Pseudospin Symmetry in Nuclear Physics

C. Bahri and J. P. Draayer

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803

S. A. Moszkowski

Department of Physics, University of California, Los Angeles, Los Angeles, California 90024

(Received 12 November 1991)

The origin and consequences of pseudospin symmetry in nuclear physics, which is exact for an oscillator potential with one-body orbit-orbit (\(v_{\Omega}\)) and spin-orbit (\(v_{\Omega}\)) interaction strengths in the ratio \(\mu = 2v_{\Omega}/v_{\Omega} = 0.5\), are considered. Specifically, the \(v_{\Omega} \approx 4v_{\Omega}\) condition is consistent with relativistic mean-field results and a pseudo LS coupling scheme. When deformation dominates, pseudospin extends to pseudo SU(3), which is applicable to superdeformation.

PACS numbers: 21.60.Fw, 21.60.Cs, 21.60.Ev

(1) Introduction.—The three-dimensional isotropic harmonic oscillator \((H_0)\) augmented with the one-body spin-orbit \((l \cdot s)\) and orbit-orbit \((l^2)\) interactions,

\[ H = H_0 + v_{\Omega}l \cdot s + v_{\Omega}l^2, \tag{1} \]

is a good approximation for the nuclear single-particle Hamiltonian. The \(l^2\) term pushes high angular momentum states down \((v_{\Omega} < 0)\) relative to those with lower \(l\) values while the \(l \cdot s\) term (coupling spatial and spin degrees of freedom) is required to achieve shell closures \((v_{\Omega} < 0)\) at the magic numbers. Unfortunately, \(v_{\Omega}\) is so large that the \(l \cdot s\) term destroys the oscillator SU(3) symmetry for all but light \((A \lesssim 28)\) nuclei, rendering it of little value in attempts at unraveling the structure of heavier systems.

This paper shows that this situation gives way to a much more favorable one, because for medium and heavy \((A \gtrsim 100)\) nuclei, \(v_{\Omega} \approx 4v_{\Omega}\) or the Nilsson parameter \(\mu = 2v_{\Omega}/v_{\Omega} \approx 0.5\). As a consequence, the level splitting generated by the \(l \cdot s\) and \(l^2\) interactions can be duplicated by a pseudo-oscillator Hamiltonian plus a pseudo \(l^2\) term, with (at most) a small symmetry-breaking residual pseudo \(l \cdot s\) interaction [1-3]. Since common residual interactions are pseudospin scalar operators, a many-particle pseudo LS-coupled shell-model scheme can be employed, and the basis truncated to leading pseudospin symmetries, without losing important physics. In addition, the pseudo LS scheme extends to pseudo SU(3) when deformation dominates [4].

(2) Spherical Nilsson scheme.—In the single-particle picture, the pseudospin concept means a division of the total particle angular momentum into pseudo \((j = \bar{l} + \bar{s})\) rather than normal \((j = l + s)\) orbital and spin parts, so \(\bar{l} \pm \frac{1}{2} = l \mp \frac{1}{2}\). The physical significance of this elementary transformation is illustrated in Fig. 1, where eigenvalues of \(H\) are plotted as a function of \(\mu\). For \(\mu = 0.5\), the pairs with \(j = l + \frac{1}{2}\) and \(j = (l+2) - \frac{1}{2}\) are degenerate for all \(l\) values. Furthermore, the splitting of the degenerate pairs follows a \(\bar{l}(\bar{l}+1)\) rule, where \(\bar{l}\) is the average angular momentum of the pair: \(\bar{l} = \frac{1}{2}[l + (l + 2)] = l + 1\). This mapping of the \((ls)j\)-coupled single-particle states onto \((l\bar{s})j\) pairs defines a special (normal-pseudo) unitary transformation: \(U = 2(n, \xi - 2l \cdot s + 3)^{-\eta}(l \cdot s)^{\xi}\), where the \(\eta\) and \(\xi\) respectively create and annihilate oscillator quanta [5,6].

The single-particle Hamiltonian transforms under this mapping as follows:

\[ \tilde{H}_0 + v_{\Omega}l \cdot s + v_{\Omega}l^2 \overset{\text{pseudo}}{\longrightarrow} \tilde{H}_0 + (4v_{\Omega} - v_{\Omega})\bar{l} \cdot \bar{s} \]

\[ + v_{\Omega}l^2 + (h \omega + 2v_{\Omega} - v_{\Omega})\bar{l}. \bar{s}. \]

Since \(h \omega + 2v_{\Omega} - v_{\Omega}\) is a constant, the pseudo form, \(\tilde{H}\)

\[ \tilde{H} = \tilde{H}_0 + v_{\Omega}l \cdot s + \tilde{V}_n, \]

where \(V_n\) is a pseudo potential.

