Scissors mode and the pseudo-SU(3) model

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The low-lying M 1 transition strength distributions observed in rare earth and actinide nuclei reflects on both the collective and noncollective aspects of the nuclear interaction. As shown in this article, the pseudo-SU(3) model can accommodate these complementary features and provides a natural interpretation for different aspects of this “scissors” mode.

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I. INTRODUCTION

The discovery of strongly enhanced M 1 transitions in heavy deformed nuclei has attracted considerable theoretical and experimental attention over the last several years. Predicted in 1978 by Lo Iudice and Palumbo [1] within the framework of the so-called phenomenological two-rotor model (TRM), these enhanced M 1 transitions were first detected experimentally in 156Gd six years later (1984), and shortly thereafter in various neighboring isotopes [2,3].

The early interpretation of this so-called scissors mode was that of rigid proton and neutron rotors with axial symmetric shapes performing rotational oscillations against one another. However, numerous experiments in the rare-earth and actinide regions using electron-scattering and nuclear resonance fluorescence techniques [4] have shown that this simple picture must be modified. While the predominantly orbital character of the magnetic dipole excitation predicted within the framework of the two-rotor model was confirmed experimentally, other features cannot be explained using this simple collective picture. One is the so-called fragmentation of the M 1 strength distribution, that is, the breakup of the M 1 transition strength among several levels closely packed and clustered around a few strong transition peaks in the energy region between 2 and 4 MeV. Recent experiments have also established the scissors mode for the γ-soft nucleus 196Pt [5] for which the total M 1 transition strength is about a factor of 5 smaller than that observed, for example, in well-deformed even-even actinide species.

As will be shown in this article, a nuclear model that is able to address all of these issues is the pseudo-SU(3) scheme. Since its introduction in the late 1960s [6,7], the pseudospin concept has been applied to various properties of heavy deformed nuclei [8–10]. However, the nucleon-nucleon interaction used in most of these pseudo-SU(3) investigations was schematic in nature because of technical difficulties associated with the calculation of SU(3) matrix elements of more general interactions. Recently a code was released [11] that lifts these limitations and allows for the introduction of interactions such as pairing, which are essential for a reasonable description of experimental results within the framework of pseudo-SU(3) model calculations [12].

The pseudo-SU(3) model is a many-particle shell-model theory that takes full advantage of pseudospin symmetry, which is manifest in the near degeneracy of the orbital pairs \[(l-1)\ell(l+1/2)\] and \[(l+1/2)l(l+1)\]. As an algebraic shell-model theory the pseudo-SU(3) scheme exploits powerful group theoretical methods in the construction of the basis states and for the calculation of required matrix elements [10,7]. Specifically, the pseudo-SU(3) is able to describe single-particle properties of a nucleus while simultaneously allowing for an interpretation of structural features in terms of collective (rotational) degrees of freedom. This is accomplished by exploiting a relation between SU(3) and the algebra of a quantum rotor.

As discussed first, the relation between SU(3) and the collective model can be used in the SU(3) symmetry preserving limit of the theory to give a geometric interpretation of 1+ states and to explain the general structure of the M 1 transition spectrum. In the second part of this paper, symmetry breaking single-particle terms and pairing are introduced to explain the more detailed structure of M 1 transition spectra, especially its fragmentation.

II. THE PSEUDO-SU(3) LIMIT

A. Geometric interpretation of 1+ states

The pseudo-SU(3) scheme allows for a very elegant generalization of the geometrical picture introduced as part of the TRM. In the TRM the scissors-like relative motion of the proton and neutron distributions is parametrized in terms of an angle \(\theta\) between the z axes of the axially symmetric distributions. However, it can be shown that an additional “twist” mode is possible [13] for triaxial shapes because rotations by \(\phi_x\) and \(\phi_y\) about the z axes of the proton and neutron distributions, respectively, emerge as additional degrees of freedom. As for the scissors mode, it is the difference of these two rotations that gives rise to a new M 1 mode. As will be discussed, this mode, together with the scissors motion, determines the basic structure of the M 1 transition spectrum.

In order to examine the nature of the scissors mode within the framework of the pseudo-SU(3) model, a general SU(3) preserving nuclear Hamiltonian can be rewritten in terms of the angles that describe the relative motion of the proton and neutron distributions. In the midshell region the dominant term in this Hamiltonian
\[ H = c_1 Q \cdot Q + c_2 \hat{L}^2 + c_3 K_L^2 + c_4 L_\pi^2 + c_5 L_\nu^2, \]

is the quadrupole deformation \( Q \cdot Q = 4C_2 - 3\hat{L}^2 \) which allows one to truncate the model space to the SU(3) irreducible representation \( \text{irrep} \) with maximum value of the second order invariant \( C_2 \) of SU(3). In addition, \( \hat{L} \) is the total angular momentum and \( K_L \) its projection on the intrinsic body-fixed axis, which generate, respectively, rotational bands and \( K \)-band splitting. Guided by the notion used in the TRM \cite{1}, this Hamiltonian can be decomposed into a rotational part and a part that describes the intrinsic motion of the protons (index \( \pi \)) and neutrons (index \( \nu \)), namely,

\[ H = H_{\text{rot}} + H_{\text{int}}, \]

where the rotational part given by

\[ H_{\text{rot}} = a\hat{L}^2 + bK_L^2, \]

and the intrinsic part by

\[ H_{\text{int}} = c\hat{L}^2 - dC_2. \]

