SHELL-MODEL PREDICTIONS FOR UNIQUE PARITY YRST
CONFIGURATIONS OF ODD-MASS DEFORMED NUCLEI

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Abstract: Features of a proposed shell-model coupling scheme for high-spin, unique parity states of odd-mass nuclei are investigated. Valence nucleons are assumed to be distributed among normal parity orbits of a major shell and a unique parity intrashell level. Basis states are built by weak coupling the leading configuration of each, the necessary further transitions being achieved by restricting the subshell configurations to leading representations of pseudo SU(3) for the normal parity space and quasispins for the unique parity part.

The appearance of $\Delta I = 2$ (intershell) and $\Delta I = 1$ (intra-shell) band sequences is shown in lowest order to be a simple function of the number of particles in the unique parity orbital and their isospin and the background deformation provided by the normal parity part of the structure. Results of a simple model study which elucidates properties of the bands including crossing, odd particle alignment, $E2$ and $M1$ rates and static moments are presented. Finally a survey of the experimental data that supports the model predictions is presented.

1. Introduction

For more than a decade there has been a rapid accumulation of experimental data on odd-mass deformed nuclei. The theoretical tool which has been most successful in leading to an understanding of these nuclei has been the particle–rotor models in which all but one of the particles are lumped together and represented by a rotor while the odd nucleon's degrees of freedom are explicitly treated. Typically the Nilsson model provides the framework for the calculations. Within this context, whenever the spin of the odd particle is large one may expect sizable Coriolis effects to occur which arise from the separation of the problem into intrinsic deformed shape plus rotation. For transitional nuclei which are not good rotors and where axial symmetry destroying vibrations may be important, the triaxial rotor-plus-particle model of Meyer-ter-Vehn and triaxial variable moment of inertia core plus particle model of Toki and Fasold have enjoyed success. For transitional weakly deformed nuclei, the particle–vibration-coupling model has been developed. Models employing a group theoretical treatment of the core include the IBFA and the particle–quadrupole phonon model. Finally self-consistent Hartree–Fock–Bogoliubov cranking calculations by Ring et al. have been used to elucidate properties of high-spin yrast configurations.

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Microscopic shell-model studies of odd-mass deformed nuclei have not kept pace due to the enormous dimensionality encountered when, as is usually the case, the multi-particle basis states are built from antisymmetrized products of valence single-particle orbitals. Recently, however, it has been demonstrated that group theoretical methods can be used to reduce such model spaces to physical subspaces of tractable size \(^1\). The method has been applied to a study of deformed even-even nuclei with valence neutrons and protons in the same major shell \(^2,3\). An important part of the description is the partitioning of the valence nucleons into two sets, those which occupy the natural (normal) \(N\) parity orbitals of a major shell and those which occupy the unique (abnormal, \(A\) parity orbital which is pushed down from the next higher shell by the spin-orbit interaction. But weak coupling of these two still yields dimensionality far in excess of even modern machine capabilities. The necessary further truncation is achieved by restricting the subspace configurations to leading representations of approximate symmetry groups of the interaction hamiltonians, pseudo SU(3) for the \(N\) subspace \(^4\) and quasispin for the \(A\) subspace \(^5\).

In this work we begin our study of odd-mass nuclei. The underlying model we shall use is an extension of that used in our study of even-even nuclei. In sect. 2 and 3 we review the pseudo SU(3) and quasi-spin geometries. We shall restrict our consideration to the unique parity levels of even-odd systems; that is, we will focus on systematics of configurations with the odd nucleon in an \(A\) orbital. In sect. 4 we use perturbation theory arguments to discover general features one would expect to emerge within the context of the model. The results from a model study designed to test the predictions are reported in sect. 5. Finally, in sect. 6, we offer an interpretation of experimental data in terms of the parameters of our model. Sect. 7 is reserved for concluding remarks.

2. Pseudo SU(3)

The technique of identifying normal parity orbitals as members of a pseudo oscillator shell grew naturally out of studies of pseudo spin-orbit doublets \(^6\). It has been used by Raja, Draayer and Hecht in a discussion of ground-state magnetic moments of odd-mass deformed nuclei \(^7\) and by Strottman in a study of Ni-Cu-Zn isotopes \(^8\). Hecht has used the scheme to investigate decoupled negative parity spectra of the Au nuclei \(^9\) while Braunschweig and Hecht used it in an analysis of core deformation for nuclei in the proton-rich Xe-Nd region \(^10\). Most recently it has been used in attempts at providing a microscopic shell-model interpretation of high-spin phenomena in deformed nuclei.

Recall that real SU(3) is a useful coupling scheme for strongly deformed nuclei when the asymptotic Nilsson quantum numbers \(|N_n,\ell,\Omega\rangle\) for single-particle states are approximately good and when members of the Nilsson spin-orbit doublet with \(\Omega = \ell \pm 1\) are nearly degenerate \(^11\). When these conditions are satisfied the filling
of the single-particle levels will be in the order \( n_s = N, n_s = N - 1, \ldots \) and will lead therefore, following the technique of Elliott, to an invariant single-particle state of highest weight for the so-called leading irreducible representation (irrep) of SU(3). For an irrep labelled \((\lambda \mu)\) the highest weight state is the one with \( n_s = 2 \lambda + \mu \) and \( n_s = n_s - \mu \); the leading irrep is the one with maximum possible \( 2 \lambda + \mu \) and for fixed value of that sum, the maximum possible \( \mu \). The leading representation is also the one with a maximum expectation value of the deformation inducing quadrupole–quadrupole interaction, \( Q \cdot \hat{Q} = 4C_2 - 3L^2 \), where \( C_2 \) is the second order Casimir invariant of SU(3) with eigenvalue \((\lambda + \mu + 3)(\lambda + \mu - \mu)\) and \( L^2 \) is the SU(3) invariant with eigenvalue \( \lambda(\lambda + 1) \).

For nuclei of the lower ds shell these conditions are satisfied sufficiently so one typically finds leading irreps of SU(3) forming 60-80% of yrast eigenstates. For the higher shells, however, the magnitude of the spin-orbit splitting is large so the Nilsson \( \ell = \lambda \pm 1 \) levels are widely separated and SU(3) is not a good symmetry. The spin-orbit interaction pushes the state of maximum \( \ell \) down into the next lower shell. The normal parity levels that remain have the same total angular momenta as levels of an oscillator shell of one less quantum. The pseudo SU(3) scheme exploits this relationship. By assigning to each level the appropriate pseudo orbital and pseudo spin labels, the valence space can be mapped onto a pseudo oscillator shell. For example, for rare earth nuclei \( (\Omega, d_z, d_{z^2}, d_{x^2-y^2}, n_s) \rightarrow \) \( \bar{L} \), \( \bar{J} \), \( \bar{\Omega} \), \( \bar{\Delta} \), \( \bar{\Delta} \), \( \bar{\Delta} \) maps the valence ghs shell orbitals onto a pseudo fp shell. Note that \( L = s - f = \ell - \frac{\bar{\Delta}}{2} \).

