On the treatment of intruder levels in strongly deformed nuclei in the framework of the SU(3) shell model

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Continued on page 3 of cover

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On the treatment of collective motion in deformed nuclei in the

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Abstract

A model system which may be constructed in order to study deformations and related collective excitation is presented. Its advantages are that the correlations generated are of a simple form and that the model makes clear the role of the collective motion in driving the reaction. The authors suggest that it is important in such cases for the model to be developed to incorporate the role of the collective motion, so that the model is as realistic as possible and as simple as possible.

1. Introduction

Given the extreme complexity of simplifying assumptions in nuclear reaction theory. The usual shell-model description of the nucleus is inadequate for the description of many reactions. In particular, the description of the collective motion of the nucleus is important in many reactions. The authors suggest that it is important in such cases for the model to be developed to incorporate the role of the collective motion, so that the model is as realistic as possible and as simple as possible.

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Abstract

A model system which mimics the shell-model dynamics of strongly deformed nuclei is constructed in order to study the role of particles in the unique parity orbitals of heavy deformed nuclei and to test the validity and applicability of some commonly used truncation procedures. Working in a truncation-free environment and including quadrupole–quadrupole, spin–orbit, orbit–orbit, and pairing forces, we find that for standard nuclear systems the correlations generated among particles in the unique parity space and by the interaction of nucleons in the normal parity orbitals with those in the unique parity orbitals play an important role in driving the many-particle system towards its maximum allowed deformation. The results suggest that nucleons in the unique parity levels contribute significantly to the overall collectivity of a nuclear system and should be taken into account explicitly whenever possible, or at least through renormalization procedures that can be justified in special cases, like for collective states below the backbending region.

I. Introduction

Given the extreme complexity of the nuclear many-particle system, the introduction of simplifying assumptions is essential for achieving a tractable microscopic theory. The usual shell-model approach, which is based on the presupposition that...
the combination of all nucleon–nucleon interactions gives rise to an average potential field and hence shell closures, reduces the problem by truncating the Hilbert space of the complete system to a valence space that is comprised of a single major shell only. While this severe truncation procedure seems to work well for light nuclei, it turns out to be insufficient for heavy systems: Not only do protons and neutrons occupy different major shells but a dominant spin–orbit interaction emerges, causing the single-particle orbital with the largest $j$ value in the $(n+1)$st shell ($j = n + 3/2$, where $j$ denotes the single-particle total angular momentum) to penetrate down into the $n$th shell.

A reasonable approach for the like-particle (proton or neutron) sub-systems found in heavy nuclei might consider the valence space to consist of the normal parity orbitals of the $n$th shell and the unique parity $j = n + 3/2$ "intruder" orbital from the $(n+1)$st shell, possibly augmented by its like-parity partners. But the latter choice implies a Hilbert space that is a direct product of the "$n$th shell space" (usually referred to as the "normal parity space") and the "$(n+1)$st shell space" (the "unique parity space") which for all but few-particle systems is still too large to be handled completely with the currently available computational tools. And combining two such systems, one for the protons and another for the neutrons, generates an additional significant increase in the size of the model space and therefore renders such calculations even further out of reach.

Algebraic approaches to the many-particle shell model, based on boson (interacting-boson model [1–3]) or fermion descriptions of the dynamics (SU(3) model [4–10], symplectic model [11,12], fermion dynamical-symmetry model [13,14]), utilize the symmetries of the problem in order to select a basis that is appropriate for the hamiltonian under consideration and in so doing arrive at a "symmetry-adapted" truncation scheme. Boson-based models, the complexity of which barely increases with particle number, may or may not single out and include unique parity components, depending on the phenomena being considered. For fermion-based models, where the complexity grows combinatorially with particle number, the large-valence-space problem is usually circumvented by introducing a model-dependent truncation scheme for the normal parity component of the full space and reducing the unique parity subspace to low-seniority (frequently even zero) configurations of the $j = n + 3/2$ level only.

The usual argument given for truncating the unique parity space to the $j = n + 3/2$ intruder level and neglecting couplings to the energetically higher orbitals ($j < n + 3/2$) of the $(n+1)$st major shell is based upon the fact that there is an energy difference between these orbitals and the intruder level which is on the order of the typical major shell-separation energy. And since in a restricted space of a single $j$-shell the short-range part of the nucleon–nucleon interaction, which generates pairing correlations, dominates, the energetically most favored state is the seniority zero ($\nu = 0$) configuration, that is, a many-particle state in which all nucleons (except the last one in the case of even–odd nuclei) are paired off to angular momentum zero. There is a significant energy difference, $\Delta E_{\text{pair}}$ ("pairing gap"), between this seniority-zero state and an excited seniority-two ($\nu = 2$) configuration which involves a single decoupled pair [15]. Since this pairing gap is large when compared to rotational numbers two and larger are core low-lying states of the nuclei.

In these models the many-particle angular-momentum-coupled pro restricted unique parity configuration U, respectively, stand for quan normal and unique part of the and $\nu$, the seniority quantum l parity space that are not con restrictions ($\nu = 0$, $J_U = 0$, $J = J_\nu$ tions; in particular, the coupin wavefunction can only make at the dynamics of the system. S, where $\mathcal{F}$ is the inertia parame system, the assumption of neg break down when $\Delta E_{\text{rotor}} = 2J$ actinide nuclei occurs for $j = 1$.

Since the seniority-zero confi tum, this strong approximation. Indeed, in most applica the only on the normal parity comp of the particles in the intruder through an overall model-dep polarizat effects which derive on to an $n$th plus $(n+1)$st sh be introduced in order to fit the

The assumption underlying a rations is that the protons and ne an adiabatic way the collective m that is, the unique parity configu parity structures track the beha moves up the yrast band. Specifi of the larger valence space is cou plage valence space. This type unava ific when dealing with system. And as long as the nnu the behavior of the complete en for properties of low-lying: the success of mean-field theore.

Most theories which make use of the model space to its normal parity va This, coupled with an ap pliy implied by the assumption, h treatment of the nucleons in t
when compared to rotational excitation energies, states with seniority quantum numbers two and larger are considered to play a minor role in the dynamics of low-lying states of the nucleus.

In these models the many-particle basis states for the valence space are given as angular-momentum-coupled products of normal parity basis states and seniority-restricted unique parity configurations: $| \phi_{jm} \rangle = (| N; J_n \rangle \ U; | J_{1u} \rangle )^{JM}$, where $N$ and $U$, respectively, stand for quantum labels that are necessary to fully classify the normal and unique part of the state in the algebraic model under consideration, and $v$, the seniority quantum label, counts the number of nucleons in the unique parity space that are not coupled pairwise to $J = 0$. When a seniority-zero restriction ($v = 0$, $J_{1u} = 0$, $J = J_n$) is invoked, there are additional major simplifications; in particular, the coupling is then trivial, as the unique parity part of the wavefunction can only make an angular-momentum-independent contribution to the dynamics of the system. Since the rotational energy goes as $J(J + 1)/2 \varepsilon$, where $\varepsilon$ is the inertia parameter and $J$ is the total angular momentum of the system, the assumption of negligible contributions from states with $v > 0$ must break down when $\Delta E_{\text{rot}} = (2J - 1)/2 \varepsilon \approx \Delta E_{\text{pair}}$, which for typical rare-earth and actinide nuclei occurs for $J \approx (10 - 14) \hbar$ and $J \approx (12 - 16) \hbar$, respectively.

