Soliton excitations as emitted clusters on nuclear surfaces

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Abstract
This work reports on calculations of the deformation energy of a nucleus for nonlinear deformations. The working hypothesis is that, beyond the usual linear approximation, the nonlinear analysis yields soliton solutions moving on its surface. The potential barrier against the emission of a soliton is calculated within the macroscopic–microscopic method. The outer turning point of the barrier determines limitations on the geometrical and kinematical parameters for the formation of a surface soliton. For large asymmetry, the two-centre shell model is used to assign a structure to the soliton. Calculations for $^{248}$No with the emission of a $^{40}$Ca soliton are reported; likewise for $^{224}$Th with the emission of $^{16}$O. Except for necked shapes at the very first stages of soliton formation, the greatest portion of the deformation path displays rather compact configurations with large neck radii. These shape sequences correspond to allowable soliton velocities. Close to and just beyond the touching point configuration, where the shape becomes concave, the width and the velocity of the soliton approaches zero. The calculations suggest that the emission of a $^{40}$Ca structure is quite probable due to a low potential barrier, whereas the emission of an $^{16}$O-type soliton is rather unlikely due to the higher penetration barrier.

1. Introduction
Up to the present, soliton structures have been analysed within the frameworks of hydrodynamics, nonlinear optics, solid state and plasma physics and elementary particle models. Experimental and theoretical results [1] suggest that solitons are non-dispersive, localized waves executing uniform motion that can be described by three inter-related parameters: amplitude $A$, half-width $L$ and velocity $V$. Furthermore, these structures arise as analytical solutions of nonlinear dynamical systems, such as the Korteweg–de Vries (KdV) equation. In order to look for the possibility of describing nuclear phenomena such as cluster emission [2] we introduce a new working hypothesis, namely the soliton formation on the nuclear surface. In this respect it is necessary to assign a microscopic structure to the parent heavy nucleus and the emitted soliton-like cluster. The microscopic substructure further allows one to add shell corrections to the usual macroscopic liquid drop energy and thus to give a
complete description of the system, from the initial nucleus with no soliton substructure to one with a soliton-like structure on its surface and on out to possible cluster emission.

A straightforward way to accomplish this is to calculate shell effects obtained from the single-particle levels of an asymmetric two-centre shell model. One of the centres is placed in the middle of a small emitted sphere and the other is the centre of the heavy fragment. This approach allows a microscopic description of the nuclear evolution from one to two independent quantum systems.

The procedure involves calculating the total potential energy as a sum of the macroscopic energy and shell corrections which is then minimized, which yields a barrier that increases as a function of the amplitude of the soliton. Calculations have been performed for two possible reactions:

$$^{248}\text{No} \rightarrow ^{208}\text{Pb} + ^{40}\text{Ca}$$

$$^{224}\text{Th} \rightarrow ^{208}\text{Pb} + ^{16}\text{O}.$$ 

We select spherical daughters and emitted clusters, in order to fit the asymmetric two-centre shell model we have constructed, which goes from one sphere (parent) to two necked spheres out to separation. These systems match the two-centre shell model we used for the shell corrections. Within this model the levels are obtained for spherical daughter and emitted cluster. The deformations of $^{248}\text{No}$ and $^{224}\text{Th}$ are obtained as a deformed minimum within the $R$, $R_2$ and $R_3$ space of deformation.

Solitary waves have been shown recently to exist on liquid drops, bubbles and shells [3]. The nonlinear hydrodynamic equations are related to the KdV equation generating localized patterns ranging from small oscillations to nonlinear ones, up through solitons. In the present model, we introduce a description of cluster emission processes, by using such soliton-like shapes on the nuclear surface of the heavy fragment. For a given cluster geometry, we calculate the corresponding soliton parameters ($A$, $L$, $V$) as functions of the separation parameter, that is along the static path of the cluster emission process.

In section 1 we define the deformation space that we work with and the shapes that can be obtained therein. In section 2 we give a short description of the macroscopic–microscopic model we used in the calculations. Emphasis has been given to the asymmetric two-centre shell model we constructed in order to approach solitonic shapes. Results for the two reactions given above are presented in section 3.