Though the mapping is exact it is only useful if the physical interaction hamiltonian is dominated by symmetry preserving operators. For pseudo SU(3) the simplest nontrivial symmetry-preserving operator is the second-order Casimir invariant \( C_2 \). But \( \hat{Q} \cdot \hat{Q} = 4C_2 - 3L^2 \) and \( \hat{Q} \cdot \hat{Q} \) is itself strongly correlated with the real quadrupole–quadrupole interaction which is known, for example through the success of Bohr–Mottelson theory, to be an important part of the residual interaction operating among the valence nucleons. We therefore expect to be able to increase, without much loss, the normal parity part of the space to a few of the leading irreps of pseudo SU(3).

A complication of course is the presence of the abnormal parity intruder level which clouds the picture of how many nucleons are in each subspace. In practice one constructs a Nilsson diagram and fills the pairwise degenerate levels with neutrons and protons in order of increasing energy using a reasonable guess for the deformation. This yields an estimate for the most probable ground-state occupancy \((n_s, n_\lambda)\) for each of the subspaces, an estimate that must then be checked by confirming that it, as compared to configurations with \((n_\lambda + 2, n_\lambda + 2)\), yields the greatest binding when a realistic interaction hamiltonian is diagonalized. Thus the Nilsson model serves as a guide for determining the dominant partition.

Usually, in applications several partitions with their own concomitant pseudo SU(3) representations are retained. It should be remembered that if all partitions
and all pseudo $SU(3)$ representations are included the model space would then be equivalent to that used in a standard shell-model calculation.

3. Quasiparticle

For a description of the abnormal parity part of the space we adopt the five-dimensional quasiparticle formalism [17]. While three-dimensional quasiparticle [18], applicable to identical nucleon configurations, allows one to factor the dependence on particle number out of many-particle matrix elements, five-dimensional quasiparticle, applicable when both neutrons and protons are present, allows one to factor out and study the dependence of many-particle matrix elements on both particle number and isospin. We shall see that it is this $(n, T)$ value of the abnormal parity configuration that determines, together with the intrinsic shape of the associated normal parity configuration, whether an odd-$A$ unique parity spectrum will exhibit $\Delta J = 2$ stretched or $\Delta J = 1$ (ordered) yrast band structure.

The quasiparticle symmetry group derives its significance from the fact that the pairing interaction is an important part of the nucleon-nucleon interaction. Within a single $j_s$ shell, where the collective quadrupole modes cannot manifest themselves fully because of the restricted geometry, pairing effects may be predominant. When both neutrons and protons are present, the $J = 0$ coupled pair creation and annihilation operators are $\Gamma = 1$ objects with components $M_s = \pm 1, 0$. These, together with the isospin and number operators, form a ten-dimensional algebra closed under commutation which can be identified as the algebra of $R(5)$. The set of all $J = 0$ odd-rank tensor operators generates the complementary $Sp(2l + 1)$ algebra [18]. In addition, the family of all one- and two-particle creation and annihilation operators together with all number-conserving operators of particle rank 1 generates an $R(4l + 3)$ algebra. Hence the group structure is

$$R(4l + 3) \downarrow$$
$$Sp(2l + 1) \otimes R(5) \downarrow$$
$$\cdots \cdots \downarrow$$
$$SU_C(2) \otimes SU_T(2) \downarrow$$
$$U(1) \otimes U(1) \downarrow$$
$$U(1, M_s) \ n \ (T, M_T)$$

The seniority $\nu$ counts the number of particles not coupled to $J = 0$, and $\tau$ is their isospin. These two, because of complementarity, label both the irreducible representations of $Sp(2l + 1)$ and $R(5)$. Basis states are labelled $(\nu, \tau, M_s, n, T, M_T)$ where $\nu$ and $\tau$ are indices used to distinguish multiple occurrences of $J$ and $T$ in $(\nu, \tau)$. The fact that $J$ and $(n, T)$ are subgroup labels in the quasiparticle scheme leads to major simplifications. As in ordinary angular momentum theory, there is a
Wigner-Eckart theorem for factoring matrix elements into a coupling coefficient and a reduced matrix element factor. For the quasiparticle model the dependence of the matrix elements on $J$ and $(n, T)$ for fixed $(\mu, i)$ is determined by an $Sp(2^\mu + 1) \rightarrow SU(2)$ coupling coefficient and an $R(5) \sim U(1) \otimes SU(2)$ coupling coefficient, respectively.

For an odd-$A$ nucleus with the unpaired nucleon in an abnormal parity orbital, the lowest configuration will have $(\mu, i)$ fixed at $(1, 1)$. For this $(\mu, i)$ there is but one $J$ equal to the single-particle value. Differences among isotopes and isotones come only from the $(n, T)$ dependent factor. To the extent the quasiparticle geometry is a reasonable symmetry; the $R(5) \sim U(1) \otimes SU(2)$ reduced coupling coefficient determines this behavior.

4. Consequences of the weak coupling model

We now wish to explore, within the context of the weak coupling picture, general features that can be expected to emerge when the valence particles in the normal and abnormal parity orbitals are allowed to interact. For low-lying states we assume the following. First, isospin in each separate subspace is a good quantum number. Second, for abnormal parity configurations of odd-mass nuclei, the basis can be restricted to seniority one states. Third, the normal parity subspace can be restricted to pseudo-spin zero. Fourth, the even-even parent nucleus has an even number of neutrons and protons in each of the separate subspaces. Finally, the interaction between the two subspaces has a quadrupole-quadrupole ($Q_{22} \otimes Q_{22}$) form. The validity of each of these assumptions is, of course, related to the properties of the true residual interaction. The first four, for example, are consistent with the choice of a two-body surface-delta interaction between particles in the separate subspaces. A $Q_{22} \otimes Q_{22}$ form for the interaction term is indicated by many theoretical studies and is an underlying feature of the Bohr-Mottelson picture of nuclear deformation.

Throughout the remainder of this paper we will assume that the number of particles in each of the two subspaces is fixed and is consistent for even-$A$ parent and odd-$A$ daughter nuclei. That is, if the dominant configuration has $N = n_n + n_A$ valence particles in an even-even parent $(n_n$ and $n_A$ are both even because of pairing), then the even-odd daughter also has $n_n$ normal parity particles but $n_n + 1$ particles in the abnormal parity subspace. It follows that if there are $\pi$ protons and $\nu$ neutrons in the abnormal parity orbital of the parent nucleus, $T_A = [\pi - \nu]$. Adding one neutron (proton) yields $T_A + 1$ for the spin of the seniority one daughter configuration. It is to be emphasized that the odd nucleon is weakly-coupled to the $n_n$ nucleons of the parent while the separate subspaces are weakly-coupled to each other. To some extent then the $n_n$ particles play a spectator role as we choose here to focus on the abnormal parity levels of even-odd nuclei. We point out however, that the results may be different from those of a model in which the odd particle is weakly coupled to an even-even parent. In our approach the
many-particle dynamics of both the N and A spaces is microscopically represented, albeit with a drastic truncation.