Since the seniority-zero configuration is unique and carries no angular momentum, this strong approximation represents a severe restriction on the dynamics. Indeed, in most applications that employ this simplification, the results are based only on the normal parity component of the total Hilbert space with the influence of the particles in the intruder states on measurable quantities included either through an overall model-dependent renormalization factor or, along with core-polarization effects which derive from the fact that the space was truncated at the onset to an $n$th plus $(n + 1)$st shell scenario, through effective charges which must be introduced in order to fit the electromagnetic transition data [16].

The assumption underlying a truncation to seniority-zero unique parity configurations is that the protons and neutrons in their respective intruder levels follow in an adiabatic way the collective motion of their partners in the normal parity space, that is, the unique parity configurations that couple to their corresponding normal parity structures track the behavior of those normal parity configurations as one moves up the yrast band. Specifically, one assumes that the normal parity subspace of the larger valence space is representative of the complete normal–unique coupled valence space. This type of simplifying assumption is both common and unavoidable when dealing with problems as complex as the nuclear many-body system. And as long as the nucleons that are considered to be active truly reflect the behavior of the complete ensemble, this type of approximation is justified, at least for properties of low-lying states. Indeed, its validity has been confirmed by the success of mean-field theories, $0 - \hbar \omega$ shell-model calculations, etc.

Most theories which make use of this “adiabatic assumption” and truncate the model space to its normal parity component have not been tested to prove its validity. This, coupled with an apparent inadequate discussion of what is involved and implied by the assumption, has led to a variety of misunderstandings about the treatment of the nucleons in the unique parity orbitals in these models. In
particular, the overall normalization factor that was introduced in some applications of the SU(3)/Sp(3, R) model in order to account for the contributions of the unique parity particles to electromagnetic transition strengths and multipole moments has been misinterpreted as an arbitrary fitting parameter and this, in turn, has been used as a basis for questioning the validity of the entire theory [17].

Indeed, in recent years the issue of intruder levels in heavy nuclear systems has received considerable attention. Their significance for correctly reproducing available data, their role in determining the deformation of nuclei, and the question of how to properly incorporate them into current models has been repeatedly debated. For example, studies in the framework of the single-shell asymptotic Nilsson and “universal” Woods–Saxon models [18], in which the deformation is put in as an external constraint and is not determined by the dynamics, imply that valence nucleons in the unique parity intruder orbitals contribute significantly to measurable quantities like $B(E2)$ strengths. Some mean-field theories claim that the particles in the intruder level play the dominant role in inducing deformation [19]. Therefore, the issue of the role of nucleons occupying unique parity levels calls for careful reconsideration. In particular, one would like to know what is the relative importance of the normal and unique parity states in determining the deformation of the nucleus, and hence in contributing to electromagnetic moments and transition strengths, and whether or not these contributions can or cannot be accommodated for low-lying states via an appropriate renormalization procedure. Furthermore, we would like to distinguish the contributions that arise due to the presence of particles in the unique parity orbitals from those that stem from core polarization. This is of particular interest in regard to the pseudo-symplectic model which explicitly includes core excitations and which ultimately aims at reproducing electromagnetic transition rates without the use of effective charges.

It is the goal of this paper to investigate these issues in the framework of the Elliott SU(3) model. To this end, we will construct a model which displays those characteristic features of heavy deformed nuclei that we are interested in and remains simple enough to be completely solvable with currently available tools and techniques. The results will then be employed to gain a deeper understanding of how the situation in realistic nuclei can be modeled. In Sections 2–5 we review the pseudo-SU(3) model, and describe the details of our approach, our Hilbert space, and hamiltonian. Furthermore, we show how the selected example relates to realistic nuclear systems and how the calculations are performed. In Section 6 the importance of correlations among normal parity particles, unique parity particles, and couplings between nucleons in the unique parity space with those in the normal parity space are studied. The angular-momentum dependence of the results and thus the validity of the adiabatic assumption is also addressed. In Section 7 the work presented in this paper is summarized, some implications of the results are investigated, and possible extensions of the study are considered.

2. Review of the pseudo-SU(3) scheme

Before constructing a model, which can be employed to investigate the role of the unique parity levels in strongly deformed nuclei, we will review some important features of deformed nuclei an SU(3)/pseudo-SU(3) model. For interaction pushes the largest-j level among the orbitals of the next oscillator symmetry – the pseud approach the largest-j level is an orbital and pseudo-spin angular m Momentum states of the set of all pseudoshell form a complete set for a j=0 symmetry of the new scheme being used.

The algebraic properties of the “real” oscillator, its abstract algebraic coupling scheme has the same 6-dimensional decomposition of the quadrupole components shows that it is precise and pseudo-shell (with small core realization is that the symmetry-b basis vector is weak enough to lead pseudo-scheme has been applied to physical phenomena, such as beta decay and in studies of the structure of heavy nuclei the valence oscillator shells. Thus, for a give protons (with major quantum number $n_p$), each of which form a pseudo-oscillator), and the associated of the $(n_p + 1)$st shell. The binding that of the pseudo-oscillator levels the $(n_p + 1)$st shell lie approximately and actinide $(A \geq 220)$ nuclei we all Rare-earth protons, for example, $n_p^U = 5$ unique parity shell, the inner size of the normal and unique par dimensionality (last column): 1 intruder level compares to that of $\alpha = 0.6$ (second-to-last column). Similar results are obtained.

In order to determine how the different orbitals of the valence space for various neutron and proton single-particle levels for [26]. The Nilsson approach is widely used in the study of deformed nuclei, and we will review some important features of the single-particle level ordering as a function of the Nilsson model parameters.
features of deformed nuclei and their description in the framework of the SU(3)/pseudo-SU(3) model. For heavy nuclei ($A \geq 100$), where the spin-orbit interaction pushes the largest-$j$ level of the $n$th oscillator shell, $j = n + 1/2$, down among the orbitals of the next lower shell – thus destroying the underlying oscillator symmetry – the pseudo-spin concept can be applied [20–22]. In this approach the largest-$j$ level is removed from active consideration, and pseudo-orbital and pseudo-spin angular momenta are assigned to the remaining single-particle states. The set of all pseudo spin–orbit levels associated with an oscillator shell form a complete set for a pseudo-oscillator shell of one quantum less, the symmetry of the new scheme being $\tilde{SU}(3)$ where the tilde denotes the pseudo-realization.

The algebraic properties of the pseudo-oscillator are identical with those of the “real” oscillator, its abstract algebra is just $SU(3)$ and the Hamiltonian in the new coupling scheme has the same form as in the normal $SU(3)$ scheme. A tensor decomposition of the quadrupole–quadrupole interaction into its pseudo-$SU(3)$ components shows that it is predominantly the quadrupole–quadrupole form of the pseudo-shell (with small corrections only) [9,10]. The advantage of the pseudo-realization is that the symmetry-breaking spin–orbit interaction in the new representation is weak enough to lead to good pseudo-$SU(3)$ quantum numbers. The pseudo-scheme has been applied with success in calculations of a variety of physical phenomena, such as backbending [23], magnetic-dipole transitions [24], and in studies of the structure of superdeformed bands [25].

In heavy nuclei the valence protons and neutrons occupy different major oscillator shells. Thus, for a given nucleus there are two open shells, one for protons (with major quantum number $n_p$) and another for neutrons (with major quantum number $n_n$). Each of these shells is comprised of a set of normal parity orbitals, which form a pseudo-oscillator shell ($n_{\sigma}^{\nu} = n_{\sigma} - 1$, $\sigma = \pi$ for protons or $\nu$ for neutrons), and the associated unique parity intruder level, which is a member of the $(n_n + 1)$st shell. The binding energy of the intruder level is comparable to that of the pseudo-oscillator levels, whereas the remaining unique parity orbitals of the $(n_n + 1)$st shell lie approximately $1\hbar\omega$ higher. For rare-earth ($A = 150–180$) and actinide ($A \geq 220$) nuclei we find the situation that is summarized in Table 1a. Rare-earth protons, for example, occupy the $n_{\nu}^{\pi} = 3$ normal parity shell and the $n_{\nu}^{\pi} = 5$ unique parity shell, the intruder orbital being the $h_{11/2}$ level. The relative size of the normal and unique parity valence spaces is given by the ratio of their dimensionalities (last column): $\Omega_{\pi}/\Omega_{\nu}^{\pi} = 42/20 = 2.1$. The degeneracy of the intruder level compares to that of the normal parity space as $\Omega_{\pi}/\Omega_{\nu}^{\pi} = 12/20 = 0.6$ (second-to-last column). Similar results are given for rare-earth neutrons and actinide nucleons.