2. Deformation space

Solitons on the surface are described by two asymmetrical spheres smoothly joined to one another through a neck region (figure 1). There are three independent geometrical parameters which form the deformation space: the distance $R$ between the centres of the two fragments, the emitted small sphere radius $R_2$, and the neck sphere radius $R_3$. The neck region is obtained by rolling a sphere of radius $R_3$ around the symmetry axis. Such shapes are generated by the following equation written in cylindrical coordinates:

$$
\rho(z) = \begin{cases} 
\sqrt{R_1^2 - (z - z_1)^2} & z_1 - R_1 \leq z \leq z_{c1} \\
\rho_3 - \sqrt{R_3^2 - (z - z_3)^2} & z_{c1} \leq z \leq z_{c2} \\
\sqrt{R_2^2 - (z - z_2)^2} & z_{c2} \leq z \leq z_2 + R_2 
\end{cases}
$$

(1)

where the quantities not shown in figure 1 are: $z_3$, the position of the centre of the neck sphere on the symmetry axis; $z_{c1}$ and $z_{c2}$, the positions of the intersection planes of the two fragment
spheres with the neck sphere; and $z_1$ and $z_2$, the positions of the two spherical fragment centres. The heavy fragment radius $R_1$ is obtained from total nuclear volume conservation. The soliton solution

$$r_{\text{surface}}(\phi, \theta) = A \left[ \text{sech} \left( \frac{\phi - Vt}{L} \right) \text{sech} \left( \frac{\theta - Vt}{L} \right) \right]^2$$

is characterized by the amplitude $A$, related to the two-centre shell model by the relation $A = R_1 + R_2$ (or relative amplitude $a = A/R_1$), and the half-width

$$L = \frac{2R_2A}{(1+a)(R_1 - R_2)}.$$  

Solitons have a special shape-kinematic dependence, $V \simeq A$ and $L \simeq 1/\sqrt{A}$; that is, a higher soliton amplitude is narrower and travels faster [1]. This relation can be used to experimentally distinguish solitons from other linear excitations (for example, by calculating the reciprocal moment of inertia). The amplitude of the soliton is approximated by $A = R - R_1 + R_2$. The half-width of the soliton is approximated by $2\rho(z_{c2})$, or the diameter of the circular surface within the separation plane between the emitted sphere and the rest of the shape.

### 3. Macroscopic energy

The deformation energy $E_{\text{def}}$ is calculated in a macroscopic–microscopic approach:

$$E_{\text{def}} = E_{\text{macro}} + \delta E_{\text{shell}}.$$  

The macroscopic part $E_{\text{macro}}$ includes the shape-dependent components of the charged liquid drop:

$$E_{\text{macro}} = E_C + E_{Y+E}.$$
where $E_C$ is the Coulomb energy and $E_{Y+E}$ is the surface or nuclear energy calculated within the Yukawa-plus-exponential model [4, 5].

The Coulomb energy general form is the double-volume integral:

$$E_C = \frac{1}{2} \int_V \int_V \rho_a(r_1) \rho_a(r_2) \frac{d^3r_1}{r_{12}}$$

which is split into four parts, two of them being equal to one another [2]:

$$E_C = \frac{n^2}{2} \int_V d^3r_1 \int_V \frac{d^3r_2}{r_{12}} + \rho_1 \rho_2 e \int_V d^3r_1 \int_V \frac{d^3r_2}{r_{12}} + \frac{n^2}{2} \int_V d^3r_1 \int_V \frac{d^3r_2}{r_{12}}$$

where $r_{12} = |r_1 - r_2|$. The first and the last terms represent the self-energies of the two fragments, and the middle term is the Coulomb interaction between the two fragments.

In cylindrical coordinates the three terms are given by:

$$E_c = \frac{n^2}{10} \int_0^{\pi} d\phi \int_0^\pi d\phi_1 \left( \frac{a^2}{2} \sqrt{\rho^2 - z \frac{\partial \rho^2}{\partial z}} \right)$$

$$\times \left[ \rho^2_1 - \rho_a \rho_1 \cos(\phi - \phi_1) + \rho \frac{\partial \rho_1}{\partial \phi_1} \sin(\phi - \phi_1) + \frac{(z - z_1) \frac{\partial \rho_1^2}{\partial z_1}}{2} \right]$$

$$\times \left[ a^2 + \rho^2 - 2 \rho_1 \rho_1 \cos(\phi - \phi_1) + (z - z_1)^2 \right]^{1/2}$$

where $\rho = \rho(z, \phi)$ is the nuclear surface equation, $z'$ and $z''$ are the intersections of the surface with the $Oz$-axis.