Quantitative conclusions regarding the results of weak-coupling the two subspaces can be obtained using first-order perturbation theory (11). For a Qa—Qc interaction effecting the mixing of the two spaces, the shift of an energy level from its unperturbed position is given by

$$
\Delta E(U) = \mathcal{O}(\eta_n, J_a, J_c) \langle \hat{a}_n \hat{a}_c \rangle \eta \frac{\Delta E(\pi, a, \pi, c)}{J_a, J_c, \pi, a, b, c, d, e},
$$

where \( \mathcal{O}(\eta_n, J_a, J_c) \) is the coefficient of the \( \langle \hat{a}_n \hat{a}_c \rangle \eta \) term in the tensor decomposition of \(-Q_a - Q_c\). In eq. (1) \( J_a \) is the angular momentum of the abnormal parity shell-model orbital, \( \eta = (\eta_n, \eta_c, \eta_p) \) is a complete set of labels for the \( \eta_n \) particle abnormal parity state vector, \( \eta \) labels the quantum numbers of the \( \eta_n \) particle normal parity state vector which for pseudo SU(3) is \( \eta = (\eta_n, \eta_c, \eta_p) \) and \( \eta_p \) labels the quantum numbers of the normal parity parts of the tensor, which for pseudo SU(3) is dominated by the \( \eta_p = (\lambda = 1, \mu = 1, \kappa = 1, \lambda = 2, \mu = 0, \kappa = 0) \) tensor (11).

The expression for \( \Delta E(U) \) factors into two parts, a structure factor which is simply the strength \( \mathcal{O}(\eta_n, J_a, J_c) \) times the product of the normal and abnormal parity one-body reduced matrix elements of \( Q_a \) and \( Q_c \), respectively, and a geometrical factor which arises from the recoupling of the angular momenta. The sign of the structure factor determines which of two characteristic spectra will dominate for abnormal parity yrast states. This can be understood by noting that typically the N-space interaction favors \( S_n = 0 \) configurations and the formation of a rotational band with \( L_n = 0, 2, 4, \ldots \) which then couples with the seniority one \( J = 1 \) A-space configuration to form states of good total angular momentum. For example, for \( J_a = \frac{1}{2}^+ \) the expected lowest energy state results from the coupling \( \frac{1}{2}^- \beta = \frac{1}{2}^+ \). For a positive valued structure factor the energy ordered splitting of the quintet that results from the coupling of \( J_a \) to \( L_n = 2 \) is \( \frac{1}{2}^+, \frac{3}{2}^-, \frac{3}{2}^+, \frac{5}{2}^- \) and \( \frac{5}{2}^+ \). That is, the \( \frac{1}{2}^- \) state lies lowest but more significantly the \( \frac{3}{2}^- \) state is pushed down whereas the \( \frac{3}{2}^+ \) state is pushed up. Similarly for the coupling \( 4 \beta = \frac{5}{2}^- \) the \( \frac{1}{2}^+ \) state is pushed down and the \( \frac{5}{2}^- \) state is pushed up. From an experimental viewpoint one expects the sequence of states \( \frac{1}{2}^+, \frac{3}{2}^-, \frac{3}{2}^+, \frac{5}{2}^- \) to be strongly coupled by a stretched E2 cascade. These are sometimes referred to as the favored states, whereas the band of states \( \frac{3}{2}^-, \frac{5}{2}^-, \frac{3}{2}^-, \frac{5}{2}^- \) which one also expects to be strongly connected by E2 transitions are called unfavored states. Whether or not the \( \frac{5}{2}^- \) state lies below the lowest \( \frac{3}{2}^- \) state depends both on the strength of the interaction and on configuration mixing and usually can only be determined through detailed analysis.

We will not focus much attention on such details. For a negative valued structure factor the energy ordered sequence is \( \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^-, \frac{3}{2}^-, \frac{1}{2}^-, \frac{5}{2}^+ \). For what we of
a better name we will call ordered. Throughout the remainder of this paper we will call an abnormal parity band sequence $J_a/J_a + 2, J_a + 4, \ldots$ a stretched spectrum. If the sequence goes as $J_a, J_a + 1, J_a + 2, \ldots$ it will be called an ordered spectrum. Other terminology found in the literature for similar sequences of levels includes decoupled and strongly coupled, respectively. The terminology stretched (ordered) will be applied regardless of the spacings between levels so, for example, even if the $J_a + 2n + 1$ states are nearly degenerate with the $J_a + 2n + 2$ states ($n = 0, 1, 2, \ldots$) the level spectrum will be called stretched (ordered) when the $J_a + 2n + 1$ states lie above (below) the $J_a + 2n + 2$ states. This is summarized in Table 1 where the spectra type and unique parity spin sequences are given for various N-A space combinations.

### Table 1

<table>
<thead>
<tr>
<th>Structure factor</th>
<th>Spectra type</th>
<th>Spin sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>(prolate)</td>
<td>+</td>
<td>stretched $J_a, J_a + 2, J_a + 4, \ldots$</td>
</tr>
<tr>
<td>(prolate)</td>
<td>-</td>
<td>ordered $J_a, J_a + 1, J_a + 2, \ldots$</td>
</tr>
<tr>
<td>(oblate)</td>
<td>+</td>
<td>ordered $J_a, J_a + 1, J_a + 2, \ldots$</td>
</tr>
<tr>
<td>(oblate)</td>
<td>-</td>
<td>stretched $J_a, J_a + 2, J_a + 4, \ldots$</td>
</tr>
</tbody>
</table>

A weak coupling of normal (N) and abnormal (A) parity shell model configurations is assumed.

Since the overall sign of the structure factor determines which of the two characteristic spectra is expected to emerge, we shall now investigate the origin of this sign. The structure factor of eq. (1) is itself a product of three factors, the overall strength of the interaction and the reduced matrix elements of the multipole operator in the N and A spaces. The choice of $Q \cdot Q$ for the interaction term is made based on the success of the Nilsson model. The $Q_N$, $Q_A$ form is known to be an essential ingredient in stabilizing the deformation and is a reasonable first approximation to the true residual interaction between particles in the normal and abnormal parity orbits. Note that the product of the strength factor and the reduced matrix element of $-Q_N$ is proportional to the contribution to the quadrupole moment from the normal parity part of the space. We have that this product is positive (negative) independent of the details of the structure of the normal parity part of the state vector. For our purpose we will use a pseudo-SU(3) description, hence an irre with $\lambda \geq \mu$ ($\lambda < \mu$) corresponds to a prolate (oblate) deformation $^{22}$. The part of the structure factor left to consider is the reduced matrix element of $Q_A$ in the abnormal parity space. Together with the prolate or oblate character of the even-even parent nucleus this sign will determine whether the abnormal parity yrast spectrum of the even-odd system is stretched or ordered. The matrix
element is given by generalized quasiparticle geometry to be \( n_A, n_A - 1, j_A = j_A, T_A = \sigma_A Y_{j_A, \sigma_A - 2, Y_{-1}}(\sigma_A, 0 = 1, j_A - j_A, T_A) \)
\[
= \sqrt{2} (2 T_A + 1)^{1/2} \frac{22 + 3}{(22 + 1)(22 + 3)} (B - n_A),
\]
(2)

where
\[
B = \begin{cases} 
(22 + 3) & \text{if } (-1)^{2 + 2, T_A, \sigma_A} + 1 \\
(22 + 1) & \text{if } (-1)^{2 + 2, T_A, \sigma_A} - 1
\end{cases}
\]
(3)