In order to determine how the valence particles are distributed among the different orbitals of the valence space one can consider the energy-level schemes for various neutron and proton shells as obtained, for example, from the Nilsson model [26]. The Nilsson approach, which has been successful in reproducing a variety of features of deformed nuclei, gives a reasonably accurate picture of the single-particle level ordering as a function of deformation. For a given deformation
Table 1a

Characterization of the valence spaces of rare-earth and actinide nuclei. The major quantum numbers ($\tilde{n}^N, n^U$) and dimensionality ($\tilde{\Omega}^N, \tilde{\Omega}^U$) of the normal parity (pseudo)subspace and the unique parity subspace are given, as well as the classification ($\tilde{j}_{\text{max}}^N$) and degeneracy ($\tilde{\Omega}_{\text{max}}^U = 2\tilde{j}_{\text{max}}^U + 1$) of the relevant intruder orbital. Neutron and proton shells are considered separately. The last two columns list the ratio of the degeneracy of the intruder level to that of the normal parity pseudo-space and the relative sizes of the unique and normal parity valence spaces.

<table>
<thead>
<tr>
<th>System</th>
<th>Normal parity space</th>
<th>Intruder level</th>
<th>Unique parity space</th>
<th>Relative sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$</td>
<td>$\tilde{n}^N$</td>
<td>$\tilde{\Omega}^N$</td>
<td>$j_{\text{max}}^U$</td>
</tr>
<tr>
<td>Rare-earth nuclei</td>
<td>$\pi$</td>
<td>3</td>
<td>20</td>
<td>$j_{11/2}$</td>
</tr>
<tr>
<td></td>
<td>$\nu$</td>
<td>4</td>
<td>30</td>
<td>$j_{13/2}$</td>
</tr>
<tr>
<td>Actinide nuclei</td>
<td>$\pi$</td>
<td>4</td>
<td>30</td>
<td>$j_{13/2}$</td>
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<tr>
<td></td>
<td>$\nu$</td>
<td>5</td>
<td>42</td>
<td>$j_{35/2}$</td>
</tr>
</tbody>
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Table 1b

Same as Table 1a, but for possible identical-particle model systems.

<table>
<thead>
<tr>
<th>Model system</th>
<th>Normal parity space</th>
<th>Intruder level</th>
<th>Unique parity space</th>
<th>Relative sizes</th>
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<tr>
<td>$(\varnothing)(\varnothing)$</td>
<td>1</td>
<td>6</td>
<td>$f_{7/2}$</td>
<td>8</td>
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<tr>
<td>$(\varnothing)(g)$</td>
<td>2</td>
<td>12</td>
<td>$g_{9/2}$</td>
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</tr>
<tr>
<td>$(\varnothing)(p)$</td>
<td>2</td>
<td>12</td>
<td>$f_{7/2}$</td>
<td>8</td>
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</table>

The above considerations show that in the vicinity of closed shells the number of nucleons in the unique parity levels is zero (for nearly empty shells) or about 50% of the total number of valence particles (for nearly filled shells). Well-deformed nuclei, however, have valence shells which are approximately half-filled. For these nuclei the valence nucleons (protons as well as neutrons) distribute among normal and unique parity states in a ratio of roughly 2 to 1.

Table 3a lists the number of valence particles in the normal and unique parity orbitals for a few representative rare-earth ($^{158}\text{Gd}$, $^{164}\text{Er}$) and actinide ($^{238}\text{U}$, $^{242}\text{Pu}$) nuclei. The last two columns give the number of normal and unique parity nucleons, respectively, divided by the dimension of the relevant space, thus yielding a measure for the "fullness" (fractional occupancy) of the space in question. For the rare earths (actinides) we find that the proton shells are filled to about (40–50)% (~20%) and (15–25)% (~10%) of their normal parity space and 10% of their unique parity space, respectively.

3. Description of the model study

The issue we are interested in is the determination of the ground state properties of levels of well-deformed nuclei coupled to a particle hole system. In order to address this problem we consider...
by filling each energy
level with a pair of protons
up to 0.25 is considered to
when the partition of $m^{\pi}$
and $m^{\nu}$ in unique purity
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closed shells the number
of filled shells). Well-de-
formed nuclei (as neutrons) distribute
by 2 to 1.

Normal and unique parity
and actinide ($^{238}$U,
while unique parity
relevant space, thus
space) of the space in
proton shells are filled to

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Table 2
Distribution of valence nucleons into normal and unique parity orbitals. The table gives the partition of
valence nucleons ($\sigma = \nu$ or $\pi$ for protons or neutrons, respectively) into $m^{\nu}$ normal parity and $m^{\nu}$
unique parity particles for the $\bar{n} = 3, 4, 5$ neutron and $\bar{n} = 3, 4, 5$ proton shells.

3. Description of the model study
The issue we are interested in studying is how the particles in the unique parity levels of well-deformed nuclei contribute to the dynamics of the whole nuclear system. In order to address this matter without prejudicing the result by trunea-
Table 3a
Particle distributions in normal and unique parity spaces for several representative rare-earth and actinide nuclei. The notation used is the same as in Table 2. The last two columns give the number of normal and unique parity nucleons, respectively, divided by the dimension of the relevant space, thus yielding a measure of the "fullness" (fractional occupancy) of the spaces in question. Note that the number of particles in the normal parity space includes only nucleons in the normal parity pseudo-shell Nucleus Valence particles Fractional occupancies
<table>
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<th></th>
<th>σ</th>
<th>m^N</th>
<th>m^U</th>
<th>m^N / Ω^N</th>
<th>m^U / Ω^U</th>
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<td>4</td>
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<td>τ</td>
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<td>6</td>
<td>0.20</td>
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<tr>
<td></td>
<td>ν</td>
<td>14</td>
<td>8</td>
<td>0.33</td>
<td>0.11</td>
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Table 3b
Same as Table 3a, but for the selected model system (d5s)^4 Θ (fp)^2

<table>
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<th>Model system</th>
<th>&quot;Valence particles&quot;</th>
<th>Fractional occupancies</th>
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<tr>
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<td>m^N</td>
<td>m^U</td>
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<tr>
<td>(d5s)^4 Θ (fp)^2</td>
<td>4</td>
<td>2</td>
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For realistic deformed nucleon low-shell configurations like the one for which the pseudorealizations are either too restricted or too handle without introducing triangular terms. An appropriate Hamiltonian

$$H = \sum_{J\pi A} \sum_{\epsilon} \epsilon_{\pi A} \hat{\pi}_{\pi A}^{J\pi} -$$

with $A = N$ or $U$ for normal and $U$ denotes the pseudorotation Hamiltonian. In the remainder of the paper, where it is needed for clarity, the angular-momentum $J$ orbital at that level, the sum being over $J^N = 1/2, 3/2, 5/2$ and the harmonic-oscillator mean such as the spin-orbit pairing interaction is explicitly included for representing the calculation. Since the "normal → pseudo-normal" transformation is now normal to the general domain of orbitals to form orbit-orbit splitting, we assume our model system to be approximated by (Recall that (d5s)^4 mimics the pseudo-nuclear system, hence the d5s/11/2 to be removed.) In realistic nucleon orbitals of the valence shell partners. Hence we fix the energy levels at 1 hω. The energy difference between successive major oscillations in the nuclei under consideration is $\Delta = 12$ MeV turn.