The general form of the Yukawa-plus-exponential energy is

$$E_{Y+E} = -\frac{a_2}{8\pi^2 r_0^2 a^4} \int_V \int_V \left( \frac{r_{12}}{a} - 2 \right) \frac{\exp(-r_{12}/a)}{r_{12}/a} d^3r_1 d^3r_2$$

where $r_{12} = |r_1 - r_2|$, $a = 0.68$ fm accounts for the finite range of the nuclear forces, $a_2 = a_1(1 - \kappa T^2)$. $\kappa$ is the asymmetry energy constant and the surface energy constant is $a_1 = 21.13$ MeV. In a similar way to the Coulomb part, one obtains three terms:

$$E_{Y+E} = -\frac{2}{8\pi^2 r_0^2 a^4} \int_V \int_V \left( \frac{r_{12}}{a} - 2 \right) \frac{\exp(-r_{12}/a)}{r_{12}/a} d^3r_1 d^3r_2$$

where $a_2 = a_1(1 - \kappa T^2)$, $I_i = (N_i - Z_i)/A_i$. For shapes with axial symmetry, each of these terms involving a double-folded integration over the nuclear volume can be reduced to a three-dimensional integral.

4. The asymmetric two-centre shell model

The two-centre shell model was developed by the Frankfurt school for symmetric splitting [7] and for low asymmetry [8]. Here we present the main steps of the two-centre shell model we developed for large asymmetry starting from another symmetrical two-centre model [9].

The general Hamiltonian describing the evolution of the level scheme of the two-centre shell model is based on two oscillators which split from an initial common oscillator. The usual spin–orbit interaction and the $l^2$ term are constructed with a dependence on the mass asymmetry. Thus, the total Hamiltonian reads

$$H = H_{\text{osc}} + V(l \cdot s) + V(l^2)$$

where $H_{\text{osc}}$ is the two-oscillator Hamiltonian and $V(l \cdot s)$ and $V(l^2)$ are the spin–orbit and the $l^2$ potentials.
4.1. The diagonalization basis

The oscillator part of the Hamiltonian, $H_{\text{osc}}$, is given in cylindrical coordinates by

$$H_{\text{osc}} = -\frac{\hbar^2}{2m_0} \left[ \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right] + V(\rho, z)$$  \hspace{1cm} (12)

where the asymmetric two-centre oscillator potential has the form

$$V(\rho, z) = \frac{1}{2} m_0 \left\{ \begin{array}{ll}
\omega_{\rho_1}^2 \rho^2 + \omega_{z_1}^2 (z + z_1)^2 & z < z_0 \\
\omega_{\rho_2}^2 \rho^2 + \omega_{z_2}^2 (z - z_2)^2 & z \geq z_0.
\end{array} \right.$$ \hspace{1cm} (13)

Here $z_0$ is the separation plane coordinate between the two asymmetric systems on the symmetry axis $Oz$. Since we consider the case of two asymmetric spheres, $\omega_{\rho_1} = \omega_{z_1} = \omega_1$ and $\omega_{\rho_2} = \omega_{z_2} = \omega_2$.

To obtain an appropriate basis for this system, we consider the intermediate case when $\omega_{\rho_1} = \omega_{\rho_2} = \omega_1$. Then the potential is

$$V(\rho, z) = \frac{1}{2} m_0 \left\{ \begin{array}{ll}
\omega_1^2 \rho^2 + \omega_1^2 (z + z_1)^2 & z < 0 \\
\omega_1^2 \rho^2 + \omega_1^2 (z - z_2)^2 & z \geq 0.
\end{array} \right.$$ \hspace{1cm} (14)