For the derivation of \( H_{\text{rot}} \) and \( H_{\text{int}} \) the identities

\[ Q \cdot Q = 4C_2 - 3\hat{L}^2, \]

\[ \hat{L} = \hat{L}_\pi + \hat{L}_\nu, \]

\[ \hat{I} = \hat{L}_\pi - \hat{L}_\nu, \]

have been used, thus defining the relative orbital momentum \( \hat{I} \) which appears as a dynamical variable in the Hamiltonian that describes the proton-neutron interaction.

By using a mapping between variables of the joint rotor and SU(3) \cite{13,14}, the intrinsic part can be rewritten as a two-dimensional harmonic oscillator potential \( H_{\text{int}} \) which describes the proton-neutron interaction in terms of the collective variable \( \theta \) and the angle \( \phi = \frac{1}{2} (\phi_\pi - \phi_\nu) \) that parametrizes the relative difference between the proton and neutron rotations about their respective \( z \) axis, that is, the ‘‘twist’’ degree of freedom

\[ H_{\text{int}} = \hbar \omega_\theta \left(n_\phi + \frac{1}{2}\right) + \hbar \omega_\phi \left(n_\theta + \frac{1}{2}\right) + E_0. \]

Here the constant \( E_0 \) has been introduced as an abbreviation for

\[ E_0 = -d\left[ (\lambda_\pi + \lambda_\nu + 2)^2 + (\mu_\pi + \mu_\nu + 2)^2 \right] \]

\times \left( (\lambda_\pi + \lambda_\nu + 2)^2 - 3 \right), \]

and the oscillator frequencies \( \omega_\theta \) and \( \omega_\phi \) are defined as

\[ \omega_\theta = \sqrt{48cd}(\lambda_\pi + 1)(\lambda_\nu + 1), \]

\[ \omega_\phi = \sqrt{48cd}(\mu_\pi + 1)(\mu_\nu + 1), \]

with the parameters \( c \) and \( d \) as given in Eq. (4).

It can be shown that different excitation modes \( (n_\theta, n_\phi) \) of this harmonic oscillator correspond to certain coupled SU(3) eigenfunctions \cite{13,14}. The leading irrep of the decomposed SU(3) tensor product

\[ (\lambda, \mu) = (\lambda_\pi + \lambda_\nu, \mu_\pi + \mu_\nu), \]

which corresponds to \( (n_\theta, n_\phi) = (0,0) \), is always associated with a minimum in the relative angular displacements. In this case the proton and neutron distributions overlap maximally, generating maximum deformation of the system. Assuming prolate shapes for the parent distributions \( \lambda_\pi > \mu_\pi \) each phonon added to the scissors mode produces a larger increase in energy than a phonon added to the twisting motion [see Eq. (10)]. This follows from the softness of the twist degree of freedom relative to the stiffer scissors mode.

The SU(3) configuration

\[ (\lambda, \mu) = (\lambda_\pi + \lambda_\nu - 2, \mu_\pi + \mu_\nu + 1) \]

is the first scissorslike mode. It is always part of the tensor product decomposition and contains a \( J^\pi = 1^+ \) state which is associated with the bandhead of a \( K = 1 \) band. This mode corresponds to an excitation of the \( \theta \) degree of freedom by one quanta and therefore corresponds to the scissors mode of the TRM.

A generalization of this picture is possible if one allows for a triaxial proton or neutron distribution. In the SU(3) model where a one-to-one correspondence between the irrep labels \( (\lambda, \mu) \) and the shape variables \( (\beta, \gamma) \) of the collective model \cite{15} can be made,

\[ \beta^2 = \frac{4\pi}{5} \left( \frac{1}{(A_r^{1/2})^2} \right)[\lambda^2 + \mu^2 + 3(\lambda + \mu + 1)], \]

\[ \tan \gamma = \frac{\sqrt{3}(\mu + 1)}{2\lambda + \mu + 3}, \]

this corresponds to either \( \mu_\pi \) or \( \mu_\nu \) or both being nonzero. In this case a second scissors state appears which is given by

\[ (\lambda, \mu) = (\lambda_\pi + \lambda_\nu - 1, \mu_\pi + \mu_\nu - 1) \]

or \( (1,1) \) in terms of \( (n_\theta, n_\phi) \). According to the underlying geometrical picture, this structure is produced by superimposing a \( \phi \) twisting motion on top of the lowest scissors mode. When only one of the nucleon distributions is triaxial, a pure twist mode \( (n_\theta, n_\phi) = (0,1) \) is not possible. However, when such a configuration occurs in combination with a scissors displacement, the rotational symmetry of the axial distribution is lifted and the twist motion is possible resulting in the ‘‘scissors plus twist’’ mode.

