Giant resonances are electromagnetic excitations in nuclei in which an appreciable fraction of the total number of nucleons participate in the motion [1]. One of the most interesting of these is the isoscalar giant quadrupole mode. Through configuration mixing this resonance gives rise to enhanced electric quadrupole transition strengths between members of low-lying rotational bands. This is especially true for the ground band. While one can account for the observed enhancements within the context of a standard shell-model calculation by introducing an effective charge [2], this approach fails to reveal the true nature of the phenomenon. The purpose of this paper is to probe the microscopic structure of collective quadrupole phenomena in light, deformed nuclei: the shell-model character of the resonances, how and to what degree mixing with low-lying configurations occurs, and the coherent nature of the excitation. The calculations presented here show that the isoscalar giant quadrupole resonance is a highly coherent excitation spanning more than a dozen major shells of the oscillator. Additionally, they show that the resonance is just one manifestation of the existence of a strongly deformed band \((\hbar \gamma = 1.5\hbar \omega_p)\) in the continuum that mirrors the low-lying rotational structure.

\[ \mathbf{r} = \mathbf{R}_0 + \mathbf{Q} \mathbf{Q}^T + \mathbf{R}_0, \]
empirical rule, $A = 1.3 \times 2.4 - 1.3 \times 2.5$ MeV, where $A$ denotes the total number of nucleons [3]. The next most important part of the hamiltonian is $Q^2 - Q^2$. The isospin $i$ denotes the "collective" quadrupole operator, $Q_i = \sqrt{\frac{\hbar}{2\pi}} Y_{2i}(\hat{r})$. This $Q$ has non-vanishing matrix elements between major shells of the oscillator that differ by up to two quanta, $n = n' + 1$ and $n = n' + 2$. This means $Q^2 - Q^2$ directly couples shells that differ by as many as four oscillator quanta. It is this part of the hamiltonian that builds collective quadrupole coherence into calculated eigenstates.

Typically, matrix elements of the $Q^2 - Q^2$ term are much smaller than the shell separation energy, $\hbar \omega (Q^2 - Q^2) \ll \hbar \omega$. For a complete theory additional interactions such as the pairing operator and one-body terms that are required to reproduce observed single-particle energies must be included. These terms, which act (horizontally) within a shell and not (vertically) between shells, are represented by the $H_1$ in eq. (1). In the calculations a special form for this residual interaction was used [4]:

$$H_1 = n^2 \sum_{i} (\Sigma_{i+1} - \Sigma_{i}) \delta_{n}$$

The $L$ in this expression is the ordinary angular momentum operator while the $\Sigma_i$ and $\Sigma_i'$ operators are higher order rotational scalars:

$$\Sigma_i = [L \times Q_i] \cdot L$$

and

$$\Sigma_i' = ([L \times Q_i] \cdot L)^*$$

The superscript "*" appended to the $\Sigma_i$ denotes the "algebraic" form of $Q_i$. The $Q^2$ operator is the $Q$ of the Elliott model which is symmetric in particle coordinate and momentum variables [7]. Its matrix elements are equal to those of $Q^2$ within an oscillator shell, $n = n'$, but vanish between different shells, $n \neq n'$. Therefore, like pairing and the single-particle-averaged operators, $H_1$ acts within and not between shells. Indeed, a more detailed analysis has shown that in even-even systems like $^{24}\text{Mg}$, $H_1$ has the same SU(3) tensor character as the dominant parts of these more familiar operators [8]. It is also important to know that matrix elements of $H_1$ vanish between $0^+$ states. Since the $L = 0^+$ configuration of a rotational band can be thought of in a semiclazzical sense as the $L = 0^+$ configuration rotating with angular frequency $\omega_L$, where $\omega$ is the moment of inertia, the physical content of the theory is determined, for the most part, by the magnitude of the single parameter $\gamma$ that multiplies $Q^2 - Q^2$.

The fermionic nature of nucleons is honored in the symplectic model. A simplification can be achieved by considering the $2\hbar \omega$ excitation modes to be generated by monopole "S" and quadrupole "D" bosons, where the boson creation and annihilation operators have matrix elements that are directly proportional to the $2\hbar \omega$ raising and lowering parts of $Q^2$ in the limit of a large number of ground-state oscillator quanta. This approximation, which was used in our calculation, is called the SU(3) boson model or the contracted symplectic model [9-11]. It is a good approximation, even for relatively light nuclei, because the total number of ground-state oscillator quanta, counting from the bottom of the well upward and remembering to include the zero-point energy and exclude spurious center-of-mass motion, is always large. For example, in the $^{24}\text{Mg}$ case the number is $62.5$. It is important to understand that the underlying fermionic character of the model is not lost in making this approximation. At the $\hbar \omega$ level it is treated exactly and for other configurations the number of particles (holes) in higher (lower) shells is small so blocking due to the Faust principle can play at most a minor role. This has been corroborated by full symplectic shell-model calculations. In particularly, there is remarkable correspondence between the BE2 values for low-lying positive parity states in $^{24}\text{Mg}$ evaluated with the boson approximation [10] and the full symplectic model [12].