As an example, suppose a given even-even nucleus is prolate. This invariably implies that the normal parity reduced matrix element is negative (due to the predominance of pairing forces). Let \( j_A = \frac{3}{2} \) and consider eight neutrons and two protons in the normal parity orbital of the even-even parent system. Adding a neutron yields \( n_A = 11, T_A = \frac{7}{2}, B = 17, \) and the matrix element of eq. (2) is negative. The abnormal parity yrast levels should therefore be ordered, that is, one expects a sequence \( \frac{7}{2}, \frac{3}{2}, \frac{1}{2}, \ldots \). If instead of a neutron a proton is added, \( T_A = \frac{5}{2}, B = 19, \) and the matrix element is positive, thereby suggesting a stretched yrast spectrum, \( \frac{5}{2}, \frac{3}{2}, \ldots \) with possibly a \( \frac{1}{2} \) state falling below the \( \frac{3}{2} \) state. If the even-even parent nucleus were oblate rather than prolate the results would be interchanged: namely, adding one neutron (proton) would result in a stretched (ordered) yrast spectrum. Thus the generalized quasiparticle formulation allows one to make specific predictions of how the odd nucleon interacts with the even-even parent. In table 2 we summarize the results by giving the theoretically expected abnormal parity yrast spectrum for various even-odd nuclei. Note that we have assumed that the number of neutrons in the abnormal parity orbital of the even-even parent exceeds the number of protons. Adding a neutron (proton) therefore increases (decreases) the value of \( T_A \). If the opposite is true, one can still use table 2 but with the neutron and proton interchanged everywhere in the second column, a case that actually not needed in any of the applications of this paper.

5. Model calculation

To illustrate properties of wave functions of stretched and ordered spectra, we report in this section the results of a simplified model calculation. We begin by considering an even-even parent with twenty-four valence nucleons in the gds shell, partitioned as eight protons and eight neutrons in the normal parity space and eight neutrons in the abnormal parity \( h_{11/2} \) level. One can map from the real gds shell...
### Table 2

**Unique parity yrast spectra for the various parent and abnormal parity configurations**

<table>
<thead>
<tr>
<th>Case no.</th>
<th>Sign $\lambda_n = -(2L_z + 1)$</th>
<th>Odd particle</th>
<th>Parent shape</th>
<th>Condition</th>
<th>Yrast spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$+$</td>
<td>proton</td>
<td>prolate</td>
<td>$\lambda_n &lt; B_z$</td>
<td>stretched</td>
</tr>
<tr>
<td>2</td>
<td>$+$</td>
<td>neutron</td>
<td>prolate</td>
<td>$\lambda_n &gt; B_z$</td>
<td>stretched</td>
</tr>
<tr>
<td>3</td>
<td>$-$</td>
<td>proton</td>
<td>oblate</td>
<td>none</td>
<td>ordered</td>
</tr>
<tr>
<td>4</td>
<td>$-$</td>
<td>neutron</td>
<td>oblate</td>
<td>$\lambda_n &lt; B_z$</td>
<td>ordered</td>
</tr>
<tr>
<td>5</td>
<td>$-$</td>
<td>neutron</td>
<td>oblate</td>
<td>$\lambda_n &gt; B_z$</td>
<td>stretched</td>
</tr>
<tr>
<td>6</td>
<td>$+$</td>
<td>neutron</td>
<td>prolate</td>
<td>none</td>
<td>ordered</td>
</tr>
<tr>
<td>7</td>
<td>$+$</td>
<td>proton</td>
<td>prolate</td>
<td>$\lambda_n &lt; B_z$</td>
<td>stretched</td>
</tr>
<tr>
<td>8</td>
<td>$+$</td>
<td>proton</td>
<td>oblate</td>
<td>none</td>
<td>ordered</td>
</tr>
<tr>
<td>9</td>
<td>$+$</td>
<td>proton</td>
<td>oblate</td>
<td>$\lambda_n &lt; B_z$</td>
<td>ordered</td>
</tr>
<tr>
<td>10</td>
<td>$+$</td>
<td>neutron</td>
<td>oblate</td>
<td>none</td>
<td>stretched</td>
</tr>
<tr>
<td>11</td>
<td>$+$</td>
<td>proton</td>
<td>oblate</td>
<td>$\lambda_n &lt; B_z$</td>
<td>ordered</td>
</tr>
<tr>
<td>12</td>
<td>$+$</td>
<td>proton</td>
<td>oblate</td>
<td>$\lambda_n &gt; B_z$</td>
<td>stretched</td>
</tr>
</tbody>
</table>

The symbols are redefined for eq. (2) of the text.

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To pseudo fp-shell orbitals and truncate to leading representations of pseudo SU(3) as done in previous studies. The quantum numbers of the weak-coupled wave functions of the parent are then given by

\[ [4444]_{\lambda \mu} \otimes L, S_n = 0, T_n = 0, J_n = L; (B; 1)_{\lambda \mu}; T = 4, J = 4, T = 4. \]  

We have chosen to consider this particular case because the SU(3) irreps with largest second-order invariant eigenvalues, and hence those expected to dominate the low-lying structure, are $[(\lambda \mu) = (4, 0)$ and $(0, 24)]$. Since representations with $\lambda > \mu$ correspond to intrinsic shapes which give prolate contributions to the nuclear shape and representations with $\lambda < \mu$ give oblate shapes, this case incorporates the possibility of both a stretched and an ordered abnormal parity yrast level sequence. If both irreps were included, the basis the interaction would preferentially select one or the other, since it is not simply proportional to the SU(3) Casimir invariant. Of course, the interaction would probably mix other representations or other partitions such as $[4444]_{\lambda \mu}$ with its own set of SU(3) irreps into the low-lying eigenstates. For this model study we have chosen to ignore such complications, as the dominant features will emerge more simply without them, and no attempt is being made to identify a particular nucleus to which the results apply. (The logical candidate is $^{176}$Tc but little is known about its deformations and therefore whether the partition selected is appropriate.) As we were primarily interested in the differences that result from a prolate versus an oblate "core", we will present two calculations, one with $[(\lambda \mu) = (24, 0)$ and the other with only the $(0, 24)$ irreps included. In this regard, note that a two-body interaction cannot directly couple the $(24, 0)$ and the $(0, 24)$ irreps, so any mixing, if it were
to occur would have to be through other intermediate irreps which we have chosen to exclude from the onset.

Our hamiltonian is,

\[ H = H_{1+} + H_{2+} + H_{\text{sd}} - Q_{0} - Q_{\pi}, \tag{5} \]

where the strength of \( Q_{0} - Q_{\pi} \) was taken to be the self-consistent value 120/(124) MeV. \( H_{1+} \) and \( H_{2+} \) are two-body surface delta interactions (SDI) with strengths equal to those used in the \(^{120}\)Ba study and \( H_{\text{sd}} \), the experimental single particle energies for real gdf shell orbitals. \(^{121}\) \( H \) was diagonalized in the weak-coupled, pseudo-SU(3)-truncated basis. The results are displayed for both the (4, 0) and (0, 24) irreps in the center of fig. 1. Both level patterns show a low-lying rotational band with the seniority two configurations lying much higher in energy.