At this point the intermediate Hamiltonian is separable and one gets the eigenfunctions [9]

$$\Phi_m(\phi) = \frac{1}{\sqrt{2\pi}} \exp(i m \phi)$$ \hspace{1cm} (15)

for the axial degree of freedom and

$$R_{n,|m|}(\rho) = \sqrt{\frac{2\Gamma(n_\rho + |m| + 1)}{\Gamma(n_\rho + |m| + 1)}} \exp \left( -\frac{1}{2} \alpha_i^2 \rho^2 \right) (\alpha_i^2 \rho^2)^{|m|/2} L_{n_\rho}^{|m|}(\alpha_i^2 \rho^2)$$ \hspace{1cm} (16)

for describing radial motion where $\alpha_i = (m \omega_i / \hbar)$, $\Gamma(x)$ is the gamma function, and $L_{n_\rho}^{|m|}(x)$ is the Laguerre polynomial. As one knows, the oscillator energy for oscillations in the plane perpendicular to the symmetry axis is

$$E_{\rho, \phi} = \hbar \omega_\rho (2n_\rho + |m| + 1)$$ \hspace{1cm} (17)

where $\omega_\rho = \omega_1$.

Solving the third equation which accounts for oscillations along the symmetry axis, we have different solutions for the two regions of the nuclear shape. According to the $z$-dependent potential,

$$V(z) = \frac{1}{2} m_0 \left\{ \begin{array}{ll}
\omega_1^2 (z + z_1)^2 & z < 0 \\
\omega_1^2 (z - z_2)^2 & z \geq 0.
\end{array} \right.$$ \hspace{1cm} (18)

where $z_1$ and $z_2$ are the centres of the heavy and light spherical fragments, respectively, one obtains the differential equations

$$\left[ \frac{d^2}{dz^2} + \frac{2m_0 E_z}{\hbar^2} - \frac{m_0^2 \omega_1^2 (z + z_1)^2}{\hbar^2} \right] Z_{\nu_1}(z) = 0 \hspace{1cm} z < 0$$ \hspace{1cm} (19)

$$\left[ \frac{d^2}{dz^2} + \frac{2m_0 E_z}{\hbar^2} - \frac{m_0^2 \omega_1^2 (z - z_2)^2}{\hbar^2} \right] Z_{\nu_2}(z) = 0 \hspace{1cm} z \geq 0.$$ \hspace{1cm} (20)
At this point it is important to mention that the $z = 0$ plane is the intersection plane between the two systems with $\omega_1 = \omega_2$, whereas the ‘real’ intersection between the asymmetric spherical systems is at $z = z_0$. The solution for the $z$-dependent Hamilton equation will be

$$Z_\nu(z) = \begin{cases} C_\nu \exp\left[-\alpha_1^2(z + z_1)^2/2\right]H_\nu[-\alpha_1(z + z_1)] & z < 0 \\ C_\nu \exp\left[-\alpha_2^2(z - z_2)^2/2\right]H_\nu[\alpha_2(z - z_2)] & z \geq 0 \end{cases}$$

(21)

where $C_\nu_1$ and $C_\nu_2$ are normalization constants and $H_\nu(z)$ are the Hermite functions.

As can be seen from these results, four quantities need to be determined: the two quantum numbers $\nu_1$ and $\nu_2$ and the two normalization constants $C_\nu_1$ and $C_\nu_2$. These quantities can be calculated from a system of four equations: for the normalization condition

$$\int_{-\infty}^{\infty} |Z_\nu(z)|^2 \, dz = 1.$$  

(22)

From the continuity of the $z$-wavefunction and its derivative at $z = 0$,

$$Z_{\nu_1}(z = 0) = Z_{\nu_2}(z = 0)$$

(23)

$$Z'_{\nu_1}(z = 0) = Z'_{\nu_2}(z = 0)$$

(24)

and from the energy-matching condition along the $O_z$-axis,

$$\hbar \omega_1(\nu_1 + 0.5) = \hbar \omega_2(\nu_2 + 0.5).$$

(25)

From this a basis for the diagonalization of the potential differences to obtain the real energy values can be calculated.