The most general setting is given if \( \mu_\pi \) and \( \mu_\nu \) are both nonzero, i.e., if both the proton and neutron distributions are triaxial. Then a third \( 1^+ \) state can be identified corresponding to a pure ‘‘twist’’ mode \( (n_\theta, n_\phi) = (0,1) \):

\[ (\lambda, \mu) = (\lambda_\pi + \lambda_\nu + 1, \mu_\pi + \mu_\nu - 2). \]
It differs from the first two excitation modes not only in its intrinsic energy but also because it belongs to a K = 0 band. In addition to the above, one obtains a fourth state which is another ‘‘scissors plus twist’’ state and differs from the first one solely in its value of the outer multiplicity parameter ρ. Because of selection rules related to the M1 transition operator and the couplings possible for two SU(3) irreps [10,13], these four M1 modes are the only ones possible in a pure pseudo-SU(3) picture.

B. Algebraic results for M1 transition strengths

To determine the M1 transition strength between two specific eigenstates, the operator

\[ T^1_{\mu} = \sqrt{\frac{3}{4\pi}} \mu_N \sum_{\sigma} g^{\sigma}_{\mu} L^{\sigma}_{\mu} + g^{\sigma}_{\sigma} S^{\sigma} \]  

with the single-particle terms

\[ L^{\sigma}_{\mu} = \sum_i l^{\sigma}_{\mu}(i), \]

\[ S^{\sigma} = \sum_i s^{\sigma}_{\mu}(i), \]

must be evaluated. Since other theories use effective g factors, it is important to note that in this work the ‘‘bare’’ orbital and spin g factors for protons and neutrons are used:

\[ g^{p}_{\pi} = 1, \quad g^{n}_{\pi} = 0 \quad \text{and} \quad g^{p}_{\sigma} = 5.5857, \quad g^{n}_{\sigma} = -3.8263. \]  

To evaluate the M1 transition operator for eigenstates given in a pseudo-SU(3) basis, the pseudo-SU(3) tensorial expansion of the expression given in Eq. (17) are used [10].

For the case of pure SU(3) eigenstates, a simple analytic expression for the M1 transition strength of the scissors state \((\lambda_\pi + \mu_\pi - 2, \mu_\pi + \mu_\pi + 1)\) can be given in terms of the proton and neutron SU(3) quantum numbers

\[ B(M1; 0^+ \rightarrow 1^+)_{\text{sc}} = \frac{3}{4\pi} \frac{\lambda_\pi \lambda_\pi + \mu_\pi + \mu_\pi + 2}{(\lambda_\pi + \mu_\pi - 1)(\mu_\pi + \mu_\pi + 1)} \mu_N^2. \]  

(19)

In the case of two prolate shapes, \(\mu_\pi = \mu_\pi = 0\), when only the scissors mode transition is possible, this reduces to

\[ B(M1; 0^+ \rightarrow 1^+)_{\text{sc}} = \frac{3}{4\pi} \frac{2\lambda_\pi \lambda_\pi}{(\lambda_\pi + \mu_\pi - 1)} \mu_N^2. \]  

(20)

In addition to the pure scissors mode, a ‘‘scissors plus twist’’ excitation \((n_\theta, n_\phi) = (1,1)\), with SU(3) quantum numbers \((\lambda_\pi + \lambda_\pi - 1, \mu_\pi + \mu_\pi - 1)\) is possible in the axial-triangular case where either \(\mu_\pi\) or \(\mu_\pi\) is nonzero. In this case an analytic expression for the M1 transition strength of the ‘‘scissors plus twist’’ mode is given by

\[ B(M1; 0^+ \rightarrow 1^+)_{\text{sc+tw}} = \frac{3}{4\pi} \frac{1}{\lambda + \mu} \left[ \mu_\pi \mu_\pi \lambda + \lambda_\pi \mu_\pi \mu_\pi + \lambda_\pi \mu_\pi \mu_\pi + \mu_\pi \mu_\pi \lambda + \mu_\pi \mu_\pi + \mu_\pi \mu_\pi \lambda \right] \mu_N^2. \]  

(24)
three selected nuclei 154Sm, 156Gd, and 160Gd are examples of pure axial, axial-triaxial, and triaxial-triaxial couplings, respectively. Each transition is labeled as a scissors (s) or twist (t) or combination (s + t) mode.

