The $^{108,110,112}\text{Ru}$ isotopes in the generalized collective model

D. Troltenier$^a$, J.P. Draayer$^a$, B.R.S. Babu$^b$, J.H. Hamilton$^b$, A.V. Ramayya$^b$, V.E. Oberacker$^b$

$^a$ Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA
$^b$ Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235, USA

Received 31 October 1995; revised 12 December 1995

Abstract

Very recently two groups reported on the measurement of excitation energies and $\gamma$-ray branching ratios for the very neutron-rich $^{108,110,112}\text{Ru}$ isotopes. The experimental data were analyzed using the rigid triaxial rotor and the rotation–vibration models resulting in basically different conclusions about the essential physics of these nuclei. In order to try to resolve this matter we applied the generalized collective model to these isotopes: Our results are compared systematically to the available experimental data and to calculations within the two aforementioned models.

PACS: 21.10.Ky; 21.60.Ev
Keywords: Ru; Triaxial shape; Generalized collective model; rotor–vibrator model; Rigid triaxial rotor model

1. Introduction

Through a series of investigations the ground state and $\gamma$ bands of the $^{108,110,112}\text{Ru}$ isotopes with spins up to $16\hbar$ were established and at least seven $\gamma$-ray branching ratios for each nucleus reported [1–6]. Remarkably it was found that the ground state bands of $^{108,110}\text{Ru}$ are nearly identical and that their $\gamma$-band transition energies are very similar, even though the respective band heads are about 100 keV apart [5]. Although the $\beta$ deformation of 0.28(2) for $^{108,110}\text{Ru}$ was established through lifetime measurements [7] about 10 years ago, very recently a discussion unfolded about the possible triaxiality of the $^{108,110,112}\text{Ru}$ isotopes. This controversy about the role of $\gamma$ deformation was triggered by two contributions:

(1) Shannon et al. [4] studied prompt $\gamma$ rays in $^{248}\text{Cm}$ fission fragments with the EUROGAM large detector array and identified levels with spins up to $10\hbar$. A number of ratios of reduced electric quadrupole transition probabilities were measured and...
compared with theoretical predictions of the rigid triaxial rotor model (RTRM) [8]. Assuming triaxial deformations, $\gamma$ of $(22.7^\circ, 23.9^\circ, 25.4^\circ)$ for $^{108,110,112}$Ru, respectively, it was found that this model describes numerous experimental $B(E2)$ ratios very well although it failed to reproduce the measured excitation energies of levels with $L > 4h$. This was interpreted as a strong hint for the existence of triaxial deformations in $^{108,110,112}$Ru, even for excited states. Shannon et al. [4] encourage further theoretical work and they express their hope that a better agreement with level spacings may be obtained by modifying the RTRM to include, ..., shape vibrations.

(2) Together with additional experimental high-spin data that include levels up to spin $16h$, a different interpretation of the important physics of the $^{108,110,112}$Ru isotopes was presented by Lu et al. [5,6]. These authors used the rotation–vibration model (RVM) [9,10] to find that a number of experimental $B(E2)$ ratios of $^{108,110,112}$Ru are well described by this model for $L > 3h$ and that the description of energies is reasonably good, at least up to spins of about $10h$. By construction the RVM assumes nuclei to have a (static) prolate ground state deformation and the centrifugal potential [10] drives the nuclear system dynamically into only relatively small triaxial deformations. Consequently the good RVM description was interpreted as a strong hint for the near axial-symmetric deformation of the $^{108,110,112}$Ru isotopes, in contradiction to the conclusions of Shannon et al. [4].

In both contributions it is recognized that the models that were used, namely the RTRM and the RVM, represent limiting cases of the generalized collective model (GCM) [10,11]. The GCM accommodates not only spherical, prolate, triaxial and oblate nuclei but also nuclei with two minima in the potential energy surface (PES) and has been successfully applied to calculate excitation energies, $B(E2)$ values and quadrupole moments of even–even nuclei from virtually all over the nuclear chart. Some results of different GCM calculations for $^{108}$Ru have been presented earlier [12,13] but the experimental data available at that time did not allow a thorough examination of the choices of parameters. For similar reasons the IBA calculations of [2,14,15] seem to be of more schematic character.

