. Only 15% of the students ranked the bulbs correctly.

Almost every possible ranking of bulbs appeared. Many of the students ranked the bulb in the circuit without the

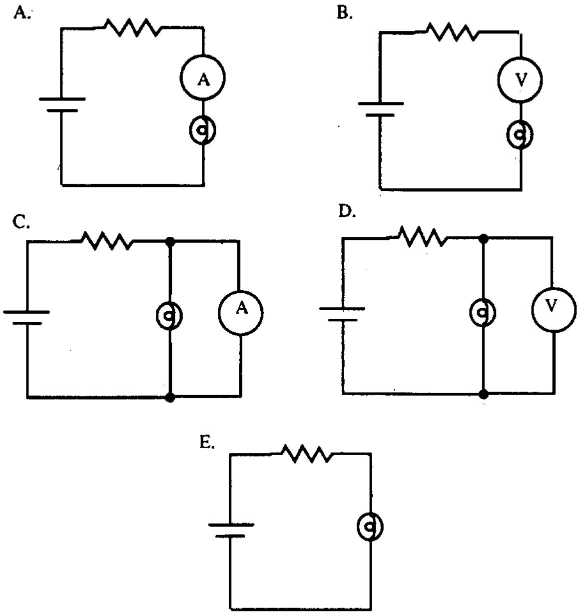


Fig. 10. Students were asked to rank the brightness of the bulbs in the five circuits shown above and to explain their reasoning.

meters as brighter than the bulbs in the circuits in which the meters were correctly connected. Other students predicted that the bulbs would be equally bright in all five circuits. Some explicitly said that the meters had no effect.

The most common type of incorrect response was based mainly on consideration of the external connections of the meters. For example, students would rank the bulbs in circuits A and B as approximately equal in brightness because both meters are in series with the bulbs. They would also use similar reasoning to reach the same conclusion for the bulbs in circuits C and D, in which the meters are in parallel with the bulbs. Sometimes the bulbs connected in series with the meters were ranked brighter than the bulbs connected in parallel. The reverse order also occurred. If students considered the internal resistance of the meters, it was in a subordinate manner. They might refer to the resistance of the meters to make fine distinctions between the brightness of the bulbs in circuits A and B, or in circuits C and D.

Although some students stated that the resistance of an ammeter is very small and that of a voltmeter is very large, they apparently did not relate this information to the function of the meters, nor did they recognize the implications for the external connections. Many students did not seem to regard the ammeter and voltmeter as instruments that should be connected so that they do not affect the current or potential difference to be measured.

The fact that the pretest, administered in one class before the material was covered in lecture and in the other class afterward, yielded similar results in both classes suggests that many students do not learn this material from the standard presentation in lecture and textbook. Overall, there was essentially no difference in the percentage of correct responses to either of the two questions for students enrolled in the laboratory course and for those who were not. In doing the prescribed experiments, the former were able to make the proper measurements, record the values and calculate resistances. Yet, the performance of these students on simple qualitative questions designed to test their functional understanding of the meters was no better than that of those who had not taken the laboratory course.

C. Inability to reason qualitatively about the behavior of electric circuits

Many difficulties that students have with electric circuits are not purely conceptual in nature but also reflect an inability to do the qualitative reasoning involved in the development and application of concepts. It is often impossible to separate difficulties with concepts from difficulties with reasoning.

a. Tendency to reason sequentially and locally, rather than holistically. When a change is made in a circuit, students often focus attention only at the point where the change occurs, not recognizing that a change made at one point in a circuit may result in changes at other points. There is a tendency to think locally or sequentially, rather than to reason holistically. Students may think of a circuit either as consisting of isolated components that can be analyzed independently of one another, or as consisting of components that can be analyzed one after another in sequence around the circuit.

In a written question based on the circuit in Fig. 5, students were asked how opening the switch would affect the brightness of Bulb A. Since the total resistance of the circuit would increase, the current through Bulb A would decrease and it would become dimmer. Many students predicted that the brightness of Bulb A would not change, arguing that "the current goes through Bulb A first." We have found that local reasoning of a sequential nature is especially common when students are asked to predict changes at points that are at a lower potential than the one at which the change is made.