A plot of calculated excitation energies for $^{24}\text{Mg}$ versus the strength of the quadrupole-quadrupole interaction with $\hbar \omega = 12.61$ MeV and the parameters of $H_1$ fixed at their best-fit values ($\omega_L = 0.1417$, $h\omega = 0.04217$, and $\gamma = -0.00553681$ for $\gamma = 0.041503$, its best-fit value, is given in fig. 1. The model space is that which is defined by applying the contracted version of generators of the symplectic algebra to the $2\hbar \omega$ oscillator configuration with maximum deformation for eight $0^+$-shell particles, namely the $(\lambda_0, \mu_0) = (8, 4)$ representation of SU(3). Vertical excitation through $2\hbar \omega$ were allowed in determining the best-fit parameter set. The movement of levels with changing $\gamma$ can best be understood by focusing on the $0^+$ levels. As everything is normalized to the ground state ($0^+$), the spreading of the three $2\hbar \omega$ excited $0^+$ levels is due entirely to the $Q^2 - Q^2$ interaction. Remember, the monopole elements of $H_1$ vanish between $0^+$ states. As can be seen from the results given in table 1, the state ($0^+$) that moves down in energy as $\gamma$ increases derives from the $2\hbar \omega$ ($10, 4$) representa-
Fig. 1. Calculated excitation energies for $^{24}$Mg as plotted as a function of the strength $g$ of the $Q^+Q^-$ term in the hamiltonian. The oscillator strength parameter which is also the shell separation distance was set at the value from 12.61 MeV, and the parameters of $H_1$ as their best-fit values for the optimum value of $g$, see eq. (1). The model space used in the calculations is defined by the symplectic basis $(\rho_0,\rho_1) = (6,4)$, which corresponds to the $2nu(4\nu)$ configuration with maximum deformation. All representations of SU(3) up to and including $\Omega = 0$ were used to form the basis. The energy ($e^+$) and strength ($e^-$) centroids are also shown with their widths ($\sigma^+$ and $\sigma^-$, respectively).

of SU(3). This representation can be thought of as the addition of one phonon ($S^+$) of excitation (2hu) along the symmetry axis of the dominant $(4,4)$ ground-state configuration. The $(10,4)$ representation is the 2hu configuration of maximal deformation. The other two 2hu $0^+$ states are dominated by admixtures of two phonons ($S^+$ and $\Delta^+$) of excitations in directions perpendicular to the ground-state symmetry axis.

Note how the spectrum expands smoothly with increasing $g$. Also note that the gradual rise in the $L=0$ states with increasing $\gamma$ follows an $L(L+1)$ rule. This comes about because $Q^+Q^-$ acting within a shell essentially behaves like the Elliott interaction $Q^+Q^- = C_2 + M_2$, where the second-order SU(3) Casimir invariant $C_2$ is a constant. The role of $H_1$ is primarily to produce the correct $K$-band splitting and reduce the moment of inertia of the system from the $Q^+Q^-$ value of $1.225 = 1.225 + 1.225 + 1.225$ to $1.2/2 = 1.2/2 + 1.2/2$. Confirmation that $H_1$ has a nearly constant effect on the spectrum is seen by noting that the splitting of pairs of states like ($27^+ - 27^+$) and ($47^+ - 47^+$) is very nearly independent of the strength of $g$. Also, the ratio $b/c$ is almost exactly what is required for $H_1$ to be rewritten in the simpler form $H_1 = -2AE - 2BR$, where $B$ is the band label, zero for the ground band and two for the first excited band [13]. Note how closely the ($27^- - 27^+$) energy separation tracks the ground-band ($27^- - 27^+$) splitting. Indeed, the calculation shows that there is a whole rotational band built on the $0^+$ structure that mirrors the ground-band sequence of states. As will be seen below, the giant quadrupole resonance derives most of its strength from the $2^-$ member of this band. Therefore we shall refer to this band as the giant resonance band. Our hope is that the existence of this structure can and will be verified by experiment. The arrows on the right in the figure indicate the energy and $E2$ strength centroids of the $2nu 2^+$ states.