It is the purpose of this contribution to analyze the $^{108,110,112}$Ru isotopes within the GCM, to provide a comparison of the available experimental energies and $B(E2)$ values with the results of GCM, RVM and RTRM calculations, and in so doing try to resolve the discussion raised in the contributions of Shannon et al. [4] and Lu et al. [5].

2. Calculations

Before discussing the results we give the important features of the version of the GCM that was used in this contribution. In the GCM low-energy nuclear excitations are interpreted as collective shape vibrations and as rotations of the nucleus as a whole, where these two modes are allowed to strongly interact with each other. In the laboratory system the center-of-mass nuclear radius,
Table 1
The parameters of the GCM hamiltonian (Eq. (1)) for the nuclei under consideration

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C₂</td>
<td>C₃</td>
<td>C₄</td>
<td>C₅</td>
<td>C₆</td>
<td>D₆</td>
<td>B₂</td>
<td>P₃</td>
<td>P₃</td>
</tr>
<tr>
<td>[MeV]</td>
<td>[MeV]</td>
<td>[MeV]</td>
<td>[MeV]</td>
<td>[MeV]</td>
<td>[10⁻⁴² MeV·s²]</td>
<td>[10⁻⁴² MeV·s²]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>¹⁰⁸Ru</td>
<td>-134.96</td>
<td>-150.10</td>
<td>-96.466</td>
<td>4.9753</td>
<td>16777</td>
<td>9891.9</td>
<td>23.730</td>
<td>17.119</td>
</tr>
<tr>
<td>¹¹⁰Ru</td>
<td>-140.06</td>
<td>-145.96</td>
<td>-98.877</td>
<td>4.9754</td>
<td>17103</td>
<td>9673.9</td>
<td>23.682</td>
<td>17.058</td>
</tr>
<tr>
<td>¹¹²Ru</td>
<td>-152.87</td>
<td>-186.32</td>
<td>-107.71</td>
<td>4.9753</td>
<td>16078</td>
<td>9545.4</td>
<td>23.914</td>
<td>16.731</td>
</tr>
</tbody>
</table>

\[ R(\alpha_2, \Omega) = R_0 \left( 1 + \alpha_0 + \sum_{\mu=-2}^{2} \alpha_{2\mu} Y_{2\mu}^*(\Omega) \right), \]

is expanded in terms of the quadrupole deformation variables \( \alpha_2 \), where \( \alpha_0 \) ensures volume conservation up to second order, \( Y_{2\mu}^*(\Omega) \) denotes a spherical harmonic, and \( R_0 \equiv 1.1A^{1/3} \) fm stands for the spherical nuclear radius [10]. The collective hamiltonian,

\[ H = T + V(\alpha_2), \] (1)

is comprised of the kinetic energy

\[ T = \frac{1}{2B_2} [\pi_2 \times \pi_2]^{[10]} + \frac{P_3}{3} [[\pi_2 \times \alpha_2]^{[2]} \times \pi_2]^{[10]}, \] (2)

where \( \pi_2 \) denotes the conjugate momentum, and the potential energy,

\[ V(\alpha_2) \equiv V(\beta, \gamma) = C_2 \frac{1}{\sqrt{5}} \beta^2 - C_3 \sqrt{\frac{2}{35}} \beta^3 \cos 3\gamma + C_4 \frac{1}{5} \beta^4 \]
\[ -C_5 \sqrt{\frac{2}{175}} \beta^5 \cos 3\gamma + C_6 \frac{2}{35} \beta^6 \cos 3\gamma + D_6 \frac{1}{5\sqrt{5}} \beta^6, \] (3)

where \( \beta \) and \( \gamma \) denote the intrinsic quadrupole deformation variables, with \( 0^\circ \leq \gamma \leq 60^\circ \) [11].

The eight real numbers \( B_2, P_3, C_2, \ldots D_6 \) are parameters which are determined for each nucleus separately in a least-square fitting procedure aimed at an optimal agreement of experimental energies and \( B(E2) \) values and the corresponding GCM results. We want to emphasize that although the model parameters are in principle adjusted for each nucleus separately, they actually change very smoothly from nucleus to nucleus (see Table 1) and that there are no additional parameters used for the electric transition operator.