The question of whether analytical results exist for the individual "scissors plus twist" M1 transition strengths remains open. In a calculation of the M1 transition strength for the two different "scissors plus twist" states, the only difference is a change in the outer multiplicity label \( \rho \) that enters through a SU(3) 9-(\( \lambda, \mu \)) coefficient [10]. This outer multiplicity label \( \rho = 1,2, \ldots, \rho_{\text{max}} \) is used to distinguish multiple occurrences of a given \((\lambda_1, \mu_1) \times (\lambda_2, \mu_2)\). A definition bearing physical significance has not been proposed to fix \( \rho \). For example, the choice made by Biedenharn and co-workers [17,18] follows a mathematically canonical definition with practical aspects associated with the vanishing of certain Wigner and Racah coefficients and simple symmetry relations under conjugation [19].

Numerical calculations based on the phase convention made by Biedenharn show that in general the M1 transition strength for the two different "scissors plus twist" states are very different, for one state it is relatively large—approximately equal in magnitude to the strength of the pure "scissors" mode—and for the other state very small. In the case of 160Gd, for example, these values are

\[
B(M1; 0^+ \rightarrow 1^+)_{\text{sc+tw}_1} = 1.82 \mu_N^2,
\]

\[
B(M1; 0^+ \rightarrow 1^+)_{\text{sc+tw}_2} = 0.0083 \mu_N^2.
\] (25)

Other phase conventions are possible. Recent results by Filippov and Lisetskyi [20] have different individual M1 transition strengths of the two "scissors plus twist" modes, while giving the same total strength.

### C. Pseudo-SU(3) symmetry limit and experiment

The analytic expressions for M1 transition strengths given above can be used to determine the total M1 transition strength in the pseudo-SU(3) symmetry limit. For three well-deformed Gd isotopes for which the SU(3) model is expected to work well, results for the total M1 strength together with the corresponding experimental results are given in Table II. The SU(3) quantum numbers used as input are from Ref. [10]. In general, a comparison of the experimental and theoretical results shows good agreement thus giving evidence for the underlying collective nature of the "scissors" mode since the pseudo-SU(3) model is closely related to the rotor model.

The underlying collective nature of the scissors mode is also reflected in the structure of the experimentally observed M1 transition spectrum. Even though the number of observed states is larger than the maximum of three states with strong transitions predicted by a model that preserves SU(3) symmetry, each cluster of states can be associated with a "scissors" or "twist" or "scissors plus twist" state as is illustrated in Figs. 1(a) and 1(b) for 160Gd. How a more realistic description of the experimental observed spectrum is possible within the framework of the pseudo-SU(3) model is the topic of the next section.

### III. CALCULATIONS WITH A REALISTIC HAMILTONIAN

To investigate the effect of the symmetry breaking terms, the following generalization of an SU(3) symmetry conserving Hamiltonian was used

\[
H_{\text{PSU}(3)} = - (a_2 + a_{\text{sym}}) C_2 + a_3 C_3 + b L^2 + c K_L^2 + D_{\text{sym}} \sum_{i,p} I_{i,p}^2
\]

\[
+ D_{\text{sym}} \sum_{i,p} I_{i,p}^2 - G_n H_p^v - G_s H_p^s,
\] (26)

where \( C_2 \) and \( C_3 \) are the second and third order invariants of SU(3), which are related to the axial and triaxial deformation of the nucleus, respectively, and \( L^2 \) and \( K_L^2 \) are the square of the angular momentum and its projection on the intrinsic body-fixed symmetry axis, which, respectively, generate rotational bands and \( K_L \)-band splitting. The parameter \( a_{\text{sym}} \) is

### TABLE II. Total \( B(M1) \) transition strengths [\( \mu_N^2 \)] from experiment [4] and pseudo-SU(3) model calculations without symmetry breaking terms.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>( \Sigma B(M1) [\mu_N^2] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>156Gd</td>
<td>Exp.</td>
</tr>
<tr>
<td></td>
<td>3.40</td>
</tr>
<tr>
<td>158Gd</td>
<td>4.32</td>
</tr>
<tr>
<td>160Gd</td>
<td>4.21</td>
</tr>
</tbody>
</table>
introduced to shift SU(3) irreps with either \( \lambda \) or \( \mu \) odd relative to those with \( \lambda \) and \( \mu \) both even, for which \( a_{\text{sym}} \) is zero, as the former belong to different symmetry types (\( B_{\alpha}, \alpha = 1, 2, 3 \), rather than \( A \)) of the intrinsic Vierergruppe (\( D_2 \)). The one-body proton and neutron angular momentum terms \( l^2 \), and \( l_y^2 \), together with the two-body pairing terms \( H^p_\mu \) and \( H^n_\nu \), are SU(3) symmetry breaking interactions.

To select an appropriate set of SU(3) basis functions, one first determines the proton and neutron occupancies by filling pairwise from below the single-particle levels of the generalized Nilsson Hamiltonian [22]

\[
h_0 = h_{\text{osc}} + Cls + Dl^2 - m \omega^2 r^2 \beta \left[ Y_{0}^2 + \frac{\sin \gamma}{\sqrt{2}} (Y_{2}^2 + Y_{-2}^2) \right],
\]

(27)

for values of \( \beta \) and \( \gamma \) that give the lowest total energy of the combined proton and neutron systems. One then determines the number of valence-space nucleons in the normal and unique parity levels, the later being intruder states that are pushed down into the valence space from the next higher shell by the strong spin-orbit interaction. An overall simplifying assumption made in most pseudo-SU(3) model calculations is that the relevant dynamics can be described by taking into account the nucleons in the normal parity sector only [23]; the nucleons in intruder states (unique parity sector) are assumed to follow in an adiabatic manner the motion of the nucleons in the normal parity sector with their effect represented through a reparametrization of the theory.