![Diagram](Fig. 1: Model study energy levels for the even-even and even-odd systems built on the (24, 0) and (0, 24) representations for the normal parity part of the model space. The odd baryonic levels are labeled by \( \lambda \). Members of the favored stretched and ended bands are circled.)

That the (0, 24) spectrum is more compressed than the (24, 0) spectrum reflects the fact that the effective moment of inertia of the (0, 24) irrep is greater than that of the (24, 0) for this choice of \( H \); but the difference is of little consequence in what follows. Our choice for \( H \) binds the (24, 0) irrep more than the (0, 24) by 4 MeV and hence favors the prolate shape in this space. The pairing part of the
SDI interaction must therefore be more effective in the (24,0) irrep than in the (0,24), producing as it does greater binding and a smaller moment of inertia for that irrep. If the calculations were realistic for $^{135}$Ce the binding energy result would suggest that the (0,24) irrep could be purged from the basis. Other irreps with direct couplings to the (24,0) would probably be more profitable to include.

If we now add one nucleon to the $h_{11/2}$ orbit we have basic wave functions for the negative parity states labelled as

$$|4444| \psi_{nL} \psi_{nL} S_n = 0, T_n = 0; J_n; (h_{11/2})^2 = 1, T_n = 4 \pm \frac{1}{2}; J_n = \frac{1}{2} \pm \frac{1}{2}; I = T_n \rangle,$$

where $T_n = \frac{3}{2}$ for an odd neutron and $\frac{1}{2}$ for a proton. The exact same $H$, eq. (5), was diagonalized in each of the four even-odd cases in the space spanned by the truncated bases, eq. (6). The results are also shown in fig. 1. (Note that the (24,0) proton and (0,24) neutron spectra are stretched whereas the (24,0) neutron and (0,24) proton spectra are ordered.) These results are consistent with cases 1 & 5 and 3 & 4 of table 2, respectively. (Note that $n_n + 1 < 2I_p + 1 = 12$ and $n_n > B - 3.4$)

Notice that for the stretched level spectra the $\frac{3}{2}^-$ state is not quite pushed below the $\frac{1}{2}^-$ state. In addition, the level ordering for the secondary levels is not quite the same as that given by the perturbation result, eq. (1). However, also notice that in each case an unfavorited stretched band is formed starting with the $\frac{5}{2}^-$ state. Comparing the energy and the relative energies of the even-even parent one finds very close agreement, a point which will be discussed in greater detail later.

The (0,24) proton and (24,0) neutron ordered level spectra are shown on the far left and right, respectively, of fig. 1. They too do not follow exactly the level ordering of the perturbation result, eq. (1), but the main features are there. Note that the $\frac{3}{2}^-$ state is pushed up, the ordered yrast sequence starts with the $\frac{1}{2}^-$ state, and there is a secondary ordered sequence associated with the $\frac{3}{2}^-$ level. The (0,24) sequence is compressed relative to the (24,0) just as for the parent cases but unlike the stretched case the ordered energies do not follow the relative separation energies of their even-even parents, being instead more uniform. This suggests that the odd nucleon disturbs the parent core more strongly in ordered cases than in stretched ones.

As far as the dependence of the results on the interaction strengths, increasing $H^0_n$ or $H^0_n$ merely increases the relative separation between levels. If the $Q_n \cdot Q_n$ term is turned off all the odd cases follow the even-even results in the sense that the 0$^-$ is replaced by the $\frac{1}{2}^-$, the $\frac{3}{2}^-$ by the degenerate quintet ($\frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-, \frac{9}{2}^-, \frac{11}{2}^-$), the 4$^-$ by the degenerate nonet ($\frac{7}{2}^-, \frac{9}{2}^-, \frac{11}{2}^-, \frac{13}{2}^-, \frac{15}{2}^-$), etc. Adding any $Q_n \cdot Q_n$ strength at all produces level patterns like those of fig. 1. If a repulsive interaction is used, e.g. $+ Q_n \cdot Q_n$, then all stretched cases become ordered and vice versa.

To get a better understanding of the nature of these spectra, we now examine the electromagnetic properties of the states. In the following, quadrupole moments
and E2 transitions are calculated using a polarization charge of 0.5 for neutrons and protons. Dipole moments and M1 transitions are calculated using the free values of the spin and orbital g-factors for the neutron and proton.

In fig. 2 we have plotted the variation of the static quadrupole moment with spin for the lowest energy states of each spin for all the even-even and even-odd cases. For the stretched examples, both the bands of favored (squares) and unfavored (circles) states follow the trends of their even-even parents. The behavior of the ordered band is more interesting. For example, the odd neutron ordered result has the (24, 0) irrep as its normal parity representation within which the interaction H favors a prolate (negative static quadrupole moment) configuration in the even-even system. The contribution to the quadrupole moment from the abnormal parity tensors is itself fairly small both in the even-even and even-odd systems. Nevertheless, for \( J = \frac{3}{2} \), the predicted \( Q(J) \) is positive for the ordered band. As the angular momentum increases we see that \( Q(J) \) approaches the even-even parent results. This behavior indicates the variable degree of polarizing influence which the odd
neutron can exert. For \( J = \frac{1}{2} \), the neutron is prevented by the Pauli principle from occupying the preferred orientations in the nucleus which maximizes the deformation of the parent. Since the neutron is not paired-off it generates its own quadrupole field which overlaps most strongly with a normal parity structure which was not the dominant structure in the even-even parent. As pairs of \( N \) parity particles are broken to generate higher angular momentum states \((I' \rightarrow I)\) increasing the ability of the odd nucleus to polarize the other nucleons is considerably reduced. Thus, gradually, the quadrupole moment tends towards the even-even value.

Fig. 3. Model study \( B(E2; I \rightarrow I - 2) \) results. Symbols have the same meaning as in fig. 2.

In fig. 3 we show the \( B(E2; I \rightarrow I - 2) \) values for the even-even and even-odd examples. Both the favored and unfavored bands for the stretched cases follow the even-even results and are strongly connected. Note the \( B(E2; \frac{3}{2} \rightarrow \frac{1}{2}) / B(E2; \frac{3}{2} \rightarrow \frac{1}{2}) \) ratio is just slightly larger than unity in comparison with the \( B(E2; \frac{3}{2} \rightarrow \frac{3}{2}) / B(E2; \frac{3}{2} \rightarrow \frac{1}{2}) \) ratio which is about 1.5. Hence we see that the stretched results are not simply the same as those from a weak coupling of an odd particle to an even-even one. The states of the ordered bands have much weaker \( B(E2; I \rightarrow I - 2) \) values. However, fig. 4 shows that the \( B(E2; I \rightarrow I - 1) \) values are large between states of the ordered bands (the \( \frac{3}{2} \) and \( \frac{5}{2} \) states are not band members) and weaker for all the stretched cases. Thus, in summary, we see that
the yrast states with spin $I$ and $I-2$ ($J-1$) for the stretched (ordered) bands are strongly correlated by the E2 operator.