Note that a full pseudo-SU(3) describes parity space, giving a pseudo-oscillator to the unique parity orbitals of the $J^N$ numbers of the two spaces differ by two.

However, the shell numbers of the normal and unique parity subspace of the (d5s)^4 Θ (fp)^2 system exhibit relative sizes, fractional occupancies, and particle distributions similar to those that occur in the present study they are not included.

In order to complete a full calculation without truncating either space we have to restrict our considerations to a few particles in low shells. We chose $m^N = 4$ particles in the $n^N = 2$ (d5s)-shell and $m^U = 2$ particles in the $n^U = 3$ (fp)-shell for our model and denote the system by (d5s)^4 Θ (fp)^2. Tables 1 and 3 confirm that the normal and unique parity subspace of the (d5s)^4 Θ (fp)^2 system exhibit relative sizes, fractional occupancies, and particle distributions similar to those that occur in the present study they are not included.
for realistic deformed nuclei in pseudo-spin applications, whereas alternative low-shell configurations like \((\hat{p})^m \otimes (\hat{f}p)^n\) or \((\hat{d}s)^m \otimes (gds)^n\) are not able to mimic these features of realistic heavy deformed nuclei. Furthermore, the latter systems are either too restrictive for our purposes \((\hat{p})^m \otimes (\hat{f}p)^n\) or too large to handle without introducing truncations \((\hat{d}s)^m \otimes (gds)^n\) \(^4\).

An appropriate Hamiltonian for the system can be written as

\[
H = \left( \sum_{j,A=N,U} \epsilon_j^A \hat{n}_j^A \right) - \frac{1}{2} \chi^A \cdot Q^A - G_\Upsilon P_\Upsilon,
\]

with \(A = N, U\) for normal and unique parity, respectively. (Here the tilde, which denotes the pseudo-realization of the normal parity subspace, has been dropped. In the remainder of the paper we will suppress the tilde, except in those situations where it is needed for clarity.) The \(\epsilon_j^A\) denotes the single-particle energy of the angular-momentum-\(j\) orbital and \(\hat{n}_j^A\) is the number operator counting the fermions in that level, the sum being over all \(j\) levels of the shell(s) under consideration (here \(j^N = 1/2, 3/2, 5/2\) and \(j^U = 1/2, 3/2, 5/2, 7/2\)). The sum \(\sum \epsilon_j^A \hat{n}_j^A\) includes the harmonic-oscillator mean field as well as all relevant single-particle effects, such as the spin-orbit and orbit-orbit terms. As the expression (1) for the Hamiltonian explicitly includes the energies of the single-particle orbitals, it is very convenient for representing the situation that one encounters in a full pseudo-SU(3) calculation.

Since the "normal \(\rightarrow\) pseudo" transformation removes the highest-\(j\) level from the normal parity valence shell of a deformed heavy nucleus, leaving the remaining normal parity orbitals to form a pseudo-oscillator shell with small spin-orbit and orbit-orbit splitting, we assume the energy levels in the normal parity space of our model system to be approximately degenerate and normalize the \(\epsilon_j^N\) to zero. (Recall that \((\hat{d}s)^m\) mimics the pseudo-SU(3) normal parity component of a realistic nuclear system, hence the \(d_{5/2}\)-level is part of the pseudo-space and does not need to be removed.) In realistic nuclei all, except for the largest-\(j\) level, unique parity orbitals of the valence shell lie about \(1 \hbar \omega\) higher than their normal parity partners. Hence we fix the energies \(\epsilon_j^U\) of all but the \(j_{\text{max}}^U = n + 3/2\) unique parity levels at \(1 \hbar \omega\). The energy difference of \(1 \hbar \omega\), corresponding to the separation between successive major oscillator shells, depends on \(A\), the number of nucleons in the nucleus under consideration as follows \([27]\): \(\hbar \omega = 41 \text{ MeV} \cdot A^{-1/3}\). For our calculations \(\hbar \omega = 12 \text{ MeV}\) turns out to be appropriate, hence we fix \(\epsilon_{1/2}^U = \epsilon_{3/2}^U = \epsilon_{1/2}^N = \epsilon_{3/2}^N = \epsilon_{5/2}^N = \epsilon_{7/2}^N = \epsilon_{9/2}^U = \epsilon_{11/2}^U = \epsilon_{13/2}^U = 0\).

\(^4\) Note that a full pseudo-SU(3) description, which introduces the pseudo-spin scheme for the normal parity space, gives a pseudo-oscillator shell with \(\hat{n}^N = n - 1\) that has nearly degenerate levels, coupled to the unique parity orbitals of the \(n^N = n + 1\) shell. Thus in real nuclei the major shell quantum numbers of the two spaces differ by two, whereas this difference is taken to be only one in our model. However, the shell numbers of the normal and unique subspaces are of no physical consequence in our study, and since the actual situation to be modeled is best represented by the \((\hat{d}s)^m \otimes (\hat{f}p)^n\) system, this is what is used in our calculations.
\( e_{7/2}^{1+} = 12 \text{ MeV} \). The energy of the \( j = 7/2 \) intruder level will be fixed at 0 corresponding to a separation \( \Delta e_{7/2} \) of 1 \( h \omega \) for the intruder orbital \((l_{7/2})\) from its \((f_{5/2},p_{3/2},p_{1/2})\) partners.

The quadrupole–quadrupole \((Q^* \cdot Q^*)\) and pairing \((P^U)\) interactions reflect the most important long-range and short-range two-body correlations, respectively. The quadrupole–quadrupole term, which dominates in deformed nuclei, is defined by

\[
Q^* \cdot Q^* = \sum_{\mu = -2}^{+2} (-1)^{\mu} Q_{2,\mu}^* \cdot Q_{2,-\mu}^* \quad \text{where} \quad Q_{2,\mu}^* = \sum_i q_{2,\mu}^*(i),
\]

and in our model the single-particle quadrupole operator \( q^* \) is taken to be the symmetrized, algebraic quadrupole operator which was first introduced by Elliott [4]:

\[
q_{2,\mu}^*(i) = \sqrt{\frac{3}{2\pi}} \left[ b^4 p_i^2 Y_{2,\mu}(\vec{r}_i) + r_i^2 Y_{2,\mu}(\vec{r}_i) \right] / b^2,
\]

where \( b \) is the oscillator-length parameter. Note that by employing \( q^* \) (i.e., \( q = q^* \)) in the calculations, as opposed to the collective quadrupole operator (i.e., \( q = q^c \)) [15], \( q_{2,\mu}^*(i) = \sqrt{16\pi/5} \left[ r_i^2 Y_{2,\mu}(\vec{r}_i) / b^2 \right] \), which couples single-particle states that differ by zero or two oscillator quanta from one another \((\mu = n,n \pm 2)\), we have restricted the dynamics of the model to a single normal parity valence shell and its associated unique parity space \(^5\).