Calculated E2 strengths for various $2^+ - 0^+$ and $4^+ - 2^+$ transitions are shown in Fig. 2. An important value is the $27^- - 0^+$ transition strength since it agrees with experiment for the optimum value of $g$ despite the fact that this is nearly three times stronger than the corresponding $0^+$ number. No effective charge was used in the calculation. The required enhancement comes about through vertical mixing induced by the $Q^+Q^-$ interaction. As shown in the figure, the $27^- - 0^+$ transitions strengths grows smoothly from about 7 to 20 Weisskopf units as the strength of $g$ increases from zero to the best-fit value. The corresponding ($47^- - 47^-$, $67^- - 67^-$, $87^- - 87^-$, $107^- - 87^+$) values are (27, 27, 20, 14), respectively. This represents a drop-off in strength from prior predictions but is in agreement with experiment. The decrease in a
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Table 1
SU(3) analysis of the calculated ground state \((4,0)\) and resonant mode \((4,0)\) in \(^{24}\)Mg for three values of the coupling constant \(x\).
result of the finite dimensionality of the $6h_{\pi}$ shell-model space. These interband transitions also agree with experiment, being (1.58, 0.86, 0.53) compared to (1.4, 1.0, 0.6) for the $2_{1}^{+} \rightarrow 0_{1}^{+}$, $4_{1}^{+} \rightarrow 2_{1}^{+}$, $6_{2}^{+} \rightarrow 4_{1}^{+}$ transitions, respectively [14-16].

A good appreciation for the shell-model character of ground-band and resonance-band states can be gained by considering the structure of the $0_{1}^{+}$ and $2_{1}^{+}$ bands. This is done in table 1 and in fig. 3. As $\gamma$ increases the amount of mixing within and between shells of the harmonic oscillator increases sharply, especially for members of the resonance band. With $\gamma = 0$ the $0_{1}^{+}$ and $2_{1}^{+}$ states are pure $6h_{\pi}$ and $(10, 4)$ representations of SU(3), respectively. With $\gamma$ optimum the ground band has significant (5%) strength beyond the $4h_{\pi}$ level while the resonance band shows out as far as $20h_{\pi}$ with approximately 7% of its strength lying beyond $10h_{\pi}$. This vertical mixing is crucial to getting the calculated $\Delta I = 2$ transition strength correct. The results given in table 1 suggest that the stretched (or $Sp(3, F)$) subspace of the full $Sp(3, F)$ space, which includes only the $(6+2n, 4)$ representation of SU(3) at the $5h_{\pi}$ level,
The isocutator giant quadrupole resonance in $^{24}$Mg lies at about 21 MeV while the excitation energy of the calculated 2$^+$ state is 17 MeV. If the beam is taken to represent the resonance, then there is a large discrepancy. However, the resonance lies in the continuum so a more appropriate comparison employs sum-rule measures. Specifically, if $S_0(L, \ell)$ denotes the sub-order energy-weighted sum,

$$S_0(J, \ell, \ell) = \sum_{\ell = 0}^{\ell} E_{\ell} T(E_{\ell}, J, \ell)$$

where $b$ is the oscillator length parameter, then the non-energy-weighted sum rule (NEWSR), the linear-energy-weighted sum rule (LEWSR) and the quadrature-energy-weighted sum rule (QEWRS) can be used [18]. The quantity $S_0$ is a measure of the total strength while $\alpha: S_0/S_0$ determines the strength centroid and $\sigma: (S_0/S_0)^{1/2}$ is its width. If the sum over final configurations in (3) is restricted to the 2$^+$ states, the sum-rule measures are:

$$S_0 = 10.4 \text{ Weisskopf units, } E_{\ell} = 23.8 \text{ MeV, and } \sigma = 5.5 \text{ MeV.}$$

The values $\alpha$ and $\sigma$ are marked in Fig. 1 along with results for the average energy (24.7 MeV) and the spreading or width ($\sigma = 5.5$ MeV) of the 2$^+$ states. If the splitting induced by $E_\ell$ is turned off, the strength centroid and width are reduced to 22.0 and 4.5 MeV, respectively. By extending the sum to include the 0$^+$ as well as the 2$^+$ states, one obtains additional useful information. In particular, the 0$^+ - 2^+$ transition exhausts 61% of the total 0$^+ - 2^+$ (all $\ell$) strength while the 0$^+ - 2^+$ transition accounts for about 14%. The 0$^+ - 2^+$ over transition adds an additional 5% to the 0$^+$ result with the remaining 20% residing in the other 2$^+$ states. This means that the resonant mode accounts for about one-third of the total ground-state strength with the other two-thirds going into low-lying 0$^+$ transitions.

This study shows that in order to reproduce the known structure of low-lying states in $^{24}$Mg and generate the observed enhanced E2 transition rates without introducing an effective charge one must go to at least an 8MeV space and more than double that to achieve an adequate representation of the giant quadrupole resonance. An important prediction of
this work is that the giant resonance is just one mani-
ifestation of the existence of a "superdeformed" ro-
tational band (\(\beta = 1.5\beta_0\)) in the continuum that men-
vers the structure of the ground-band. Experimental
verification of the latter would serve to further un-
derscore the connection between low-lying collectiv-
ity and high-lying resonance phenomena in nuclei.
Moreover, its observation would serve as further en-
dorsement of the symplectic model and provide
strong motivation for pushing forward with extent-
ions of the theory, especially with the addition of
horizontal mixing between symplectic bands so gain-
ing can be directly included and with new applica-
tions like using calculated eigenstates to probe nu-
clear currents which is important not just to the
phenomenology of rotations but also in determining
the true nature of nucleon-nucleon interactions in
nuclei.

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