In fig. 5 the $g(I)$ factors defined from the magnetic dipole moment by
\[ \mu(I) = g(I)I \]
are given for all the yrast states. The even-even results are roughly constant at a value of approximately $Z/A$ which is characteristic of a rotational band. The odd-$A$ results merely reflect the large positive (negative) contribution of the odd proton’s (neutron’s) magnetic moment. Note that the behavior of the stretched and ordered cases is similar and all slowly approach the even-even value as $I$ increases. These results, in comparison to results given in fig. 2, reflect the difference between the large single-particle value for the magnetic dipole moment relative to the collective value versus the small single-particle quadrupole moment contribution relative to the parent quadrupole moment. Thus changes in structure of the parent nucleus induced by the odd nucleon are not as discernible in the magnetic moment results.

Fig. 6 displays the magnetic dipole transition probabilities as a function of $I$ for the even-odd cases. The transitions between states of an ordered band increase with spin, with the proton (0, 24) ordered band transitions being stronger than the
Fig. 5. Model study static magnetic dipoles envelopes for the lowest state of each spin in fig. 3. Labelling is the same as in fig. 2.

Fig. 6. Model study odd-parity $BM_{1/2} (J = 1/2)$ results. Symbols have the same meaning as defined in the legend and caption for fig. 5.
neutron (24, 6) transitions. The latter result reflects the larger absolute values of the magnetic dipole moment for the odd proton states of an ordered band than for the odd neutron states (see fig. 5). The irregularities in the ordered neutron results at high spin may be an effect of basis truncation. For the stretched cases in fig. 6, note that the favored unresolved transitions (squares) are larger than the unfavored unfavorably transitions (circles) and this discrepancy becomes larger as $I$ increases. This result is consistent with the investigations of Namimoto [7] on the effects of rotation on M1 rates using the particle rotor model.

An interesting feature of the wave functions is displayed if one considers the alignment of the abnormal parity angular momentum, namely $I - j_z$ where $j_z$ is a unit vector in the direction of the total angular momentum. In fig. 7, we see that the favored states (open squares) all have maximum alignment. Hence as $I$ increases, the increase in $I$ comes solely from the collective contribution from the normal parity space. This is consistent with the decoupled picture given by Stephens [7]. The unfavored states do not exhibit complete alignment. Nevertheless, as the angular momentum increases most of that increase arises from the collective correlations in the normal parity space. This is why the $Q(I)$ and $B(E2; I \rightarrow I - 2)$ in figs. 2 and 3 were slightly stronger for the favored relative to the unfavored.

![Fig. 7. Degree of alignment of the abnormal parity $j_z = \frac{3}{2}$ configuration with the total angular momentum vector plotted as a function of the total spin of the system. Symbols are the same as in fig. 2. Solid boxes and circles overlap the empty boxes and circles respectively, and have therefore been omitted.](image)
strewn states. The results for the ordered spectra in fig. 7 indicate that the alignment decreases as \( I \) increases. Thus the individual identity of the odd particle is gradually leached as the nucleus responds to the collective increased quadrupole particle-hole excitation which forms the rotational band. The approach of the ordered \( Q(\lambda) \), \( B(E2; I \rightarrow I - 2) \) and \( g(I) \) results (figs. 2, 3, 5) to the even-even values are thus qualitatively understood due to the greater instability of the odd nucleus to polarize the nucler configurations in the normal parity space as spin increases. Comparing fig. 6 with fig. 7 we see that the ordered \( B(M1; I \rightarrow I - 1) \) results mirror the alignment trends fairly closely.

### Table 3

<table>
<thead>
<tr>
<th>( \Delta I = 2 \Delta I )</th>
<th>Proton ((24,0))</th>
<th>Spin-rotation ((9,24))</th>
<th>Proton ((0,24))</th>
<th>Neutron ((24,0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 11</td>
<td>-0.16</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.16</td>
</tr>
<tr>
<td>15 13</td>
<td>0.09</td>
<td>0.05</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>17 15</td>
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<td>-0.12</td>
<td>-0.39</td>
<td>-0.18</td>
</tr>
<tr>
<td>19 17</td>
<td>0.07</td>
<td>0.04</td>
<td>0.09</td>
<td>0.18</td>
</tr>
<tr>
<td>21 19</td>
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<td>-0.14</td>
<td>-0.00</td>
<td>-0.18</td>
</tr>
<tr>
<td>23 21</td>
<td>0.05</td>
<td>0.04</td>
<td>0.00</td>
<td>-0.17</td>
</tr>
<tr>
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<td>-0.15</td>
<td>-0.08</td>
<td>-0.10</td>
</tr>
<tr>
<td>27 25</td>
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<td>0.03</td>
<td>-0.06</td>
<td>-0.19</td>
</tr>
<tr>
<td>29 27</td>
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<td>-0.17</td>
<td>-0.08</td>
<td>-0.21</td>
</tr>
<tr>
<td>31 29</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.06</td>
<td>-0.23</td>
</tr>
</tbody>
</table>

The left (right) two cases have stretched (ordered) spectra.

In Table 3, we give our calculated \( B(E2/M1) \) mixing ratios \( 0 \). We find, similar to the results of Chen et al., \( 25 \), that the states of the ordered spectra have identical signs of \( \lambda \) whereas the stretched spectra exhibit an alternating sign pattern. This is a relatively simple consequence of the states of our basis states for the two cases. For the ordered spectra, the states have identical phases for all basis vectors (arbitrarily positive). For the stretched spectra, the sign alternates (with the first being arbitrarily positive). Note the basis states are just the members of the \( K = 0 \) band from the \((24,6)\) or \((0,24)\) steps coupled to the seniority one abnormal parity configurations. This leads to a strongly cohered \( E2 \) build up between states with \( \Delta I = 1 \) (or \( \Delta I = 1 \)) for the stretched (ordered) spectra. The \( E2 \) operator receives its dominant contributions from the normal parity \((\lambda_{\text{N}}) = (11)\), \( L = 2 \) tensor linking primarily basis states with \( E_{\text{N}} \) and \( E_{\text{P}} \). This leads to identical signs for \( E2 \) reduced matrix elements for \( \Delta I = 1 \) transitions. The M1 operator is dominated by the \( J = 1 \), \( T = 1 \) and \( 1 \) A parity tensors which contribute coherently to the M1 transition but which have a sign opposite to that of the small normal parity contribution. These two tensors must act only between states with identical \( L_{\text{N}} \) in the normal parity part. This leads to opposite (identical) signs for the \( B(M1; J \rightarrow J - 1 = 0) \) and
In this section we will review some of the experimental evidence for stretched and ordered abnormal parity level spectra in odd-A deformed nuclei and try to interpret them according to table 2. In particular we will primarily be looking at whether the $j_A+1$, $j_A+3$, ... states lie energetically above or below the $j_A+2$, $j_A+4$, ... states, respectively. For example, if the abnormal parity orbit is the $h_{11/2}$ orbit, then a positive parity spectrum such as $1^+$, $3^+$, $5^+$, $7^+$, $9^+$, $11^+$, $13^+$ would be interpreted as a stretched case, whereas $1^-$, $3^-$, $5^-$, $7^-$, $9^-$, $11^-$, $13^-$ would be an ordered spectrum. This identification is made in spite of the fact that in our model study of stretched examples the $j_A+2$ level was not pushed below the $j_A$ level in energy (though it still was linked by a strong $B(E2)$). Also for ordered cases, we should not be surprised to find the $j_A+2$ and $j_A+1$ states lying below the $j_A$ state even though they did not in the model study.