For the nuclei investigated here, the occupation numbers and the corresponding deformations \( \beta \) and \( \gamma \) are given in Table III. In accordance with experimental results, the Gd isotopes show a relatively large axial deformation and \(^{196}\text{Pt}\) is triaxial and \( \gamma \) soft. Whereas the pseudo-SU(3) model is expected to work well for the Gd case, \(^{196}\text{Pt}\) has been included to explore the limits of the theory.

Since the quadrupole-quadrupole interaction \( Q \cdot Q = 4C_2 \) dominates for deformed nuclei, only basis states with \( C_2 \) larger than a certain value are expected to give a significant contribution in the low-energy region. In the present application, for both proton and neutron distributions, all SU(3) basis states with \( C_2 \geq C_{2\text{min}} \) were selected with \( C_{2\text{min}} \) set so that all irreps lying below approximately 6 MeV excitation energy were included in the analysis. (Increasing this cutoff to 7 MeV made no difference in the results; on the other hand, the effect of this truncation started to manifest itself when the cutoff was reduced to 5 MeV.) Then all possible couplings of these proton and neutron SU(3) irreps were taken to give coupled SU(3) irreps that form basis states of the model space. Only states with \( J \leq 8 \) and \( S = 0 \) were considered.

The parameters for the Hamiltonian given in Eq. (26) and the effective charges \( e_\pi = 1 + q_{\text{eff}} \) and \( e_\nu = q_{\text{eff}} \) used in the \( E2 \) transition operator

\[
T^2_M(E2) = A^{1/3} \sum_{\sigma = \pi, \nu} \sum_{i} e_\sigma r^2_{\sigma}(i) Y_{2M}^* \left[ r_{\nu}(i) \right]
\]

(28)

were determined through a fitting procedure that included as input all known levels with \( J \leq 8 \) up through 2 MeV excitation energy and all strong \( B(E2) \) transition strengths between these levels. The Hamiltonian parameters for the cases studied are given in Table IV. As shown in Figs. 1, 2, and 3 as well as Table V, this procedure yielded good agreement between the theory and experiment, even for the \(^{196}\text{Pt}\) case.

From values for the Hamiltonian parameters in Table IV, one can see that the pairing interaction (the proton and neutron strengths were set equal) and single particle terms in the Hamiltonian are correlated. In particular, for larger pairing (\(^{156,164}\text{Gd}\)) \( D_\pi \) is positive and \( D_\nu \) is negative whereas for a
weaker pairing strength \( ^{158}\text{Gd} \) the opposite is true. A fit to the \( ^{158}\text{Gd} \) case that is only slightly less good than the one generated with the parameters given in the table can be achieved with parameters similar to those used for \( ^{156,160}\text{Gd} \), and vice versa. This will be the topic of another follow-on paper that takes a closer look at this matter. For the present purpose of studying \( M1 \) strength distributions, it suffices to know that these sets gives very similar results which are in reasonable agreement with experiment observations.

After determining eigenstates of the system, the \( M1 \) transition \( \frac{\partial}{\partial t} \) operator for the \( S=0 \) case

\[
T^1_{\mu}(M1) = \sqrt{\frac{3}{4}} \mu_N (g^0_{\pi} L^\pi + g^0_{\nu} L^\nu)
\]

with orbit \( g \) factors

\[
g^0_{\pi} = 1, \quad g^0_{\nu} = 0
\]

(i.e., with no effective \( g \) factors) was used to determine transition strengths between the \( 0^+ \) ground state and \( 1^+ \) states.

Because of the \( SU(3) \) symmetry breaking terms \( t^2_{\pi,\nu} \) and \( H^P_{\pi,\nu} \), the simple picture of the pure \( SU(3) \) case with a maxi-