Sequential reasoning about current underlies the belief that the direction of the current and order of the bulbs affect bulb brightness. The error of associating the value of the potential at a terminal of the bulb with bulb brightness can also lead to an inappropriate use of sequential reasoning when students rank the brightness of bulbs according to their sequence in the circuit, rather than the potential difference between their terminals. The students who did not consider the effect on the circuit as a whole of connecting an ammeter or voltmeter in different ways were thinking only in local terms. Still another example of a failure to think globally involves the difference between parallel branches connected across a battery and parallel branches connected elsewhere in the circuit. The students who claimed that the brightness of Bulb B in Fig. 5 would not change when the switch was opened because Bulb B was part of a parallel connection were not considering how the relationship with the battery differs for these two types of parallel connections.

b. Lack of a conceptual model for predicting and explaining the behavior of simple dc circuits. The performance of students on several of the tasks that have been described in this article indicated that most had not synthesized the basic electrical concepts into a coherent framework. Lacking a conceptual model that they could use as a basis for predicting the relative brightness of the five bulbs in Fig. 2, the students resorted to formulas, relied on intuition, or attempted to do both. Many students immediately tried to use Ohm's law but could not apply it properly. Some, who may have had difficulty with the algebra, tried to solve the problem by arbitrarily assigning numerical values to the variables, often choosing inconsistent values for the current, potential difference and resistance.

In the absence of numerical values, students are likely to rely on intuition. Many of the student responses used as illustrations in this paper were the consequence of an intuitive approach. We found that, even when they used formulas, some students were reluctant to abandon their intuition. They might perform a calculation, but if the result contradicted their expectations, they would ignore or modify the mathematics. Below, a student who followed this pattern explained how he had arrived at his unusual ranking for the bulbs in Fig. 2.

> "I assigned an arbitrary resistance to each bulb, then applied the series and parallel rules, assuming that the bulb the current flows through first will be brightest."

The student used Ohm's law to calculate the equivalent resistance for the network of bulbs in each circuit. He then associated the brightness of each bulb with the current through the network of bulbs, an error that was discussed in the section on equivalent resistance. Most of the students who made this error gave their ranking as $D=E>A>B=C$. This student, however, was not willing to base his response on mathematical formalism alone. His final ordering of the bulbs was $D>E>A>B>C$. Lacking a proper model on which to base his reasoning, he revised the ranking he calculated to incorporate his intuitive belief that the "bulb the current flows through first will be brightest." Since he thought of Bulbs D and B as ahead of Bulbs E and C, respectively, he replaced the "$=$" signs by "$>$" signs.

As has been discussed earlier, the prediction that the first bulb in a series arrangement is the brightest is a common error that may be a symptom of more than one underlying difficulty. Whatever the origin, it is clear that the student quoted above had not overcome a serious misconception. Faced with an unfamiliar situation, he modified the physics involved to reconcile what he had learned with what he believed.

IV. CONCLUSION

As the research described in this paper indicates, most students in an introductory physics course do not develop the type of functional understanding that enables them to apply the basic electrical concepts.[14] We found some serious conceptual and reasoning difficulties present at the beginning of instruction that were just as prevalent at the end. Apparently, these difficulties were not successfully addressed by the standard presentation of material in the traditional lecture and laboratory format.

To bring about a significant conceptual change, it is necessary to engage students at a sufficiently deep intellectual level. However, the typical introductory physics course is a passive learning experience for many students. The criterion most often used as a measure of academic mastery is the ability to solve standard quantitative problems. In the study of dc circuits, the attention of students is primarily directed toward the solution of quantitative circuit problems through the use of Ohm's law and Kirchhoff's rules. The type of questions that might strengthen concept development and scientific reasoning ability are generally not posed. However, providing students with physical "hands-on" experience by having them perform standard experiments does not insure that concept development will take place as laboratory skills are learned. It is unrealistic to expect laboratory instruction that does not take student difficulties into account to accomplish what lectures fail to do.

