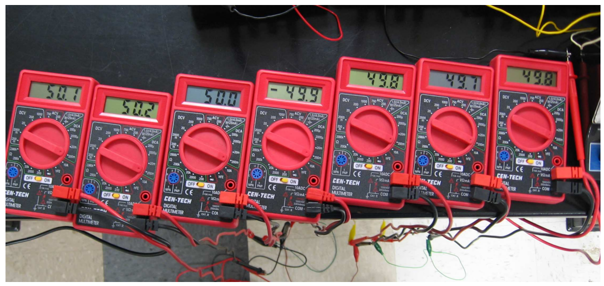
1 Statistics

We have a set of N measurements of some quantity x. The measurements are x_1, x_2, $\ldots x_N$. The values are not all the same because of errors that we cannot control. The picture below shows the measured electrical current through a series of meters. They should all display the same value and they do not.



How do we decide what the real value is, and how reliable is our estimate of the real value? There are two issues here

- Our answer could be very close to the right answer (it's accurate)
- Our answer could only be in a very small range (it's precise)

In a real experiment, you can be lucky and get an accurate answer and it may or may not be precise. Also, you can be unlucky and get an inaccurate answer, and it may be precise or imprecise.

We will always have an uncertainty in our measurements. Therefore we cannot quote a single value, but only a range of values. We write $x \pm \sigma$ for our answer, meaning we believe the real answer is in the range of $(x - \sigma, x + \sigma)$. For example, if we believe the actual mass of an object is in the range of 2.9g to 3.1g we would write that as 3.0 ± 0.1 g.

If we have N repeated measurements of a quantity, our best estimator of the real value is the mean value

$$\overline{x} = \frac{1}{N} \sum_i x_i$$

Now the question is: how reliable is our estimate? The first thing we will compute is an estimate of how much our measurements bounce around the mean value. First we compute a number called the variance

$$\sigma^2 = \frac{1}{N-1} \sum_i (x_i - \overline{x})^2$$

that is, we take the difference between the mean value and each measurement, square it, add up all the squares, and divide by one less than the number of measurements.

The typical difference between a randomly chosen measurement and the mean value is given by the square root of the variance, and we all that the standard error σ, or RMS error.

Here is why we did all the measurements. The RMS error tells us how imprecise our apparatus is when we do one measure. But we did lots of measurements. Don't we get something back from all that extra work? The answer is yes. The mean value is actually less uncertain than the uncertainty in any one measurement. The estimated uncertainty in the mean value (also called the error in the mean) is

$$\sigma_m = \sigma/\sqrt{N}$$

Therefore, as we do more measurements, we expect that our mean value becomes more and more precise. Of course, if we have some systematic error where our meter is always reading wrong, this repeated measurement trick will never fix that. It will only reduce random errors. Another downside is that our uncertainty in the mean get smaller rather slowly as we increase the number of measurements. If you want to cut your uncertainty in half, you need to take four times as many measurements. If you want to reduce your uncertainty to 10% of its current value, you need to increase your number of measurements by a factor of 100.

Let's do a simple example from a physics lab. A simple pendulum of length 0.40 m has a period of 1.27 sec according to the standard formula.

Item	Data	$d_i = x - \overline{x}$	$\lvert d_i \rvert$	d_i^2
1	1.07	-0.16	0.16	0.026
2	1.20	-0.03	0.03	0.001
3	1.30	0.07	0.07	0.005
4	1.40	0.17	0.17	0.029
5	1.15	-0.08	0.08	0.006
6	1.60	0.37	0.37	0.137
7	1.40	0.17	0.17	0.029
8	1.30	0.07	0.07	0.005
9	1.22	-0.01	0.01	0.000
10	1.00	-0.23	0.23	0.053
11	1.20	-0.03	0.03	0.001
12	0.92	-0.31	0.31	0.096
sum	14.76	0.00	1.70	0.387
$\div N$	1.23	0.00	0.14	0.032

Mean:	1.23
Variance:	3.52E-02
Std. deviation	0.188
Uncertainty of mean	5.42E-02

1.1 Are two values the same?

In this program you will learn the sophisticated way to compare two sets of measurements to see if they are the same or they are different. You will hear phrases like "t-test," "chi-squared", "ANOVA," or others. Here is the simple bone-headed shortcut that physicists use.

- Find the mean answer $\overline{x}_1$ and the uncertainty σ_1 in the mean value of the first measurement.

- Think of this as a range of possible actual answers $(\overline{x}_1 - \sigma_1, \overline{x}_1 + \sigma_1)$ where the real answer might lie.

- Find the mean answer $\overline{x}_2$ and the uncertainty σ_2 in the mean value of the second measurement.

- Think of this as a second range $(\overline{x}_2 - \sigma_2, \overline{x}_2 + \sigma_2)$ where the real answer might lie.

- If this second interval overlaps the first interval alot, the numbers are "probably" the same.

- If the two intervals don't overlap at all, they are "probably" different.

- If they just miss overlapping, or overlap a bit, we cannot conclude something easily.

You will learn how to make the word "probably" much more precise in your statistics class. Often we just want a rough idea to check with, and the procedure above will work just fine for that.