<table>
<thead>
<tr>
<th>Nucl.</th>
<th>( a_2 )</th>
<th>( a_{sym} )</th>
<th>( a_3 )</th>
<th>( b )</th>
<th>( c )</th>
<th>( D_\pi )</th>
<th>( D_\nu )</th>
<th>( G_{\pi,\nu} )</th>
<th>( q_{eff} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{156}\text{Gd} )</td>
<td>0.0230</td>
<td>0.0008</td>
<td>77.2 \times 10^{-6}</td>
<td>0.0121</td>
<td>0.1435</td>
<td>0.0756</td>
<td>-0.0724</td>
<td>0.1052</td>
<td>1.3119</td>
</tr>
<tr>
<td>( ^{158}\text{Gd} )</td>
<td>0.0245</td>
<td>0.0006</td>
<td>80.4 \times 10^{-6}</td>
<td>0.0080</td>
<td>0.2259</td>
<td>-0.0738</td>
<td>0.0478</td>
<td>0.0685</td>
<td>1.3634</td>
</tr>
<tr>
<td>( ^{160}\text{Gd} )</td>
<td>0.0224</td>
<td>0.0004</td>
<td>39.4 \times 10^{-6}</td>
<td>0.0085</td>
<td>0.1871</td>
<td>0.0271</td>
<td>-0.0817</td>
<td>0.1096</td>
<td>1.2361</td>
</tr>
<tr>
<td>( ^{196}\text{Pt} )</td>
<td>0.0113</td>
<td>0.0010</td>
<td>145 \times 10^{-6}</td>
<td>0.0155</td>
<td>0.0953</td>
<td>0.0215</td>
<td>0.0001</td>
<td>0.2295</td>
<td>0.7181</td>
</tr>
</tbody>
</table>

FIG. 2. Low-energy and \( M1 \) transition spectra for the even-even \( ^{156-160}\text{Gd} \) isotopes.
A. Results for even-even Gd isotopes

As a first test of the theory, calculations for the even-even $^{156-160}$Gd isotopes for which the experimental data are well established [4] were carried out. As these are deformed nuclei in a midshell region they are expected to be well suited for a description by the pseudo-SU(3) model. The results shown in Fig. 2 confirm this assumption, giving, in general, a good description of the different rotational bands. A noteworthy feature is that the rotational structure of the low-energy spectrum given by the pure-SU(3) model survives the mixing induced by the pairing. The experimental and theoretical results for the $E(2)$ transition strength that were used in the fitting routine can be found in Table V. The eigenvectors given by this fitting procedure were then used to determine the structure of the $M1$ transition spectrum.

As expected, the SU(3) symmetry breaking terms in the Hamiltonian lead to a break-up of the $M1$ transition spectra into relatively closely packed levels centered around the sharp peaks of the pure-SU(3) limit of the theory. In particular, it seems that pairing is essential for a proper description of the fragmentation of the $M1$ strength. In Fig. 1(c) this fragmentation is clearly exhibited for $^{160}$Gd. Notice that the energy centroids coincide with those found using the pure pseudo-SU(3) Hamiltonian, shown in Fig. 1(b), while the overall distribution of $M1$ intensities is close to the experimental one [Fig. 1(a)].

TABLE V. Total $B(M1)$ transition strength as given by experiment and theory.

| Nucleus | $\Sigma B(M1) [\mu_N^2]$ | $B(E2|0^+ \rightarrow 2^+_1) [e^2 b^2]$ |
|---------|----------------------------|-------------------------------------|
| 156Gd   | 3.40 (Exp.) 2.91 (Theo.) | 4.66 (Exp.) 4.79 (Theo.) |
| 158Gd   | 4.32 (Exp.) 3.02 (Theo.) | 5.02 (Exp.) 5.23 (Theo.) |
| 160Gd   | 4.21 (Exp.) 3.29 (Theo.) | 5.19 (Exp.) 5.00 (Theo.) |
| 196Pt   | 0.69 (Exp.) 1.27 (Theo.) | 1.40 (Exp.) 1.56 (Theo.) |

More specifically, one finds a number of transitions that, in general, are close to the experimentally observed ones. For $^{156}$Gd there are eleven such states which turns out to be in good agreement with the experimental result. Also, for most of the nuclei, the centroid of the experimental and theoretical $M1$ transition strength distribution lie at about the same energy, so that good overall agreement is obtained. The total $M1$ strength, which for the full Hamiltonian is a bit lower then for its pure-SU(3) limit due to destructive interference associated with the mixing (see Table V), also shows reasonable agreement with the experimental results, in most cases only slightly underestimating them. One possible reason for this discrepancy is the missing spin 1 admixture in the wave functions.

A detailed analysis of the wave function for $^{156}$Gd and $^{160}$Gd (see Tables VI and VII) shows a relatively small mixing for the $0^+$ ground state which contain more than 80% of the leading SU(3) irrep. Also listed are the $1^+$ states which have the largest contribution from the SU(3) eigenstates that

TABLE VI. Decomposition of wave function for $^{156}$Gd for its ground state and $1^+$ states dominated by "scissors" and "scissors plus twist" modes. For each state the four most important contributions are given with their proton, neutron, and coupled SU(3) quantum numbers.