3. Results and discussion

The parameters of the GCM hamiltonian (Eq. (1)) were determined in the following way: Initially, we started with a number of different parameter sets which correspond to prolate, oblate, triaxial and other intermediate types of PESs. Comparing the GCM
results (energies and $B(E2)$ values) that were obtained in minimization procedures with experimental data, we successively ruled out different types of parameter sets. Finally, the best agreement was obtained using the PESs which are depicted in Fig. 1 as contour plots where dashed lines (equipotential line distance equal to 0.5 MeV) denote the part of the potential below the respective ground state energies. The potential energy surfaces of $^{108,110,112}$Ru all show a triaxial minimum that becomes more pronounced with increasing neutron number. The ground state wave function is widely spread in the $\gamma$ degree of freedom and has a tendency towards oblate deformations. These GCM results are partly supported by earlier theoretical work [17] which used the Strutinsky method and identified triaxial ground state deformations for the nuclei under consideration and by Ref. [3] where shallow triaxial minima were found at smaller $\gamma$ values in Hartree–Fock calculations of the ground state PESs of $^{108,110,112}$Ru.

Using the GCM parameters in Table 1 we calculated the spectra of $^{108,110,112}$Ru which
are depicted in Fig. 2 in comparison to the available experimental data and the results obtained in the RVM. The total of four parameters of the RVM calculation used in this contribution are identical to the ones of Ref. [5] and were determined in quite a unique procedure: In the absence of the rotation-vibration interaction the numerical values of the three energy parameters $\varepsilon$, $E_{\beta}$ and $E_{\gamma}$ are fixed by the experimental positions of the first excited $2^+$ state, and the band heads of the $\beta$ and $\gamma$ band respectively (see Refs. [9,10]). Since no experimental $\beta$ band data were available for $^{108,110,112}$Ru and because the presented RVM results are not very sensitive to its actual value, $E_{\beta}$ was chosen to equal about $2E_{\gamma}$. In the full calculation which includes the rotation-vibration interaction the values of $\varepsilon$ and $E_{\gamma}$ were only slightly changed such as to give a good overall agreement. Finally, the ground state deformation, $\beta_0$, was fixed to exactly reproduce the measured $B(E2; 0_1 \rightarrow 2_1)$ value. Note that the latter is a separate calculation since all eigenenergies are completely determined by $\varepsilon$, $E_{\beta}$ and $E_{\gamma}$.

The version of the RTRM that was used has in total three parameters: the ground state deformation, $(\beta_0, \gamma_0)$, and the collective mass, $B$, which induces an overall scaling of the energy eigenvalues and does not influence the $B(E2)$ values. The actual values
of \((\beta_0, \gamma_0)\) were taken from Ref. [4]. We do not discuss RTRM energies because their deviations from experimental data are not competitive for angular momenta larger than \(4\hbar\) as was already emphasized in the contribution by Shannon et al. [4]. Fig. 2 shows that both the RVM and GCM account for the experimental energies reasonably well, although the RVM results become worse for higher angular momenta in the ground state band. This holds for the \(\gamma\) band, too, and also it should prove interesting to find out whether the inverted level structure found in the RVM results will be confirmed by the experimental data\(^1\). It seems that the experimental level staggering in the \(\gamma\) band in the case of \(^{108}\text{Ru}\), i.e. \((3, 4), (5, 6) (7, 8)\), is slightly better described by the RVM whereas in the case of \(^{112}\text{Ru}\) with an experimental staggering like \((4, 5), (6, 7), (8, 9)\), the GCM results are clearly superior. This might indicate that \(^{108}\text{Ru}\) is slightly less oblately deformed than rendered in the GCM description.

Before discussing the \(B(E2)\)-value ratios, we list in Table 2 the experimental yrast intra-band \(B(E2)\) values in comparison to the RTRM, GCM and RVM results. The

\(^1\) An experimental example for this type of extreme level staggering was, for example, reported for \(^{128}\text{Ba}\) in Ref. [16].
Fig. 2. The experimental ground state and γ-band energies of $^{108,110,112}$Ru are shown in comparison to GCM and RVM results. The numbers next to the bars denote the angular momenta of the positive parity states and the dashed lines are supposed to guide the eye between corresponding levels.