4.2. The asymmetric oscillator system

Once we have total wavefunctions, we have to determine the differences between the diagonal Hamiltonian and the real one. First, the oscillator Hamiltonian has to provide the initial oscillator potential when there is only one heavy sphere (starting point). For this initial configuration the difference that needs to be diagonalized is

$$\Delta V^\text{sphere}(z) = \begin{cases} \frac{1}{2}m_0[\omega_1^2(z + z_1)^2 - \omega_2^2(z - z_2)^2] & z \geq 0 \\ 0 & z < 0 \end{cases}$$

(26)

For the next stages of deformation, the difference between the $z$-dependent oscillator potentials that needs to be diagonalized is

$$\Delta V(z) = \begin{cases} 0 & z < 0 \\ \frac{1}{2}m_0[\omega_1^2(z + z_1)^2 - \omega_2^2(z - z_2)^2] & 0 \leq z \leq z_0 \\ 0 & z > z_0 \end{cases}$$

(27)

As for the difference in the $\rho$-dependent oscillator potential, this only exists for intersecting spheres and is given by

$$\Delta V(\rho) = \begin{cases} 0 & z \leq z_0 \\ \frac{1}{2}m_0(\omega_1^2 - \omega_2^2)\rho^2 & z > z_0 \end{cases}$$

(28)

or, if written as an operator, the quantity to be diagonalized is given by

$$\Delta V(\rho) = \frac{1}{2}m_0(\omega_1^2 - \omega_2^2)\rho^2 \Theta(z - z_0)$$

(29)
where $\Theta(z)$ is the Heaviside function. The difference $\Delta V(\rho)$ is zero for the initial spherical configuration.

Once $\Delta V(z)$ and $\Delta V(\rho)$ are diagonalized and added to the oscillator energy of the sphere + ellipsoid system, which is

$$ E = \hbar \omega_1 [2n_p + |m| + v_1 + 1.5] $$

the level schemes of the two intersecting asymmetric oscillators with frequencies $\omega_1$ and $\omega_2$ are obtained.

### 4.3. Spin–orbit and orbit–orbit interactions

The spin–orbit ($l \cdot s$) and orbit–orbit ($l^2$) interaction terms generate the necessary single-particle level splitting to obtain the correct schemes of the individual fragments after separation.

The use of a deformation-dependent form of these operators was introduced in [10] for the Nilsson model and in [8] for the two-centre shell model; instead of the $l$ operator one introduces

$$ l = \frac{\nabla V \times p}{m_\mu \omega^2} $$

where $V$ is the asymmetric two-centre oscillator potential. The usual expressions for the two operators are

$$ V(l \cdot s) = -2\kappa \hbar \omega l \cdot s \quad V(l^2) = -\kappa \mu \hbar \omega l^2. $$

Since one obtains the level schemes of two nuclei which lay in different mass regions, the strength parameters of the interactions $\kappa$ and $\mu$ will be different. The values we used for these parameters are

$$ \kappa_n = 0.0588 \quad \kappa_p = 0.0592 $$

$$ \mu_n = 0.328 \quad \mu_p = 0.335 $$

for the actinide region, and for the light nuclei region:

$$ \kappa_n = \kappa_p = 0.0601 $$

$$ \mu_n = \mu_p = 0.448. $$

Since the strength parameters are different for the asymmetric regions of the nuclear shape, they became $z$-dependent operators as follows:

$$ \kappa \cdot \hbar \omega(z) = \kappa_1 \cdot \hbar \omega_1 + (\kappa_2 \cdot \hbar \omega_2 - \kappa_1 \cdot \hbar \omega_1) \Theta(z - z_0) $$

$$ \kappa \mu \cdot \hbar \omega(z) = \kappa_1 \mu_1 \cdot \hbar \omega_1 + (\kappa_2 \mu_2 \cdot \hbar \omega_2 - \kappa_1 \mu_1 \cdot \hbar \omega_1) \Theta(z - z_0). $$

To obtain a Hermitian operator for $V(l \cdot s)$ and $V(l^2)$ one has to use the anticommutator [8]

$$ V(l \cdot s) = -\left[ \kappa \cdot \hbar \omega(z), \frac{\nabla V \times p}{m_\mu \omega^2} \right] $$

$$ V(l^2) = -\frac{1}{2} \left[ \kappa \mu \cdot \hbar \omega(z), \left( \frac{\nabla V \times p}{m_\mu \omega^2} \right)^2 \right]. $$
For the variation of $\kappa$ and $\mu$ with elongation we chose the linear dependence on oscillator frequency along the $z$-axis:

$$\kappa_i = \kappa_0 + \frac{\omega_i - \omega_0}{\omega_{if} - \omega_0} (\kappa_{if} - \kappa_0)$$

and the same law of variation for $\mu$. Here $i = 1, 2$, $\kappa_0$ is the value for the initial nucleus, $\kappa_{if}$ for the final one.