<table>
<thead>
<tr>
<th>$J$ Energy [MeV]</th>
<th>$c$ [%]</th>
<th>$(\lambda,\mu)$</th>
<th>$(\lambda_+,\mu_+)$</th>
<th>$(\lambda_-,\mu_-)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+_g.s.$</td>
<td>0.00</td>
<td>84.4 (28.4)</td>
<td>(10.4)</td>
<td>(18.0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.2 (24.6)</td>
<td>(10.4)</td>
<td>(14.2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.5 (30.0)</td>
<td>(12.0)</td>
<td>(18.0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.7 (24.6)</td>
<td>(10.4)</td>
<td>(14.2)</td>
</tr>
<tr>
<td>$1^+_{wc}$</td>
<td>2.59</td>
<td>60.2 (26.5)</td>
<td>(10.4)</td>
<td>(18.0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.45 $\mu_3^2$)</td>
<td>16.4 (20.11)</td>
<td>(10.4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.1 (26.5)</td>
<td>(8.5)</td>
<td>(18.0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.1 (20.11)</td>
<td>(8.5)</td>
<td>(12.6)</td>
</tr>
<tr>
<td>$1^+_{s+t}$</td>
<td>3.46</td>
<td>29.8 (27.3)</td>
<td>(10.4)</td>
<td>(18.0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.257 $\mu_3^2$)</td>
<td>41.7 (23.8)</td>
<td>(8.5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.2 (20.11)</td>
<td>(8.5)</td>
<td>(15.3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.1 (26.5)</td>
<td>(8.5)</td>
<td>(12.6)</td>
</tr>
</tbody>
</table>
TABLE VII. Decomposition of wave function for $^{160}$Gd for the ground and $1^+$ states dominated by the “scissors,” “twist,” and “scissors plus twist” modes. In each case the four most dominant contributions are given with proton, neutron, and coupled SU(3) quantum numbers.

<table>
<thead>
<tr>
<th>$J$</th>
<th>$E$ [MeV]</th>
<th>$c$ [%]</th>
<th>$(\lambda, \mu)$</th>
<th>$(\lambda, \mu, \nu)$</th>
<th>$(\lambda, \mu, \nu, \rho)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0_{g.s.}^+$</td>
<td>0.00</td>
<td>83.4</td>
<td>(28, 8)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.6</td>
<td>(30, 4)</td>
<td>(10, 4)</td>
<td>(20, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.7</td>
<td>(30, 4)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.2</td>
<td>(32, 0)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td>$1_{0w}^+$</td>
<td>1.84</td>
<td>98.6</td>
<td>(29, 6)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td>(0.21 $\mu_3^2$)</td>
<td>1.1</td>
<td>60.3</td>
<td>(26, 9)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td></td>
<td>(1.02 $\mu_3^2$)</td>
<td>21.1</td>
<td>(26, 9)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.7</td>
<td>(27, 7)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12.8</td>
<td>(26, 9)</td>
<td>(8, 5)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td>$1_{sc}^+$</td>
<td>2.72</td>
<td>60.3</td>
<td>(26, 9)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td>(0.21 $\mu_3^2$)</td>
<td>21.1</td>
<td>(26, 9)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.27</td>
<td>(27, 7)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td>(0.65 $\mu_3^2$)</td>
<td>5.0</td>
<td>(27, 7)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.3</td>
<td>(27, 7)</td>
<td>(9, 3)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td>$1_{s+1l}^+$</td>
<td>3.58</td>
<td>56.7</td>
<td>(27, 7)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td>(0.04 $\mu_3^2$)</td>
<td>30.5</td>
<td>(27, 7)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.9</td>
<td>(27, 7)</td>
<td>(8, 5)</td>
<td>(18, 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2</td>
<td>(27, 7)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
</tr>
</tbody>
</table>

B. Results for the $\gamma$-soft $^{196}$Pt case

As mentioned in the Introduction, very recent experimental results [5] have also confirmed the existence of the scissors mode in the $\gamma$-soft $^{196}$Pt system. To explore the limits of the pseudo-SU(3) model for describing $M1$ strengths, this nucleus was also included into our investigation.

Following the same procedure as used for the even-even Gd isotopes, a fitting procedure was used to determine the Hamiltonian parameters and low-lying eigenstates up to about 2 MeV. The results (see Fig. 3) show that the pseudo-SU(3) model is able to give a reasonable description of the $^{196}$Pt spectrum which does not show a pure rotor structure but rather more the characteristics of a quantum vibrator. Also, a relatively small value for the $B(E2, 0^+_1 \rightarrow 2^+_1)$ transition strength was obtained, one that is about a factor three smaller than the corresponding results for the Gd and Dy isotopes but one that still overestimates the experimental value by about a factor of 2 (see Table V).

In this case a relatively strong pairing interaction $G_{\pi,\nu} = 0.23$ MeV, together with a $C_2$ parameter that is lower than for the Gd isotopes (see Table IV) was found to mix different SU(3) irreps and produce the required soft rotor results with its weaker $E2$ transition strengths. An analysis of the $0^+$ ground state, whose dominant irreps are listed in Table VIII, shows that it is a mix of three SU(3) irreps that are derived by coupling the leading proton irrep $(\lambda, \mu, \nu) = (2, 8)$ and the neutron irrep $(\lambda, \mu, \nu) = (4, 18)$ such that the “stretched” representation (6,26) is not the dominate one. This result is in agreement with the analysis of Bahri [25,26] who found that the pairing interaction drives the nucleus towards a more triaxial shape, thus softening it.