FIG. 1. Eigenvalues of the single-particle Hamiltonian \(H/\hbar \omega = m - \kappa(2l \cdot s + \mu l^2)\), where \(\mu = 2v_{\Omega}/v_{\Omega}\) and \(\kappa = -v_{\Omega}/2\hbar \omega \approx 1/2\). For the specific value \(\kappa = 0.05\) and \(0.0 \lesssim \mu \lesssim 1.0\). The \(j = (l + 2) - \frac{1}{2}\) and \(j = l + \frac{1}{2}\) levels are degenerate for \(\mu = 0.5\), which can be duplicated by the simpler pseudo-oscillator Hamiltonian \(H/\hbar \omega = \bar{n} - \kappa \bar{l}^2\) when \(\hbar \omega = h \omega\) and \(\bar{n} = n - 1\) with \(\bar{l} = l - 1\) and \(\bar{s} = 2, \ldots, 1\) or 0 and \(\bar{s} = \frac{1}{2}\). Each pseudoshell is accompanied by a unique parity intruder level (shown as dashed) with \(j = (n + 1) + \frac{1}{2} = n + \frac{1}{2}\) from the shell above. As indicated, empirical results place medium and heavy mass nuclei close \((m_\mu \sim 0.6\) and \(m_\mu \sim 0.4\)) to the \(m = 0.5\) value required for exact pseudospin symmetry.

© 1992 The American Physical Society 2133
leads to the same conclusion when the kinetic energy is a small fraction of the nucleon mass.

Next, consider the strength of the spin-orbit coupling. Starting with the usual Dirac equation (with only the time component of the scalar and vector potentials taken into account) and using a nonrelativistic reduction of the relativistic mean-field theory, the spin-orbit interaction is given by

\[ V_{ls} = \frac{h^2}{2M} \frac{2}{r^2} \frac{d}{dr} \left( \frac{1}{1 - Bp/\rho_0} \right) I \cdot s. \]  

(5)

In this expression, \( \rho \) and \( \rho_0 \) are respectively the nucleon density at radius \( r \) and the nuclear matter density. The dimensionless quantity \( B \) in (5) is related to the strength of the scalar and vector coupling constants. The spin-orbit strength \( V_{ls} \) can be obtained from the average of \( V_{ls} \) over the region inside radius \( R \).

\[ V_{ls} = \frac{-h^2}{2MR^2} \frac{6B}{1-B}. \]  

(6)

In determining this result, the fact that \( dp/dr \) vanishes everywhere, except near the surface of the nucleus, has been used.

It follows from Eqs. (4) and (6) that the ratio

\[ \mu = \frac{2V_{ls}}{V_{ls}} = \frac{1 - B}{3B}. \]  

(7)

is independent of mass number. Furthermore, to obtain \( \mu = 0.5 \) requires \( B = 0.4 \). In the simplest version of the theory, \( B = \frac{1}{2} (B_s + B_v) \), with its scalar \( (i=s) \) and vector \( (i=v) \) components given by \( B_i = g_i \rho_i/\mu_i M c^2 \), where \( \mu_i \) and \( g_i \) respectively denote meson masses and coupling constants. Using this expression for \( B \), the Nambu–Jona-Lasinio (NJL) model [8,9]—which in its modern form starts with massless quarks and generates hadron masses out of the vacuum by spontaneous symmetry breaking, and which has also been used to predict the coupling constants and masses appearing in a relativistic nuclear field theory—gives the result \( \mu = 0.686 \) shown in Table II. As also shown in the table, results for the original Walecka model [10] and a derivative coupling model due to Zimanyi and Moszkowski [11]—which gives a more realistic equation of state for nuclear matter, which

<table>
<thead>
<tr>
<th>( n )</th>
<th>( l )</th>
<th>( x_{ul}/\pi )</th>
<th>( x_{ul} )</th>
<th>( x_{ul} - x_{u} )</th>
<th>( l(l+1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>3.000</td>
<td>1.5</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2.895</td>
<td>1.45</td>
<td>6.11</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>2.605</td>
<td>1.35</td>
<td>21.85</td>
<td>20</td>
</tr>
</tbody>
</table>

TABLE II. Comparison of \( \mu = 2V_{ls}/V_{ls} \) values for various relativistic mean-field theories. Exact pseudospin symmetry requires \( \mu = 0.5 \). Results given are for \( \rho_0 = 0.16 \) nucleon/fm\(^3\) and a nuclear binding energy of \(-16\) MeV.


\[ ^a B = \frac{1}{2} (B_s + B_v). \]

\[ ^b B = \frac{3}{2} B_s + 2B_v \ (including \ recoil \ and \ exchange \ effects). \]
includes the effect of nucleon recoil, and when extended to include exchange correlations—also yield reasonable results for \( \mu \).