In ordered cases the odd particle preferred to couple with a configuration from the even-even parent which was not part of the yrast band. The results from our drastically truncated model study should therefore be only qualitatively indicative of the ordered structures found in real nuclei. The theoretical consequences of this will be discussed in more detail in the next section. Most of the experimental results discussed in this section are summarized in ref. 25). This also contains Nilsson diagrams from ref. 27 that are helpful in estimating occupancies of abnormal parity orbitals without doing any calculations.

Let us first look at nuclei where both neutrons and protons are filling the $h_{11/2}$ orbit. In fig. 8, a comparison of yrast positive parity levels of isotopes 34) of Ba with the negative parity stretched states of the corresponding La isotopes 35) shows close agreement of energy spacings between levels. Assuming the Ba isotopes are prolate and that there are roughly six to ten neutrons in the parent nuclei then the La spectra should be stretched by case 1 of table 2. Knowing this, then the addition of a neutron to the even-even Ba should result in nuclei which have ordered negative parity level spectra by case 3 of table 2. Indeed, experimental 36) information for 127-130Ba seem to confirm this prediction. The Xe isotopes for the same mass ranges should follow the same pattern, namely ordered spectra. For 132-135Xe this has also been observed 37). Using similar logic we expect the negative parity spectra of the odd mass Cs nuclei to be stretched. However, there is not enough available experimental data. From table 2, we note that when the number of neutrons in the $h_{11/2}$ is five we should expect a transition from ordered
to stretched level spectra for the odd neutron nuclei. Simple counting suggests this should occur in the vicinity of $^{199-212}$Xe and $^{120-127}$Ba. This is almost borne out in $^{126}$Xe ($\gamma$, $\gamma'$, $\gamma''$, $\gamma'''$, $\gamma''''$, $\gamma'''''$) (ref. 41) though the $\gamma''$ state (which is nearly degenerate in energy with the $\gamma'''$ state) is out of place. We conclude that the even-even Ba and Xe isotopes are prolate, since only in that case are the observed odd particle spectra consistent with table 5. In addition, from the same analysis we obtain estimates of the $\beta_{1/2}$ occupancies in the dominant shell-model configurations of these nuclei.

The mass 80 region is another where both neutrons and protons are actively filling the same orbital. Here the positive parity states arise from an odd number of neutrons in the $g_{9/2}$ orbital. Taking the $^{76}$Se parent nucleus as mildly prolate we would expect to see from a Nilsson diagram that at least four neutrons and zero protons reside in the $g_{9/2}$ orbit. From table 2, adding 5 neutron (proton) should result in an ordered (stretched) level spectrum by way of case 3 (13). In fig. 9 this expectation is upheld. Note the close agreement between the energies $\gamma''$ of $^{170}$Br and $^{176}$Se. For the lower mass Se isotopes we would expect the transition to stretched spectra (case 2) to long as a prolate to oblate transition does not occur for the even-even Se parents. With obtain Se parents case 5 would apply and imply ordered spectra. For slightly heavier nuclei, it appears $^{76,78,80}$Kr are all ordered and $^{78,80,82}$Rb are all stretched (with energy spacings closely corresponding to $^{76,78,80}$Kr respectively, $^{78,80,82}$Rb). This is consistent with prolate shapes for the even-even Kr parent nuclei and four neutrons and zero protons in the $g_{9/2}$ orbit for $^{76}$Kr (the latter being consistent with $c > 0.15$ in the Nilsson diagram of ref. 23).
Up to now, for all cases in this paper, valence neutrons and protons were assumed to be from the same major shell. For nuclei with mass greater than around 140 however, valence neutrons are filling a higher major shell than the valence protons, due to, of course, there being a greater number of neutrons than protons. Thus, there are two abnormal parity levels (each abnormal relative to their respective major shell). For even-odd nuclei, we define the abnormal parity level $J_a$ in table 2 to refer to the abnormal parity orbit which the odd unpaired nucleon occupies. For example, the abnormal parity level $J_a$ in $^{152}$Er refers to the neutron $i_{13/2}$ orbit; whereas in $^{152}$Er, $J_a$ refers to the proton $h_{11/2}$ orbit. For the purpose of qualitative discussion, the effect of the other abnormal parity orbital (the one which contains only paired nucleons for low energy eigenstates) can be ignored. Implicitly then, we are assuming that the nucleons in such an orbit couple to the corresponding normal parity configuration and play the same type of spectator role. In this type of situation the designation normal parity configuration refers to a strong coupling of neutron and proton normal parity configurations.

In the rare earth region, consider first the positive parity states of the odd Er isotopes which arise from neutrons in the $i_{13/2}$ level. The even-A Er isotopes are known to be examples of well-deformed prolate nuclei. Thus a Nilsson deformation parameter of $a > 0.25$ is reasonable and therefore one can expect three to seven neutrons in the $i_{13/2}$ orbit for $^{167-181}$Er. This results in a transition from stretched to ordered level spectra in going from $^{162}$Er to $^{180}$Er as shown in fig. 10 and as predicted by cases 2 and 3 of table 2. A similar transition occurs going from $^{184,186}$Se, $^{188,190}$Gd, and $^{192,194}$Dy. Thus one finds that the Sm, Gd, Dy, and Er
isotopes undergo this transition at neutron number 89, 91, 93 and 95, respectively. Hence the number of valence protons, though not themselves filling the $\frac{11}{2}^{-}$ level, nevertheless influence the neutron population of this level, probably by altering the degree of deformation in the parent nucleus. A little higher up, around mass 180, one is still in a region of significant prolate deformation. One expects at least seven neutrons and zero protons in the $\frac{11}{2}^{-}$ orbit and hence should find ordered spectra via case 3 of table 1. The $\frac{11}{2}^{-}, \frac{17}{2}^{-}, \frac{23}{2}^{-}, \frac{29}{2}^{-}, \frac{35}{2}^{-}, \frac{41}{2}^{-}, \frac{47}{2}^{-}, \frac{53}{2}^{-}$ (all seem to fail) to exhibit ordered spectra for the positive parity states. Also the static quadrupole moment of the $\frac{3}{2}^{-}$ ground state of $^{166}$Er is 5.1 ± 0.5 e·b, which assuming $^{166}$Er has a prolate shape, is in qualitative agreement with the prediction of fig. 2 and indicates the polarizing influence of the odd neutron.