Finally, one has to diagonalize the potential

$$\Delta V(\rho, z) = \Delta V(z) + \Delta V(\rho) + V(I \cdot s) + V(I^2)$$

(38)

together with the diagonal term of the two-centre oscillator potential.

The model provides evolution from an initial level scheme to two asymptotically independent single-particle schemes. With the introduction of a large asymmetry between fragments, the shapes can simulate the existence of a soliton on the nuclear surface and assign it a microscopic structure.

The level scheme of a soliton-like shape is used to obtain the shell corrections of the system. As the soliton is assimilated with an emerging fragment, it will provide the shell correction value of the independent nucleus it is shaping.

Shell corrections are obtained by means of the Strutinsky procedure [11].

5. Results

A first look at the energetic behaviour of a soliton-like shape is given in figure 2 in terms of the macroscopic energy surfaces. The initial stage of the soliton formation is the parent nucleus. At this moment $R_1 = R_2$, meaning the two spheres overlap. This hypothesis is sustained by the favourable macroscopic energy, which is lower when the first step of deformation (small distance between centres) takes place with a larger radius $R_2$. Then the emerging sphere develops towards the final $R_2$ radius. This is the reason why at the beginning we always

Figure 2. Macroscopic potential energy surfaces for $^{40}$Ca emission from: (a) $^{238}$No and (b) for $^{16}$O emission from $^{234}$Th, as a function of elongation $R$ and neck radius $R_3$. The first maximum appears close to the touching point configuration in both cases ($R_3 = 0$).
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Figure 3. Total potential energy surfaces (macroscopic plus shell corrections) for $^{40}\text{Ca}$ emission from: (a) $^{248}\text{No}$ and (b) $^{16}\text{O}$ from $^{224}\text{Th}$. The deformed ground state of $^{248}\text{No}$ is revealed as a minimum along the $R_3$-axis at its origin. For both surfaces the closest energy maximum occurs at the tangent sphere configuration. The maximum energy value for larger $R_3$ is not reached in the figure.

have $R = R_1 - R_2 = 0$. The left-hand side macroscopic energy surface corresponds to the formation of $^{40}\text{Ca}$ on the surface of $^{248}\text{No}$, whereas the right-hand side represents the $^{16}\text{O}$-like soliton on the surface of $^{224}\text{Th}$. Variation along the elongation $R$ corresponds to an increment in the soliton amplitude along the symmetry axis. A larger neck radius $R_3$ corresponds to a larger half-width $g(\theta)$. The rear plane at $R = 0$ is the spherical state of the system. Then the energy increases monotonically with a higher slope for small values of the neck radius. The increase is smoothed by the necking as $R_3$ increases. With the enhancement of the kinetic energy of the soliton, besides the very first stages of the process where the neck radius is very small, the half-width becomes larger. The ridge in energy has a maximum at the near touching spheres configuration for both reactions. The slope continues to increase for large $R_3$ beyond the touching point value of elongation $R$.

The addition of shell corrections yields the total deformation energy shown in figure 3. As a first observation note the pronounced deformed ground state of $^{248}\text{No}$ as the first minimum in energy moves to $R > 0$, and a much less but still deformed ground state for $^{224}\text{Th}$. For both emerging solitons it is obvious that the energy path corresponds to large half-width values up to the top of the energy ridge, then they abruptly turn towards rupture point shapes ($R_3 = 0$). Hence these potential energy surfaces suggest a three-dimensional curve as the path of minimum energy in the cluster emission. The potential barrier formed along the path of minimum energy values is obtained by minimization of the total energy in the multidimensional deformation space.

The static paths which a soliton with the internal structure of an emitted cluster has to follow have been plotted on the contour maps of the energy surface in $(R, R_3)$ coordinate space in figure 4. Again the left-hand side plot is the $^{40}\text{Ca}$ emission and the right-hand side one corresponds to $^{16}\text{O}$. Apart from the first $R$ values where $R_3$ is small, the solitons bypass the first energy peak by taking large neck radius values, i.e. large half-widths. As the energy increases on the large $R_3$ side, the static path for both cases changes direction reaching the scission point where the clusters are emitted.
Figure 4. Contour plot of figure 3 with a static path (broken curve) for: (a) $^{40}$Ca emission and (b) $^{16}$O emission. As the elongation increases the amplitude of the soliton increases together with the half-width which is proportional to the neck radius $R_3$. Once the touching point maximum is bypassed, both shapes decrease rather abruptly through necking towards scission.