(4) Pseudospin dynamical symmetry.—A consequence of good pseudospin symmetry is that a \( LS \) coupling scheme (with distinct \( S \) multiplets that are decoupled and ordered) is expected to be a good starting point for describing heavy nuclei, many-particle phenomena. The \( \tilde{m} \)-particle valence spaces \( (\tilde{m} = \tilde{m}_e \) for protons and \( \tilde{m} = \tilde{m}_n \) for neutrons, which occupy different major shells) divide into subspaces: \( \tilde{S} = 0, 1, 2, 3, \ldots, \tilde{S}_{\text{max}} \) for \( \tilde{m} \) even or \( \tilde{S}_e = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots, \tilde{S}_{\text{max}} \) for \( \tilde{m} \) odd, with \( \tilde{S}_{\text{max}} = \min(\frac{1}{2} \tilde{N}, \tilde{N} + \frac{1}{2} \tilde{m}) \), where \( \tilde{N} = \frac{1}{2} (n + 1)(\tilde{n} + 2) \) is the pseudoshell degeneracy. The proton-neutron \( LS \)-coupled states (\( \tilde{a} \) labeling \( L \) multiplicity), \( |\psi\rangle = |L_S \tilde{a} \rangle \times |\tilde{S}_n \ldots \tilde{S}_1 \rangle \), with \( \tilde{S}_e = 0, \tilde{S}_n = \tilde{S}_{\text{min}}, \tilde{S}_e = \tilde{S}_{\text{max}}, \) and \( \tilde{S}_n = \tilde{S}_{\text{min}} \), are expected to dominate, because realistic interactions favor pseudospaces symmetric (pseudospin antisymmetric) configurations.

This truncation to the lowest \( \tilde{S}_a \) \( \tilde{S}_s \), \( \tilde{a} \) \( \tilde{S}_{\text{max}} \) \( \tilde{a} \) multiplets is usually insufficient to reduce the model space to a reasonable and workable size. Fortunately, another symmetry for strongly deformed nuclei can be invoked to effect a further truncation. Just as for light nuclei, \( \tilde{SU}(3) \) of the pseudo oscillator which lies between \( \tilde{U}(N) \) and \( \tilde{SO}_{\text{L}}(3) \) can be used to organize the states within each \( \tilde{S} \) multiplet according to their deformation [12]. In this case the deformation is realized in terms of the pseudo (not normal) space symmetry. Nonetheless, this gives rise to strongly enhanced \( B(L2) \) transition strengths, because the electric quadrupole operators \( Q_2 \) and \( Q_E \) are known to differ very little from one another [13].

Of the various coupling schemes that can be built with these group structures, the \( \tilde{SU}(3) \) strong-coupling limit shown in Fig. 2 is the most natural (cf. Ref. [14]). In the dynamical symmetry limit, when the interaction is expressed solely in terms of group invariants, the corresponding eigenvalue spectrum is given by

\[
E = m_e S_e + m_n S_n + C(m_e S_e + m_n S_n) K (L S) J = C m_e \tilde{m}_e + C m_n \tilde{m}_n + C \tilde{S}_e \tilde{S}_n (\tilde{S}_e + 1) + C \tilde{S}_e \tilde{S}_n (\tilde{S}_n + 1) + C \tilde{S}_e \tilde{S}_n (\tilde{S}_e + \tilde{S}_n + 1) \]

\[
- \frac{1}{2} C \tilde{S}_e \tilde{S}_n (\tilde{S}_e + \tilde{S}_n + 1) - \frac{1}{2} C \tilde{S}_e \tilde{S}_n (\tilde{S}_e + \tilde{S}_n + 1) - \frac{1}{2} \tilde{S}_e \tilde{S}_n (\tilde{S}_e + \tilde{S}_n + 1) \]

\[
+ C \tilde{S}_e \tilde{S}_n (\tilde{S}_e + \tilde{S}_n + 1) + C K^2 + C J (J + 1).
\]  

(8)