There is a need for instructional materials that foster the active mental participation of students in the learning process. The development of curriculum that fulfills this need should be guided by knowledge of what students know and can do, rather than by assumptions about what they should know and should be able to do. In this article, we have illustrated how our investigation of student understanding in the limited domain of simple dc circuits has contributed to the building of a research base for curriculum development. In a companion article, we describe how we have used results from this investigation to guide the development of curriculum that more closely matches the needs and abilities of students than does traditional instruction on this topic.[1,15,16]

ACKNOWLEDGMENTS

The work reported in this article has extended over many years and has been a joint effort by many members of the Physics Education Group, past and present. In particular, E. H. van Zee and D. Greenberg conducted some of the early interviews and descriptive studies. R. Harrington made a substantive contribution through a recent study of student learning in the laboratory. The assistance of J. Valles has also been invaluable. In addition, the authors would like to thank A. B. Arons and E. L. Jossem (The Ohio State University) for helpful comments on the manuscript. Partial support was provided by a series of grants from the National Science Foundation, of which the most recent is MDR 8950322.

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[1] For the companion article, see P. S. Shaffer and L. C. McDermott, "Research as a guide for curriculum development: An example from introductory electricity. Part II: Design of instructional strategies," Am. J. Phys. **60**, 1003–1013 (1992).

[2] For a discussion of the mismatch between students and the traditional introductory curriculum, see L. C. McDermott, "What we teach and what is learned—Closing the gap," Am. J. Phys. **59**, 301–315 (1991).

[3] In this paper, the term *misconception* is used to refer to an idea for which the student's interpretation is in conflict with the formal concept as understood by a physicist.

[4] See, for example, L. C. McDermott and E. H. van Zee, "Identifying and addressing student difficulties with electric circuits," in *Aspects of Understanding Electricity*, Proceedings of an International Workshop, Ludwigsburg, Germany, edited by R. Duit, W. Jung, and C. v. Rhöneck (Verlag Schmidt & Klaunig, Kiel, Germany, 1984), pp. 39–48.

[5] An extensive listing of research reports worldwide appears in the bibliography *Students' Alternative Frameworks and Science Education*, edited by H. Pfundt and R. Duit (Institute for Science Education, Kiel, Germany, 1991), 3rd ed. Articles readily accessible to readers of American journals include: N. Fredette and J. Lochhead, "Student conceptions of simple electric circuits," Phys. Teach. **18**, 194–198 (1980); N. Fredette and J. Clement, "Student misconceptions of an electric circuit: What do they mean?" J. Col. Sci. Teach. **11**, 280–285 (1981); R. Cohen, B. Eylon, and U. Ganiel, "Potential difference and current in simple electric circuits: A study of students' concepts," Am. J. Phys. **51**, 407–412 (1983); J. Dupin and S. Johsua, "Conceptions of French pupils concerning electric circuits: structure and evolution," J. Res. Sci. Teach. **24**, 791–806 (1987); P. M. Heller and F. N. Finley, "Variable uses of alternative conceptions: a case study in current electricity," J. Res. Sci. Teach. (in press).

[6] Reports of research are included in the Proceedings of several international conferences. See, for example, *Research on Physics Education*, Proceedings of the First International Workshop, La Londe Les

Maures, France, organized by G. Delacôte, A. Tiberghien, and J. Schwartz (Éditions du CNRS, Paris, 1983); *Misconceptions in Science and Mathematics and Misconceptions and Educational Strategies in Science and Mathematics*, Proceedings of the first and second international seminars, edited by H. Helm and J. D. Novak (Cornell U. P., Ithaca, NY, 1983 and 1987); *Aspects of Understanding Electricity*, Proceedings of an International Workshop, Ludwigsburg, Germany, edited by R. Duit, W. Jung, and C. v. Rhöneck (Verlag Schmidt & Klaunig, Kiel, Germany, 1984).

[7] See, for example, D. M. Shipstone, C. v. Rhöneck, W. Jung, C. Kärrqvist, J. Dupin, S. Johsua, and P. Licht, "A study of students' understanding of electricity in five European countries," Int. J. Sci. Educ. **10**, 303–316 (1988). Reports of cross-cultural studies in several Asian countries and Australia are presented in *Research for Students' Conceptual Structures and Changes in Learning Physics*, Proceedings of ASPEN-AAPTEA Workshop II, Manila, Philippines (Asian Physics Education Network-ASPEN, 1991).