One can show that a quadrupole–quadrupole interaction strength of \( \chi^2 = 0.1 \) MeV is realistic for calculations involving a single (normal parity) ds-shell [16,27–29]. Since \( \chi^2 \) is not expected to deviate significantly from this value when considering the next higher shell (fp-shell), we have assumed that two particles in the unique parity \( n^U = 3 \) space interact with the same strength as two particles in the normal parity \( n^N = 2 \) space, and, furthermore, that the quadrupole–quadrupole force between any two nucleons does not depend on the parity of the orbitals that those nucleons occupy. Hence we have chosen to represent the quadrupole correlations in the model space by the term \(- \frac{1}{2} \chi^2 \cdot \sum (Q^* \cdot Q^*) \cdot (Q^N + Q^U) \cdot (Q^N + Q^U)\), where \(- \frac{1}{2} \chi^2 \) couples the spaces of different parity \( Q^N \) and \( Q^U \) act on the normal and unique parity states, respectively.) Since \( \chi^2 = 0.1 \) MeV is considered to be a realistic value, we will vary \( \chi^2 \) from 0 to about 0.25 MeV, where the latter approaches the asymptotic limit in which the hamiltonian is dominated by the quadrupole–quadrupole interaction and the eigenstates of the system are expected to become approximately eigenstates of \( Q^* \cdot Q^* \).

Shell-model studies for light (ds)-shell nuclei show that a pairing force in the presence of a strong quadrupole–quadrupole interaction has little effect on the structure of calculated ground-state wavefunctions. Since we are interested in strongly deformed nuclei, whose dominant role, we expect pair deformations. However, since it is unknown, we will include not only parity subspace and an additional shell couplings of the pairing type number of particles in each such system from one subspace to the other.

A reasonable value for the p subspace of this model can be found. If the \( j^{2,2} \) state of the \( \langle j^{2,2} \rangle \) has a \( J = 1/2 \), \( 100 = 0.12 \) MeV and \( 100/20 \), and \( 100/10 \) moderate pairing, respectively.

Here \( \chi^2(n_m) \), \( \langle f_n \rangle \), \( \langle \lambda_{\mu} \rangle \) are SU(3) symmetries of the wavefunctions, \( S_4 \) and \( T_4 \) denote multiple occurrences of the quark system can then be written as a single unique parity basis states [6].

\[ |\psi_{JM}\rangle = \langle \{ f_n \} \alpha_{\mu} (\lambda_{\mu}) \rangle \]

An alternative choice for the

\(^5\)These inter-shell correlations are explicitly taken into account in the symplectic/pseudo-symplectic extension of the SU(3)/pseudo-SU(3) model.
strongly deformed nuclei, where quadrupole correlations are known to play a dominant role, we expect pairing terms in the hamiltonian (1) to be of minor importance. However, since it has been argued in the past that the large energy difference between the intruder orbital and its like-parity partners from the same major oscillator shell serves to enhance pairing correlations relative to quadrupole deformations, we decided to test this assumption by explicitly introducing such a force in the unique parity space of our model. If this term turns out to significantly affect our results we will have to reconsider our current approach and extend our hamiltonian to include not only $G_U P^U$ but also a pairing interaction in the normal parity subspace and an additional two-body interaction which accounts for cross-shell couplings of the pairing type.\footnote{Note that the hamiltonian (1) does not include a pairing force which acts between nucleons that occupy shells of opposite parities. There is no evidence suggesting that pairing is enhanced through cross-shell couplings as is known to be the case for the quadrupole–quadrupole interaction, which is included in the model study.} Also note that for simplicity we have fixed the number of particles in each subspace, that is, we do not allow for pair scattering from one subspace to the other.

A reasonable value for the pairing-strength parameter $G_U$ for the unique parity subspace of this model can be determined from a knowledge of realistic interactions and energy spectra. If the pairing operator, $P$, is defined in the usual way by $\langle j^2, j', J', N | P | j, J, N \rangle = \frac{1}{2} \frac{(2J + 1)}{(2j' + 1)(2j + 1)} \delta_{J',j'} \delta_{J,j} \delta_{J',j},$ we find that $G_U = 0, h\omega/100 \approx 0.12$ MeV and $h\omega/50 \approx 0.24$ MeV leads to a pairing energy gap of roughly 0, $h\omega/20$, and $h\omega/10$, and thus represents the cases of no, weak, and moderate pairing, respectively, in the unique parity space.

For both the normal ($A = N$) and unique ($A = U$) parity subspaces we introduce Elliott’s SU(3) scheme to label the many-particle states [8–10]:

$$\left| \begin{array}{cc} |m^*\rangle [f_{\lambda} \alpha_{\lambda} (\lambda_{\mu} \mu_{\lambda}) \kappa_{\lambda} L_{\lambda} \beta_{\lambda} (S_{\lambda} T_{\lambda}) J_{\lambda} M_{\lambda}] \end{array} \right>.\] (4)

Here $|m^*\rangle, [f_{\lambda}], (\lambda_{\mu} \mu_{\lambda})$ characterize the particle permutation, spatial (U($\Omega$)), and SU(3) symmetries of the wavefunction, $L_{\lambda}$ and $J_{\lambda}$ are the orbital and total angular momenta, $S_{\lambda}$ and $T_{\lambda}$ denote the spin and isospin, and $\alpha_{\lambda}, \beta_{\lambda}, \kappa_{\lambda}$ distinguish multiple occurrences of the quantum labels. The basis functions for the complete system can then be written as angular-momentum-coupled products of the normal and unique parity basis states [30]:

$$\left| \phi_{JM} \right> = \left| \{1|m^*\} |f_{\lambda}\rangle \alpha_{\lambda} (\lambda_{\mu} \mu_{\lambda}) \kappa_{\lambda} L_{\lambda} \beta_{\lambda} (S_{\lambda} T_{\lambda}) J_{\lambda} M_{\lambda} \right>.$$ (5)

An alternative choice for the basis involves SU(3)-coupled wavefunctions, which
will turn out to be useful for calculating the matrix elements of the quadrupole–quadrupole interaction:

$$|\phi_{JM}\rangle = \{[1^{m_n}] [f_n] \alpha_N(\lambda_N \mu_N) \beta_N(S_N T_N);$$

$$[1^{m_u}] [f_u] \alpha_U(\lambda_U \mu_U) \beta_U(S_U T_U) \rho(\lambda \mu) \kappa LSJM\}.$$  \hspace{1cm} (6)

For our purposes, the SO(3)-coupled basis states, Eq. (5), are more convenient, but we will need to employ the following basis transformation to obtain the matrix elements of $Q^a \cdot Q^a = (Q^{an} + Q^{au}) \cdot (Q^{an} + Q^{au})$:

$$\langle (\lambda_1 \mu_1) \kappa_1 L_1 S_1 T_1; (\lambda_2 \mu_2) \kappa_2 L_2 S_2 T_2 | J \rangle = \sum U_{L_S} \left[ \begin{array}{ccc} L_1 & S_1 & J_1 \\ L_2 & S_2 & J_2 \end{array} \right] \sum_{\rho(\lambda \mu) \kappa} \langle (\lambda_1 \mu_1) \kappa_1 L_1; (\lambda_2 \mu_2) \kappa_2 L_2 || (\lambda \mu) \kappa L \rangle_{\rho}$$

$$\times \langle (\lambda_1 \mu_1) S_1 T_1; (\lambda_2 \mu_2) S_2 T_2 | \rho(\lambda \mu) \kappa L J \rangle$$

where $\langle \cdot; \cdot || \cdot \rangle_{\rho}$ is a SU(3) → SO(3) Wigner coupling coefficient with $\rho$ being the associated multiplicity label and $U(\cdot;\cdot;\cdot)$ denotes a SU(2) Jahn–Hope coefficient.