This choice is consistent with a deformation reinforcement principle which recognizes the favored configuration to be the one with maximum overlap of maximally deformed proton and neutron spatial orbitals. In (8), \( C_2(\lambda, \mu) \) is the second-order \( SU(3) \) invariant with eigenvalue \( \lambda^2 + \mu^2 + \lambda \mu + \mu \). An explicit form for an operator that has \( K^2 \) as its eigenvalue is known in the limit \( L \ll \sqrt{C_2(\lambda, \mu)} \) [12]. The constants in (8) are related to the effective interaction. For example, the \( \chi ' \)s are given by the strengths of the quadrupole-quadrupole interactions; \( C \tilde{S}_e \tilde{S}_n \) is related to centroid separations of the pseudospin multiplets; \( C \tilde{S}_e \tilde{S}_n \) is the inertia parameter; \( C \tilde{S}_e \tilde{S}_n \) determines the band splitting; etc. The \( J^2 \) term can be replaced by \( \tilde{L} \tilde{S} = \frac{1}{2} (L^2 - \tilde{S}^2) \) and used to fine tune the placement of the \( \tilde{L}(\tilde{L} + 1) \) bands with respect to \( \tilde{S} \).

When the pseudospin dynamical symmetry picture applies, there are \( \tilde{L}(\tilde{L} + 1) \) bands—one for each pseudospin orientation—that differ in total angular momenta \( (J = \tilde{L} + \tilde{S}) \) by integer (even \( A \) compared with even \( A \)) or half-integer (odd \( A \) with even \( A \)) amounts. This seems to affirmatively answer the question of "whether low-lying collective states having alignment 1 would occur in a nucleus with rather good pseudospin symmetry" [16]. In considering this matter, it is important to emphasize that the alignment can be either proton or neutron in origin, or a combination. In particular, a consequence of good pseudospin symmetry is the prediction of \( 2S + 1 \) \( \text{identical} \) \( \tilde{L}(\tilde{L} + 1) \) bands with \( J \) values given by \( J = \tilde{L} - \tilde{S} \) in the first, \( \tilde{L} - \tilde{S} + 1 \) in the second, \( \ldots \), \( \tilde{L} + \tilde{S} \) in the last. The model further predicts (since on the average \( \mu_\alpha > 0.5 \) and \( \mu_\nu < 0.5 \)) that in odd-\( A \) proton nuclei the \( J = \tilde{L} + \frac{1}{2} \) series should fall below the \( J = \tilde{L} - \frac{1}{2} \) sequence and vice versa for odd-\( A \) neutron systems. Indeed, for the \( \text{Tb}_{65} \) case (\( \tilde{S} = \frac{1}{2} \)) an excited superdeformed band of the \( J = \tilde{L} + \frac{1}{2} \) has been reported and taken as evidence for the goodness of the pseudospin SU(3) picture [17].

(5) Conclusions.—The origin and consequences of the
\[ \mu = 2v_{11}/v_{11} \approx 0.5 \] result was examined. Actual estimates for \( \mu \) are (0.60 and 0.65) for protons with (50 < Z < 82 and \( Z > 82 \)), and (0.42 and 0.33) for neutrons with (82 < N < 126 and \( N > 126 \)), respectively. These values are sufficiently close to \( \mu \approx 0.5 \) that the many-particle extension of the single-particle picture is expected to have good total pseudospin symmetry, provided the residual interaction is a pseudospin scalar operator. Examples include pairing, the surface delta interaction, and \( \tilde{Q} \cdot \tilde{Q} \), which generates \( \tilde{L} \tilde{L} + 1 \) rotational sequences in the decoupled pseudo spaces. At a more fundamental level, good pseudospin symmetry was shown to be consistent with relativistic mean field results for \( v_{11} \) and \( v_{11} \).

Further consequences of good pseudospin symmetry were noted; particularly, the appearance of identical bands. Strong deformation in the pseudo-space part of the many-particle basis gives rise to \( \tilde{L} \tilde{L} + 1 \) rotational sequences for each of the \( 2S+1 \) orientations of the pseudospin. That these bands yield strongly enhanced \( B(E2) \) strengths follows because \( \tilde{Q}_E \approx Q_E \). A prediction of the theory is that many additional, strongly deformed bands should be found when the detectors with high efficiency and multiple-coincidence capability that are currently under construction come on line.

We have assumed that the particles in the intruder levels do not affect the dynamics in a significant way. This assumption is justified as long as intruder pair alignment (common in normally deformed systems) which results in backbending, accompanied by weakened \( E2 \) transition strengths, is not observed. Superdeformed rotational sequences appear to conform to this assumption, namely, intruder level particles apparently keep an internal structure that changes (at most) slowly as a function of increasing angular momentum of the system.

This work (J.P.D.) was supported in part by the U.S. National Science Foundation.