Figure 5. The barriers along the static path for: (a) $^{40}$Ca emission and (b) $^{16}$O emission, together with macroscopic energy (dotted curves) and shell corrections (broken curves). Shell corrections increase the total energy of the first energy minima for $^{248}$No, thus the two-humped barrier is not higher than about 1.2 MeV. $^{16}$O emission from $^{224}$Th has a barrier of about 11 MeV.

The two static barriers are plotted in figure 5 with a full curve, together with the macroscopic energy (dotted curve) and the shell corrections (broken curve). One can see how the deformed ground state of $^{248}$No is formed (a): due to shell corrections, the first minimum is at about 6.8 MeV of the total energy. This point becomes the ground state and the whole barrier in front of the emerging soliton is shifted with respect to this value. A two-humped barrier no higher than about 1.2 MeV blocks the $^{40}$Ca emission.

The situation is different for the emission of $^{16}$O from $^{224}$Th. The ground state is only slightly deformed. A rather high one-hump barrier of about 11 MeV extends along the whole range of elongation $R$ up to the scission point. Shell corrections decrease the macroscopic
energy values slightly. The decrease is mainly due to the double-magic character of $^{208}\text{Pb}$ which forms as the cluster emerges.

One can state that $^{40}\text{Ca}$-like solitons are energetically favoured to form on the nuclear surface of a very heavy nucleus as $^{248}\text{No}$. The formation of a $^{16}\text{O}$-like soliton on $^{224}\text{Th}$ is not energetically favoured due to the high and large potential barrier it has to penetrate.

The relative velocity distribution $V$ of the two presumed solitons along the minimum energy path together with the scaled values of the half-width $L$ and relative amplitude $a = A/R_1$ are plotted in figures 6. We investigated the evolution of these soliton parameters (as defined in section 1), depending on the static energy evolution parametrized by the distance between centres $R$.

The tendency in the first stages showed that the amplitude and half-width increase with the elongation parameter, when the emitted cluster is emerging out from the parent nucleus (since their non-overlapping sector is increasing). During the formation of the cluster the half-width
remains practically constant, since the surface energy controls this stage. When the two nuclei are well separated and the limit when the soliton envelope hardly fits the two spheres, the half-width approaches a zero value. These values of the half-width $L$ (full curve) are compared with those obtained analytically in [3] directly from the soliton amplitude, within the framework of the nonlinear liquid drop model ($L$, broken curve). We notice a good agreement for the half-widths within the range $R \simeq 4.5–14$ fm. The hydrodynamic soliton model is no longer valid for a separation parameter $R$ smaller than 4–5 fm, because of the dominant shell effects in this range. This can be noted in a comparison between figures 2 and 3 for $R \leq 4–5$ fm. For the first $R$ values the static paths follow the first energy peak, and jump from small toward large values for $R_3$ (figure 4) providing small half-widths ($L$, full curve), while a pure hydrodynamic soliton would have larger half-widths for this range ($L$, broken curve). In the above range of validity of the soliton model, we calculate the relative velocity of the soliton ($V$, broken curve) [3]. The velocity is increasing with the amplitude of the soliton, and hence with the elongation of the cluster-like emission shape. This gives the limiting configuration for this soliton model. Figure 6(a) displays the $^{40}$Ca emission and figure 6(b) represents the $^{16}$O emission. Soliton shapes at the beginning and the end of the process are also shown. For lighter nuclei (such as oxygen) the evolution of the parameters is smooth and monotonic. In the case of heavier nuclei (Ca) we obtained some oscillations in the width and velocity during the first half of the emission process, which can be related to the oscillations in the $R_3$ parameter because of the shell effects.

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**References**

    Ludu A and Draayer J P 1998 Physica D 123 82
    Wexler C and Dorsey A T 1999 Phys. Rev. Lett. 82 620
    Wexler C and Dorsey A T 1999 Phys. Rev. B 60 10971