4. Quadrupole collectivity as a relevant measure

A proper (frame-independent) measure for the quadrupole collectivity of a nuclear system in an eigenstate $|s\rangle$ is the expectation value of $Q^a \cdot Q^a$ in that state: $\langle Q^a \cdot Q^a \rangle_s = \langle s | Q^a \cdot Q^a | s \rangle$. Under the usual assumption that the proton and neutron distributions track one another, this expectation value also measures the non-energy-weighted sumrule [31] for the reduced electric-quadrupole transition probability of the system in the eigenstate $|s\rangle$. Specifically, when the tracking assumption applies $Q^{au}_{2\mu} \propto Q^{an}_{2\mu}$, and from this one finds that the electric-quadrupole operator $Q^{E}_{2\mu}$ – which follows from the expression for the electric-multipoles transition operators in the long-wavelength approximation [32],

$$Q^{E}_{2\mu} = \sum_{\sigma} \sum_{i=1}^{n_i} \alpha \tau_{i}(i) Y_{2\mu}(\phi_{i})$$

where $b$ is the oscillator-length parameter and the $\alpha$ are proton ($\sigma = \pi$) and neutron ($\sigma = \nu$) effective charges – is proportional to $Q^{an}_{2\mu}$:

$$Q^{E}_{2\mu} = b^2 (e_{\pi} Q^{an}_{2\mu} + e_{\nu} Q^{an}_{2\mu}) \propto Q^{an}_{2\mu}.$$  \hspace{1cm} (9)

The constant in the last expression takes into account the fact that the matrix elements of $Q^c$ and $Q^a$ (Eq. 2) with $q^c \rightarrow q^a$) are identical within an oscillator shell, but $Q^c$ differs from $Q^a$ since it, like $q^c$, has non-vanishing matrix elements coupling different ($n' = n \pm 2$) major shells. So with the approximation $q^c \rightarrow q^a$, that is $Q^c \rightarrow Q^a$, comes the need $\chi^c \rightarrow \chi^a$, in the Hamiltonian $Q^a \cdot Q^a$ compared to $Q^c \cdot Q^c$. For example, for requiring the final $Q^{c}\rangle$, where $\langle \cdot | \cdot \rangle$ denotes the average matrix element

$$Q^{c}\rangle = \langle \langle Q^a \cdot Q^a \rangle \rangle^1/2.$$  \hspace{1cm} (10)

Note that the general expression for effective charges, $e_{\pi} = e + e_{\text{eff}}$ and $e_{\nu} = e - e_{\text{eff}}$, from the proton and neutron, $e_{\text{eff}} = \alpha \rho \kappa L J$ from suppressed (due to truncation effect). From this one can derive the well-known expression and reduced transition strength

$$\langle Q^a \cdot Q^a \rangle_{\alpha \beta} \alpha \langle Q^c \cdot Q^c \rangle_{\alpha \beta}$$

$$\propto \sum_{\alpha \beta} \sum_{J'(J+1)} (2J' + 1) (2J + 1)$$

Here the eigenstates are charactarized by $\alpha, \beta$ and $J'$ denote the coupling of the unique parity of the coupled nuclear.

For the ground-state, $J' = 1$, $J' = 2$ states for each nucleon, $J' = 1$, $J' = 2$ term dominating [3]. Hence $\langle \alpha | J' \rangle$ measures $|B(E2)\rangle$ strength and $\langle \alpha | J' \rangle$.

Since $Q = Q^{N} + Q^{U}$, where $Q^{N}$ is the basis function, the quadrupole couplings of the unique parity nuclei

$$\langle Q^c \cdot Q^c \rangle = \langle (Q^N + Q^U) \cdot Q^c \rangle$$

$$= \langle Q^{N} \cdot Q^{N} \rangle + \langle Q^{U} \cdot Q^{c} \rangle$$

(The superscript $c$, which distinguishes the collective counterpart from the quadrupole interaction, has been dropped for indices we will denote the quadrupole interaction $Q^c$ with the unique parity orbitals obvious probabilities which are too low. However, the quadrupole collectivity is re-estimated according to Eq. (10), we conclude an excellent measure for studying the unique parity nuclei, as well as coupled with those in the normal parity strengths.
that is $Q^c \rightarrow Q^a$, comes the need to introduce a new (stronger) coupling constant, $\chi^c \rightarrow \chi^a > \chi^c$, in the hamiltonian to compensate for the reduced strength of $Q^a$. $Q^a$ compared to $Q^c$. $Q^a$. An appropriate rescaling factor can be determined, for example, by requiring the following equivalence: $\chi^a \langle \langle Q^a \cdot Q^a \rangle \rangle = \chi^c \langle \langle Q^c \cdot Q^c \rangle \rangle$, where $\langle \langle \cdot \rangle \rangle$ denotes the trace, which apart from a dimension factor is just the average matrix element. Thus we find: const. = $(\chi^a/\chi^c)^{1/2} = (\langle \langle Q^a \cdot Q^a \rangle \rangle/\langle \langle Q^c \cdot Q^c \rangle \rangle)^{1/2}$.

Note that the general expression for $Q^E_{\mu \nu}$ given in Eq. (8) accommodates effective charges, $e_{\nu} = e + e_{\text{eff}}$ and $e_{\rho} = e_{\text{eff}}$. These are used in place of the real nucleon charges, $e_{\nu} = e$ and $e_{\rho} = 0$, to account approximately for contributions to $Q^E_{\mu \nu}$ from suppressed (due to truncation) core configurations (the core-polarization effect). From this one can derive a sum-rule result which relates quadrupole coupling strengths and reduced transition probabilities ($B(E2)$ values):

$$\langle Q^a \cdot Q^a \rangle_{aJ} \propto \langle Q^E_2 \cdot Q^E_2 \rangle_{aJ}$$

$$\propto \sum_{a'J'} \frac{(2J'+1)}{(2J+1)} B(E2, (aJ) \rightarrow (a'J')). \quad (10)$$

Here the eigenstates are characterized by their total angular momenta $J$ and $J'$, and $a$ and $a'$ denote further quantum numbers necessary to specify these states. For the ground state, $(aJ) = (1,0)$, of an even–even system the sum is only over $J' = 2$ states, and for all practical purposes this reduces to a few terms with the $(a'J') = (1, 2)$ term dominating [33,34]. Therefore the expectation value of $Q^a \cdot Q^a$ measures the $B(E2)$ strength as well as the quadrupole deformation.

Since $Q = Q^N + Q^U$, where $Q^N (Q^U)$ acts only on the normal (unique) part of the basis function, the quadrupole collectivity exhibits a dependence on the couplings of the unique parity nucleons with each other as well as with the normal parity nucleons:

$$\langle Q \cdot Q \rangle = \langle (Q^N + Q^U) \cdot (Q^N + Q^U) \rangle_s$$

$$= \langle Q^N \cdot Q^N \rangle_s + \langle Q^U \cdot Q^U \rangle_s + 2\langle Q^N \cdot Q^U \rangle_s. \quad (11)$$

(The superscript $a$, which distinguishes the algebraic quadrupole operator from its collective counterpart has been dropped here. Specifically, to avoid a proliferation of indices we will denote the algebraic quadrupole operator with $Q$ and the quadrupole interaction strength with $\chi$ from now on.) Neglecting the particles in the unique parity orbitals obviously leads to estimates for the reduced transition probabilities which are too low. Indeed, from this equation and from the fact that the quadrupole collectivity is related to the reduced $E2$ transition probabilities according to Eq. (10), we conclude that the expectation value of $Q \cdot Q$ provides an excellent measure for studying the influence that correlations between the unique parity nucleons, as well as couplings between particles in the unique parity space with those in the normal parity space, have on the electromagnetic transition strengths.
5. Influence of the unique parity particles

To obtain an estimate for the influence of the unique parity particles, recall that in Elliott’s SU(3) labeling scheme the quadrupole–quadrupole interaction $Q \cdot Q$ is diagonal and that its expectation value $\langle Q \cdot Q \rangle$ in a basis state depends solely on the SU(3) quantum labels $(\lambda \mu)$ and on the orbital angular momentum $L$ of the state:

$$\langle Q \cdot Q \rangle = 4C_2(\lambda \mu) - 3L(L+1),$$

(12)

where the second-order Casimir invariant $C_2$ of SU(3) is given by

$$C_2(\lambda \mu) = (\lambda + \mu)(\lambda + \mu + 3) - \lambda \mu.$$  

(13)

In the SO(3)-coupled basis, $Q^N \cdot Q^N$ and $Q^U \cdot Q^U$ are easily obtained, whereas $Q \cdot Q$ is diagonal in the SU(3)-coupled basis; hence the calculation of the total quadrupole collectivity of the system requires a basis transformation to the SU(3)-coupled form. Also note that $Q^N \cdot Q^U$ is not diagonal in either coupling scheme.