[8] E. Mazur, "Qualitative vs. quantitative thinking: Are we teaching the right thing?" Optics and Photonics News **2**, 38 (1992).

[9] See, in addition to some of the articles cited above, A. B. Arons, "Phenomenology and logical reasoning in introductory physics courses," Am. J. Phys. **50**, 13–20 (1982).

[10] For examples of the use of laboratory-based interviews in research by the Physics Education Group, see D. E. Trowbridge and L. C. McDermott, "Investigation of student understanding of the concept of velocity in one dimension," Am. J. Phys. **48**, 1020–1028 (1980); D. E. Trowbridge and L. C. McDermott, "Investigation of student understanding of the concept of acceleration in one dimension," Am. J. Phys. **49**, 242–253 (1981); R. A. Lawson and L. C. McDermott, "Student understanding of the work-energy and impulse-momentum theorems," Am. J. Phys. **55**, 811–817 (1987); F. M. Goldberg and L. C. McDermott, "An investigation of student understanding of the real image formed by a converging lens or concave mirror," Am. J. Phys. **55**, 108–119 (1987).

[11] For examples of the use of computer-based interviews in research by the Physics Education Group, see D. J. Grayson, *Use of the Computer for Research on Instruction and Student Understanding in Physics*, Ph.D. dissertation, Department of Physics, University of Washington, 1990 (unpublished); L. C. McDermott, "Research and computer-based instruction: Opportunity for interaction," Am. J. Phys. **58**, 452–462 (1990).

[12] For an example of a descriptive study by the Physics Education Group, see L. C. McDermott, M. L. Rosenquist and E. H. van Zee, "Student difficulties in connecting graphs and physics: Examples from kinematics," Am. J. Phys. **55**, 503–515 (1987).

[13] For additional research on student difficulties in interpreting circuit diagrams, see S. Johsua, "Student's interpretation of simple electrical diagrams," Eur. J. Sci. Educ. **6**, 271–275 (1984).

[14] P. S. Shaffer, Ph. D. dissertation, Department of Physics, University of Washington, 1992 (unpublished).

[15] L. C. McDermott *et al.*, *Electric Circuits from Physics by Inquiry* (Physics Education Group, University of Washington, Seattle, WA, 1982-1992). *Physics by Inquiry* consists of a set of laboratory-based instructional modules soon to be available through a commercial publisher.

[16] L. C. McDermott *et al.*, *Tutorials in Physics* (Physics Education Group, University of Washington, Seattle, WA, 1991–1992). This is the tentative title of a set of tutorials currently under development.

Research as a guide for curriculum development: An example from introductory electricity. Part II: Design of instructional strategies

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This is the second of two closely related articles that together describe how results from research can be used as a guide for curriculum development. The first article shows how the investigation of student understanding of electric circuits by the Physics Education Group has contributed to the building of a research base. This second article describes how the group has drawn on this resource both in developing a curriculum for laboratory-based instruction and in adapting this curriculum to fit the constraints of a traditional introductory course. Also discussed is how, in turn, development and implementation of the curriculum have enriched the research base.

I. INTRODUCTION

During the past several years, investigations of student understanding in physics have contributed to a steadily growing research base. This is the second of two closely related articles that together describe how this resource can be used to guide the development of curriculum.[1] The subject matter context is dc circuits.

As implemented by the Physics Education Group, the process of using research to guide curriculum development has three parts: (1) conducting systematic investigations of student understanding; (2) applying the results in the development of specific instructional strategies to address specific difficulties; and (3) designing, testing, modifying, and revising the materials in a continuous cycle on the basis of classroom experience with the target population. In the first article, we examine part (1) of this process by summarizing results from our long-term investigation of student difficulties with simple electric circuits. In the present article, we illustrate parts (2) and (3) of the process with examples from two types of curriculum developed by our group. The first is a set of laboratory-based instructional modules, collectively entitled *Physics by Inquiry*.[2] These modules have been specially designed for the preparation of teachers of physics and physical science, but they have also been used successfully with other student populations. The second type of curriculum consists of tutorial materials intended for use in conjunction with the lectures and textbooks that form the core of the typical introductory course.[3]