In the preceding discussion of odd-$A$ $\frac{11}{2}^{-}$ nuclei, one has that for seven neutrons and no protons $B = B_{0} = \frac{1}{2}$. This means that the factor $B - N_{A}$ in eq. (2) is zero and hence the whole first-order prediction yields a vanishing result. In practice this first-order degeneracy prediction is lifted by secondary factors. These include, among other things, configuration mixing terms in the interaction. It is unlikely that any eigenstate is a pure $N_{A} = \frac{3}{2}$ configuration when we say $N_{A} = \frac{3}{2}$ we refer to the dominant configuration only. Nonetheless, because of the vanishing of the first order result for $N_{A} = \frac{3}{2}$ or $N_{A} = \frac{3}{2}$ configuration of is no importance can be crucial in predicting the expected characteristic spectrum. In such cases only full matrix calculations can indicate the exact origin of the degeneracy splitting.
The negative parity levels of odd proton nuclei in the rare earth region furnish another interesting check. Here the neutrons have already filled the $h_{11/2}$ orbit, and we have configurations of twelve $h_{11/2}$ neutrons plus some number of protons. Since we are adding a proton to a presumably prolate even-even nucleus cases 8 and 9 should apply (the transition occurring when we go from five to seven protons in the $h_{11/2}$ shell). Experiment [25] shows some indication for the transition from stretched to ordered level spectra. $^{143}$Tb is possibly stretched and $^{151,157,159}$Tb are ordered whereas $^{158,161,163}$Ho are all ordered. We see, however, that there is not a unique proton value $Z'$ such that all isotopes for $Z''<Z'$ must have stretched spectra whereas for $Z''>Z'$ the spectra of all isotopes are ordered. Similar to the previous examples of odd $h_{11/2}$ neutron configurations, this indicates that the relative occupancies of proton levels is subtly affected by the number of valence neutrons in the nucleus.

An interesting example occurs for the lutetium isotopes. Here we find stretched spectra for $^{173,179}$Lu in apparent disagreement with case 9 of table 2. However the spin sequence is $(1/2, 3/2, 5/2, 7/2, \cdots)$ [ref. 25] which indicates that the $h_{9/2}$ orbit is involved. In that case we would estimate its occupancy as one proton and around four neutrons. This would imply by case 1 of table 2 a stretched spectrum and hence would agree with experiment. We should then point out that an $h_{9/2}$ proton occupancy corresponds to a proton excitation out of the $Z=50-82$ major shell. Since neutrons are actively filling the $h_{9/2}$ shell, this might favor an induced proton population of this shell similar to the work by Federman and Pittel [19]. It is clear, nevertheless, that cases where a higher-lying abnormality orbit (such as $h_{9/2}$) can compete with the lower energy orbit ($h_{11/2}$) may not necessarily fit into

![Fig. 11: Comparison of the experimental energy levels of the isotopes of Lu. Results are from ref. 25. The odd nuclei are labelled by 2J.](image-url)
the scheme of table 2 due to the possibility of mixing between the two orbits. The Au isotopes furnish another example of this type of confusion. There, the negative parity states are interpreted \(^{17}\) to consist of two separate bands at roughly the same energy arising from the \(h_{11/2}\) and \(h_{15/2}\) orbits. Such cases (though still qualitatively understandable from table 2) lie outside the assumptions underlying the development of the results given in table 2.

To investigate the even-even parent shape dependence we will consider the Hg isotopes which due to their closeness to the proton closed shell probably have oblate deformed even-even parents. Since the number of neutrons in the \(h_{11/2}\) shell should be at least seven or greater we would expect stretched spectra from case 6 of table 2. In fig. 11, a comparison between \(^{192,194}\text{Au}\) and \(^{198,200}\text{Hg}\) shows close agreement with the experimental \(^{20}\) energies of the pairs. Furthermore, the measured static quadrupole moments (1.08 and 1.27 \(\varepsilon\cdot\text{b}\) \[^{17}\]) and \(g\) factors (-0.16 and -0.15) \[^{20}\] for the \(3^+\) state of \(^{198,200}\text{Hg}\) respectively, agree with the qualitative predictions of figs. 2 and 3, thereby giving us more confidence in our assumption of oblate intrinsic shapes for the even-even parents.

7. Discussion

The abnormal parity levels of odd mass nuclei have been the subject of numerous theoretical studies. In terms of the Nilsson model, extensive investigations have focused on the role of the Coriolis force \(^{5,16}\) in producing a transition from strong coupling (ordered) to rotation aligned or decoupled (stretched) spectra. The role of the deformation in determining the appearance of a stretched or ordered spectrum was also investigated in detail by Alaga and Paar \(^{16}\) using the weak coupling of the abnormal parity particle to the vibrational excitations of the parent nucleus.

Our results are generally in agreement with the analyses based on the Nilsson model. Although we use a shell-model framework, configuration assignments were guided by the Nilsson model. The details of the results of table 2 were generated primarily by the distribution of nucleons in the abnormal parity space. The normal parity particle structure plays a background role, only through their tendency to favor a prolate or an oblate shape. Then given that \(Q - Q\) is the important part of the interaction between the two spaces our results emerge quite simply. The present study examines the general consequences of strong-coupling the odd nucleon to the particles already in the abnormal parity level. The description of the abnormal parity nucleus was given in terms of the generalized quasiparticle formulation which yields an exact handling of the Pauli principle within that orbit when both neutrons and protons are involved. Systematic changes in spectra depending on the species of the odd nucleon, the number and isospin of the particles already in the abnormal parity level, and the deformation favored by the interactions for the even-even
core have been explored. We have seen by looking at specific examples that the simple results of table 2 seem to be borne out throughout the periodic table.

It is important to recognize that, when describing collective effects in even-even nuclei the shell-model structure is usually washed out by the large number of coherently superimposed excitations involved in forming the collective excitation. The present work indicates that the odd nucleon interacts with the other nucleons in its own orbit very significantly. For the conditions leading to a stretched spectrum, the odd nucleon interacts with the other abnormal parity nucleons in such a way as not to destroy the collective properties of the even-even parent. As seen in the model study, energy level spacings and transition probability systematics are very similar in the parent and daughter nuclei. For an ordered spectrum, however, the odd nucleon is not permitted by the Pauli principle to be accommodated into the orbital structure in a way which reinforces the even-even parent structure. It thus tends to destroy the yrast structure of the parent, by preferring to interact with an orbital configuration which was not the same as the yrast configuration of the parent. Thus a significant change in energy spacings and quadrupole moment was found for such cases in the model studies. The theoretical consequence of this finding for microscopic descriptions is that odd-even nuclei which have ordered spectra require a more accurate description of the even-even parent than do stretched odd mass nuclei. Thus the basis necessarily cannot be treated as severely as one might hope for the ordered nuclei. For analyses where one weak-couples the odd particle to the even-even core wave functions (ignoring the explicit microscopic commutation properties between the odd particle and the core) one may expect to find that there is a greater consistency between interaction parameters for the even-even core calculation and the full odd-even calculation when the odd mass nucleus is stretched than when it is ordered.

A challenging problem which remains is to find out what features of our severe truncation in the model study in sect. 5 must be removed in order to describe the experimentally observed spectra more precisely. Namely, how does a \( \frac{1}{2} + \) sequence arise rather than a sequence starting with \( \frac{3}{2} - \) as in the model study. Our finding in the ordered case that the abnormal parity nucleons prefer to couple more strongly with a normal parity configuration which was not dominant for the even-even parent complicates matters. Possibly a simple solution such as seniority three mixing in the abnormal parity space and/or \( K = 2 \) mixing in the normal parity space will be adequate.

Since the predictions of table 2 are seemingly consistent with experiment, we now plan to apply the microscopic formalism to realistic studies of various odd-\( A \) nuclei. We expect to calculate both normal and abnormal parity levels within a given nucleus.

The authors are indebted to Prof. K.T. Hecht and Prof. E.F. Zganjar for comments and suggestions during the course of this work.