Each space is spanned by a set of basis functions with different $(\lambda \mu)$ and $L$ labels, thus the quadrupole collectivity of the system in a given eigenstate depends on the decomposition of the eigenstate into basis states. The expectation value of $Q \cdot Q$ is largest if the system is in the $L = 0$ eigenstate of $Q \cdot Q$ which maximizes $C_2(\lambda \mu)$. The $(\lambda \mu)$ of that state is referred to as the leading irreducible representation (irrep) of the Hilbert space under consideration, and will be denoted by $(\lambda \mu)_{\text{max}}$. Therefore, if we know the leading irrep of the Hilbert space of a nuclear system we may place an upper limit on the quadrupole collectivity which the system can display under the most favorable circumstances: $\langle Q \cdot Q \rangle_{\text{max}} = 4C_2((\lambda \mu)_{\text{max}})$. Given the leading irreps we can then determine the maximum contributions of the normal $(\langle Q^N \cdot Q^N \rangle_{\text{max}} = 4C_2((\lambda \mu_N)_{\text{max}}))$ and unique $(\langle Q^U \cdot Q^U \rangle_{\text{max}} = 4C_2((\lambda \mu_U)_{\text{max}}))$ parity spaces to the maximum possible total quadrupole collectivity $(\langle Q \cdot Q \rangle_{\text{max}})$ of the system. To evaluate $\langle Q \cdot Q \rangle_{\text{max}}$, the leading irrep $(\lambda \mu)_{\text{max}}$ of the system has to be determined in the SU(3)-coupled basis. It is not difficult to show that coupling two spaces with leading irreps $(\lambda \mu_N)_{\text{max}}$ and $(\lambda \mu_U)_{\text{max}}$ yields an SU(3)-coupled space with leading irrep $(\lambda \mu)_{\text{max}} = (\lambda_N + \lambda_U, \mu_N + \mu_U)$, and hence $\langle Q \cdot Q \rangle_{\text{max}}$ and thus $2\langle Q^N \cdot Q^U \rangle_{\text{max}}$ are easily obtained.

Table 4 lists the leading irreps of the normal and unique parity spaces and of the SU(3)-coupled product space for two representative rare-earth nuclei ($^{159}$Gd and $^{165}$Er) and two actinides ($^{238}$U and $^{242}$Pu). It also gives the Casimir invariants associated with those irreps, thus allowing an estimate of the relative importance of the $Q^N \cdot Q^N$, $Q^U \cdot Q^U$, and $Q^N \cdot Q^U$ interactions in determining the total quadrupole collectivity $\langle Q \cdot Q \rangle$. The ratios $R^A_T = \langle Q^A \cdot Q^A \rangle_{\text{max}}/\langle Q \cdot Q \rangle_{\text{max}}$ for $A = N$ and U and $R^N_T = 2\langle Q^N \cdot Q^U \rangle_{\text{max}}/\langle Q \cdot Q \rangle_{\text{max}}$ are listed in the last three columns of Table 4. We find that, on the average, these ratios are $R^N_T \approx 25\%$, $R^U_T \approx 30\%$, and $R^N_T \approx 45\%$. Hence, in the limit of weak spin–orbit and orbit–orbit forces and a strong quadrupole–quadrupole interaction (fixed particle numbers),
Table 4

Detailed characterization of the normal–unique coupled space for several representative rare-earth and actinide nuclei and for the $(\bar{d}t)^4@((fp)^2$ system. Listed are the leading irreps of the normal and unique parity subspaces, $(\lambda_{N\mu_N})_{\text{max}}$ and $(\lambda_{U\mu_U})_{\text{max}}$, and of the complete, coupled, Hilbert space, $(\lambda\mu)_{\text{max}}$, as well as the second-order SU(3) Casimir invariants associated with these irreps, $C_2[(\lambda_{N\mu_N})_{\text{max}}]$, $C_2[(\lambda_{U\mu_U})_{\text{max}}]$, and $C_2[(\lambda\mu)_{\text{max}}]$, and the expectation value of $\langle Q^N, Q^U \rangle / 2 = C_2[(\lambda_{N\mu_N})_{\text{max}}] - C_2[(\lambda_{N\mu_N})_{\text{max}}] - C_2[(\lambda_{U\mu_U})_{\text{max}}]$. Neutron and proton shells of the rare-earth and actinide examples are considered separately. The last three columns show the ratios $R^N_Q$, $R^U_Q$, and $R^{NU}_Q$, defined in the text, which provide a measure of the relative importance of the $Q^N, Q^U$, and $2Q^N, Q^U$ interactions in contributing to the total quadrupole collectivity $\langle Q^Q \rangle$.

<table>
<thead>
<tr>
<th>System</th>
<th>$\alpha$</th>
<th>$(\lambda_{N\mu_N})$</th>
<th>$(\lambda_{U\mu_U})$</th>
<th>$(\lambda\mu)$</th>
<th>$C_2[(\lambda_{N\mu_N})]$</th>
<th>$C_2[(\lambda_{U\mu_U})]$</th>
<th>$C_2[(\lambda\mu)]$</th>
<th>$\langle Q^N, Q^U \rangle / 2$</th>
<th>$R^N_Q$ (%)</th>
<th>$R^U_Q$ (%)</th>
<th>$R^{NU}_Q$ (%)</th>
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<tbody>
<tr>
<td>$^{153}$Gd</td>
<td>$\pi$</td>
<td>(10,4)</td>
<td>(18,0)</td>
<td>(28,4)</td>
<td>198</td>
<td>378</td>
<td>432</td>
<td>19.6</td>
<td>37.5</td>
<td>42.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu$</td>
<td>(18,4)</td>
<td>(16,2)</td>
<td>(34,6)</td>
<td>478</td>
<td>346</td>
<td>692</td>
<td>31.5</td>
<td>22.8</td>
<td>45.7</td>
<td></td>
</tr>
<tr>
<td>$^{156}$Er</td>
<td>$\pi$</td>
<td>(10,4)</td>
<td>(18,4)</td>
<td>(28,8)</td>
<td>198</td>
<td>478</td>
<td>504</td>
<td>16.8</td>
<td>40.5</td>
<td>42.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu$</td>
<td>(20,4)</td>
<td>(24,0)</td>
<td>(44,4)</td>
<td>568</td>
<td>648</td>
<td>1056</td>
<td>25.0</td>
<td>28.5</td>
<td>46.5</td>
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<tr>
<td>$^{238}$U</td>
<td>$\pi$</td>
<td>(18,0)</td>
<td>(16,2)</td>
<td>(34,2)</td>
<td>378</td>
<td>346</td>
<td>612</td>
<td>28.3</td>
<td>25.9</td>
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<tr>
<td></td>
<td>$\nu$</td>
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<td>(34,4)</td>
<td>(70,4)</td>
<td>1404</td>
<td>1422</td>
<td>2592</td>
<td>25.9</td>
<td>26.3</td>
<td>47.8</td>
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<tr>
<td>$^{242}$Pu</td>
<td>$\pi$</td>
<td>(18,0)</td>
<td>(24,0)</td>
<td>(42,0)</td>
<td>378</td>
<td>648</td>
<td>432</td>
<td>20.0</td>
<td>34.3</td>
<td>45.7</td>
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<tr>
<td></td>
<td>$\nu$</td>
<td>(34,6)</td>
<td>(34,4)</td>
<td>(68,10)</td>
<td>1516</td>
<td>1422</td>
<td>2700</td>
<td>25.2</td>
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</tbody>
</table>

$(\bar{d}t)^4@((fp)^2$ (4,2) (6,0) (10,2) 46 54 160 60 28.8 33.8 37.5
the contributions of the unique parity nucleons to the total quadrupole collectivity of the system dominate over the contributions arising from the normal-parity particle correlations.