Table 2
The available absolute experimental $B(E2)$ values (in units of e$^2$b$^2$) in comparison to RTRM, GCM and RVM results

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Transition</th>
<th>$B(E2)_{\text{exp}}$</th>
<th>$B(E2)_{\text{RTRM}}$</th>
<th>$B(E2)_{\text{GCM}}$</th>
<th>$B(E2)_{\text{RVM}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{108}$Ru</td>
<td>$0_1 \rightarrow 2_1$</td>
<td>1.1 ± 0.1</td>
<td>1.07</td>
<td>0.98</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>$2_1 \rightarrow 4_1$</td>
<td>0.56 ± 0.04</td>
<td>0.56</td>
<td>0.54</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>$4_1 \rightarrow 6_1$</td>
<td>0.55</td>
<td>0.54</td>
<td>0.54</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>$6_1 \rightarrow 8_1$</td>
<td>0.55</td>
<td>0.56</td>
<td>0.56</td>
<td>0.53</td>
</tr>
<tr>
<td>$^{110}$Ru</td>
<td>$0_1 \rightarrow 2_1$</td>
<td>0.97 ± 0.1</td>
<td>0.97</td>
<td>1.04</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>$2_1 \rightarrow 4_1$</td>
<td>0.50</td>
<td>0.58</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4_1 \rightarrow 6_1$</td>
<td>0.50</td>
<td>0.57</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$6_1 \rightarrow 8_1$</td>
<td>0.50</td>
<td>0.59</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>$^{112}$Ru</td>
<td>$0_1 \rightarrow 2_1$</td>
<td>1.00</td>
<td>1.15</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2_1 \rightarrow 4_1$</td>
<td>0.51</td>
<td>0.62</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4_1 \rightarrow 6_1$</td>
<td>0.51</td>
<td>0.62</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$6_1 \rightarrow 8_1$</td>
<td>0.51</td>
<td>0.64</td>
<td>0.56</td>
<td></td>
</tr>
</tbody>
</table>
RTRM results were obtained by assuming intrinsic quadrupole deformations, \((\beta_0, \gamma_0)\), of \((0.37, 22.72^\circ)\), \((0.346, 23.93^\circ)\) and \((0.345, 25.42^\circ)\) for \(^{108,110,112}\text{Ru}\) (see Ref. [5]), respectively. For the specific case of transitions within the ground state band there is virtually no difference between the \(B(E2)\) values of the three models which are also in good agreement with the few available experimental data points. This is very interesting because the respective model assumptions about the nuclear shape are fundamentally different: The RTRM assumes the same rigid triaxial shape for all states while the GCM allows for vibrations in both \(\beta\) and \(\gamma\) directions with a triaxial potential minimum at clearly more oblate deformations. This has to be compared to the RVM where only harmonic oscillations around a statically prolate deformation are possible due to the model assumptions. Therefore it seems useful to look more closely at the available data for \(B(E2)\) values: Fig. 3 depicts \(B(E2)\)-value ratios as measured by Lu et al. [5] \(^2\), Shannon et al. [4], and as calculated within the GCM, RVM and RTRM divided by the data of Lu et al. [5] (see caption of Fig. 3 for additional explanations). Both experimental and theoretical data were derived by assuming all transitions to be pure E2.