In contrast with these results for $^{196}$Pt, for a well-deformed nucleus such as $^{150}$Gd the pairing interaction is smaller by about a factor of 2, $G_{\pi,\nu} = 0.105$ MeV, and the contribution of the leading irrep to the $0^+$ ground state is 85% with the next largest contribution being 4%.

The structure of the theoretical $M1$ transition spectrum compares favorably with the experimental results, giving a relatively large transition strength in the energy region around 2.5 MeV and two smaller transitions at 3.1 and 3.6 MeV. The calculated total $M1$ strength of 1.27$\mu^2$, however, is almost a factor of 2 too large (see Table V). Keeping in
mind that for this case the total $M1$ strength is only about one sixth of that for good rotors such as the Gd isotopes, this is not unreasonable because the starting point of the theory is the assumption of a reasonably well-deformed system which $^{160}$Pt is not. Nevertheless, it is important to emphasize that in this case the theory predicts a significant reduction [by about a factor of $1/3$ from the value of $3.55 \mu^2$ predicted in the pure SU(3) symmetry limit of the theory] in the $M1$ strength, in agreement with what is observed experimentally.

To summarize, the description of the $\gamma$-soft rotor shows some of the limits of the pseudo-SU(3) model but still gives a reasonable description of the experimentally observed low-energy features and the $M1$ transition spectra, thus being the first fully microscopic nuclear model to address the scissors mode in the transition region.

**IV. CONCLUSION AND OUTLOOK**

The goal of this article was to describe the collective and noncollective features of the so-called scissors mode within the framework of the pseudo-SU(3) model. By capitalizing on a linear connection between the invariants operators of the rotor group and SU(3), a generalization of the original geometrical picture of the two rotor model that assumes axial deformations by adding an additional ‘‘twist’’ mode for triaxial proton or neutron distributions. In the most general case, i.e., for triaxial proton and neutron distributions, four modes are realized: In addition to the ‘‘scissors mode’’—the only one possible in the axial-axial case—a ‘‘twist’’ mode, as well as a doublet ‘‘scissors plus twist’’ mode. Each of these modes corresponds to a well-defined SU(3) irrep that includes a $J^\pi = 1^+$ state with nonzero $M1$ transition strength to the $J^\pi = 0^+$ ground state. These irreps then define the basic structure of the transition spectrum. This feature, and the ability of the pure pseudo-SU(3) model to reproduce the total $M1$ strength in good agreement with the experimental results (less than 3% difference for $^{150}$Gd and $^{160}$Gd), are strong indicators of the underlying collective structure of the scissors mode.

No effective $g$ factors were used to compute the $M1$ transition strengths. It was shown that a simple analytic expression gives the $M1$ transition strength for each of the different modes in terms of the SU(3) parameters $(\lambda_\pi, \mu_\pi)$ and $(\lambda_\nu, \mu_\nu)$, respectively, for proton and neutron distributions.

As a many-particle theory, the pseudo-SU(3) model can be extended by adding noncollective one-body and two-body residual interactions to the Hamiltonian. This allows one to describe the experimentally observed fragmentation of the $M1$ strength, thus giving a natural explanation for the observed breakup of the transition strength into clusters of closely packed transitions [27]. While the underlying collective structure of the pseudo-SU(3) Hamiltonian dictates the general structure of the $M1$ transition spectrum, the addition of a pairing interaction appears to be essential for a realistic description of the experimentally observed fragmentation [27]. An important feature of the pairing interaction is that even though it is relatively strong and introduces fragmentation it does not destroy the rotational structure of the low energy spectrum.

As a first test of a realistic Hamiltonian with collective and noncollective features, results were generated for the well-deformed even-even $^{156-160}$Gd isotopes, for which the pseudo-SU(3) model is expected to work well. It is important to realize that a description of the $M1$ transition spectrum, which requires the correct $M1$ transition strength in a small energy region, poses a much greater challenge for a microscopic theory than the rotational structure and $E2$ transitions of the low-energy spectrum. The results found in our investigation showed good agreement with the experimentally observed values and appear to be of equal or better quality than comparable calculations using the QRPA model [24].

To demonstrate the ability of the pseudo-SU(3) model to address a large variety of cases, we also investigated the nature of the scissors mode for $^{190}$Pt. Experimental results for the $M1$ strength distribution of this $\gamma$-soft rotor were obtained only very recently and have not yet been addressed by other nuclear models. Our results show some of the limits of the pseudo-SU(3) model that starts with the assumption of well-deformed, rotorlike structures. However, the configuration mixing introduced by the pairing interaction ‘‘softens’’ the rotor enough to give a reasonable description of the low-energy spectrum as well as of the $M1$ transition spectrum. These results are another example for how the interplay of collective and noncollective features in the nuclear interaction is reflected in the nuclear structure.

**ACKNOWLEDGMENTS**

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