\(^2\) Note that there is an typographical error in Table IV of this reference: The last column lists the ratios \(B(E2;\gamma_2 \rightarrow 4_2)/B(E2;\gamma_2 \rightarrow \gamma_1)\) and not \(B(E2;\gamma_2 \rightarrow 4_1)/B(E2;\gamma_2 \rightarrow \gamma_1)\).
The data of Fig. 3 demonstrate that the RVM describes the experimental results well for transition ratios with an initial angular momentum not smaller than $L = 5\hbar$. The experimental ratios $B(E2;42 \rightarrow 2_2)/B(E2;42 \rightarrow 2_1)$ and $B(E2;42 \rightarrow 2_1)/B(E2;42 \rightarrow 4_1)$ are excellently described for $^{108}\text{Ru}$, worse for $^{110}\text{Ru}$, and deviate for $^{112}\text{Ru}$ by more than an order of magnitude. The ratios involving smaller angular momenta disagree typically by a factor of 2–3 from the experimental data. In comparison the overall agreement for the RTRM results is better although the systematic disagreement in the case of $B(E2;5_1 \rightarrow 3_1)/B(E2;5_1 \rightarrow 4_1)$ is about an order of magnitude or larger. In Ref. [4] this deviation was attributed to a small M1 admixture in $\Delta L = 1$ transitions. This explanation was criticized by Lu et al. [5] and Fig. 3 shows that there are also a few other ratios that are of similar disagreement. For example, compare the $B(E2;42 \rightarrow 2_2)/B(E2;42 \rightarrow 2_1)$ and $B(E2;42 \rightarrow 2_1)/B(E2;42 \rightarrow 4_1)$ ratios in $^{110,112}\text{Ru}$ or the $B(E2;6_2 \rightarrow 4_2)/B(E2;6_2 \rightarrow 6_1)$ ratio in $^{112}\text{Ru}$. However, it does not seem as if the quality of the RTRM description changes drastically for transition ratios including angular momenta $L > 3\hbar$, as was suggested by Lu et al. [5]. At least for $^{110,112}\text{Ru}$ it is for those cases, i.e. for $L > 3\hbar$, not obvious whether the RVM description is even superior to the results obtained in the RTRM. For $^{108,110,112}\text{Ru}$ all GCM $B(E2)$-ratios under consideration disagree by less than a factor of about two from the experimental.
Fig. 3. On a logarithmic scale this figure depicts $B(E2)$-value ratios as measured by Lu et al. [5] (full circles) and Shannon et al. [4] (full squares), and as calculated in the GCM (diamonds), the RVM (crosses) and the RTRM (plus signs) divided by the $B(E2)$ ratios observed by Lu et al. In cases where Lu et al. measurements were not available the data are divided by the results obtained by Shannon et al. The error bars for the Shannon et al. results range between 3% for strong lines and 20% for weak transitions and were numerically not available. The theoretical data points which are out of range are encircled and marked by an upward or downward arrow.

In summary, we presented the results of a GCM calculation that accounts well for the available low-lying energies and $B(E2)$ values in the $^{108,110,112}$Ru isotopes. The RVM describes experimental energies reasonably well for states below $10\hbar$ while the RTRM energies are not competitive for angular momenta $L > 4\hbar$. The $B(E2)$ ratios of the GCM are within a factor of about two of the experimental data, while the RTRM and the RVM seem to show some systematic deviations which increase with neutron
number. The few available absolute experimental $B(E2)$ values of these isotopes are about equally well described by all three models which also give almost exactly the same intra-band transition rates within the respective ground state bands. We consider the GCM results as strongly suggesting a triaxial shape for the $^{108,110,112}$Ru isotopes, a conclusion which is consistent with previous microscopic calculations.

Acknowledgements

Work at Louisiana State University was supported by the National Science Foundation under Grant No. PHY-93-12628. Two of us (D.T. and J.P.D.) thank the Institute for Nuclear Theory at the University of Washington for its hospitality and the Department of Energy for partial support during the completion of the work. Work at Vanderbilt University was supported in part by the US Department of Energy under Grant No. DE-FG05-88ER40407. One of the authors (V.E.O.) acknowledges the support from the Department of Energy under Grant No. DE-FG05-87ER40376.
Fig. 3 — continued.

References

E. Cheifetz, H.A. Selic, A. Wolf, R. Chechik and J.B. Wilhelmy, in: Nuclear Spectroscopy of Fission
D. Troltenier, P.O. Hess and J. Maruhn, in: Computational Nuclear Physics, vol. 1, Nuclear Structure,
[12] K. Sümmerer, N. Kaffrell, E. Stender, N. Trautmann, K. Broden, G. Skarnemark, T. Björnstad,