If we consider the corresponding ratios for our model (four particles in the $\tilde{d}s$-shell and two in the fp-shell), we find similar values (see last row in Table 4): $R^N_1 = 29\%$, $R^U_1 = 34\%$, and $R^{NU}_1 = 37\%$. Furthermore, since $\langle Q^A \cdot Q^A \rangle$ is a measure of the deformation of the $A$-space, we conclude from the above comparison that the model approximately reproduces the relative deformations of the normal and unique parity spaces of heavy deformed nuclei.

The above comparisons of real nuclei and our model show that in a situation where SU(3) symmetry-breaking interactions are absent the unique parity particles are not negligible. The question that we are seeking to answer with the help of our model is whether the same holds true for realistic heavy deformed nuclei, namely, in circumstances where there is a strong spin–orbit force. Towards that end we define the ratios $r^A = \langle Q^A \cdot Q^A \rangle / \langle Q \cdot Q \rangle_{\max}$ for $A = N$ or $U$, $r = \langle Q \cdot Q \rangle / \langle Q \cdot Q \rangle_{\max}$, and $r^{NU} = 2 \langle Q^N \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\max}$, where $\langle Q^A \cdot Q^A \rangle$ denotes the expectation value of $Q^A \cdot Q^A$ for non-zero spin–orbit and orbit–orbit forces. We will investigate the behavior of $r$, $r^N$, $r^U$, and $r^{NU}$ as a function of the interaction parameters. This study clearly indicates that the particles in the unique parity band neither the SO(3) nor the SU(3) are eigenstates of $Q^N \cdot Q^U$. However, the help of the transformation between the SU(3)-coupled basis states and the angular momenta of the $(\tilde{d}s)^3 \otimes \text{fp}^2$ and the angular momenta of the $(\tilde{d}s)^3 \otimes \text{fp}^2$ and the angular momenta of the $(\tilde{d}s)^3 \otimes \text{fp}^2$.

Fig. 1. Quadrupole collectivities for the yrast state of $(\tilde{d}s)^3 \otimes \text{fp}^2$, $J = 0$, as a function of $\chi$ for $\Delta E_{1/2} = 1\ h\omega$, for (a) $G_U = 0$, (b) $G_U = 0.12\ MeV$, and (c) $G_U = 0.24\ MeV$. Plotted are $\langle Q \cdot Q \rangle / \langle Q \cdot Q \rangle_{\max}$, the total quadrupole collectivity normalized with respect to the maximum possible value of $Q \cdot Q$ in the coupled space ($\langle Q \cdot Q \rangle_{\max} = 4 \langle Q^N \rangle_{\max} = 640$), as well as the contributions arising from the normal parity nucleons, $\langle Q^N \cdot Q^N \rangle / \langle Q \cdot Q \rangle_{\max}$, the unique parity nucleons, $\langle Q^U \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\max}$, and the correlations between particles which occupy orbitals of opposite parities, $2 \langle Q^N \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\max}$. For comparison we have indicated the expectation values of $Q \cdot Q$, $Q^N \cdot Q^N$, $Q^U \cdot Q^U$, and $2Q^N \cdot Q^U$ in the state $|\xi_{1/2} \rangle = |N; J_\pi \rangle (|f_{1/2} \rangle F, \nu = 0)^{J_{1/2} - \nu}$ in the right margins.
parameters. This study clearly involves the full model space since $Q^N \cdot Q^U$ couples the particles in the unique parity levels to those in the normal parity orbitals; neither the SO(3)- nor the SU(3)-coupled product functions (see Eqs. (5) and (6)) are eigenstates of $Q^N \cdot Q^U$. However, Eq. (11) allows us to calculate $r_{NU}$ with the help of the transformation between the angular-momentum-coupled basis functions and the SU(3)-coupled basis states (see Eq. (7)). To investigate how the $r$ values depend on the interaction parameters, we consider the full model space $((ds)^1 \otimes (fp)^2)$ and take the angular-momentum-coupled functions (5) as our basis.
6. Results of the calculation

Figs. 1a, b, and c show the quadrupole collectivity of the lowest eigenstate, \(|\Psi_0\rangle = (|N; J_N\rangle |U; J_U\rangle)^{JM}\), for \(\Delta E_{1/2} = 1\ h\omega\), \(G_U = 0, 0.12\ MeV,\) and \(0.24\ MeV\), respectively, with \(\chi\) ranging from 0 to 0.25 MeV. The results plotted are \(\langle Q \cdot Q \rangle / \langle Q \cdot Q \rangle_{\text{max}}\), the total quadrupole collectivity normalized with respect to the maximum possible value of \(Q \cdot Q^\ast\) in the coupled space \((Q \cdot Q^\ast)_{\text{max}} = 4C_2[(10, 2)] = 640\), as well as the individual contributions arising from the normal parity nucleons, \(\langle Q^N \cdot Q^N \rangle / \langle Q \cdot Q \rangle_{\text{max}}\), the unique parity nucleons, \(\langle Q^U \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\text{max}}\), and the correlations between the two sets, \(2\langle Q^N \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\text{max}}\). As expected, the total quadrupole collectivity rises with increasing \(\chi\); for \(G_U = 0\) the coupled \((2s)^4 \otimes (fp)^2\) system displays about 61\% of the maximum possible quadrupole collectivity at small \(\chi\) values, rises to 67\% for \(\chi = 0.05\ MeV,\) and reaches 75\% for \(\chi = 0.1\ MeV.\) If we now introduce a weak \((G_U = 0.12\ MeV)\) or moderate \((G_U = 0.24\ MeV)\) pairing interaction in the unique parity space, we find that this lowers the quadrupole collectivity at small \(\chi\) values, but for realistic interaction parameters, pairing has little influence on the expectation value of \(Q \cdot Q^\ast\), as can be seen from Figs. 1b and c. In the asymptotic limit, \(\chi / \Delta E_{1/2} \rightarrow \infty\), we have for all three cases, \(\langle Q \cdot Q \rangle / \langle Q \cdot Q \rangle_{\text{max}} \rightarrow 100\%\), as expected.

Besides the dependence of the total quadrupole collectivity of the \((2s)^4 \otimes (fp)^2\) system on the interaction strength \(\chi\), Figs. 1a, b, and c also show the relative importance of the correlations among the normal parity nucleons, the unique parity nucleons, and the couplings between particles differing in parity in determining the total quadrupole collectivity of a nucleus: For \(G_U = 0\) and small \(\chi\) values we find \(\langle Q^N \cdot Q^N \rangle / \langle Q \cdot Q \rangle_{\text{max}} = 27\%\), \(\langle Q^U \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\text{max}} = 15\%\), and \(2\langle Q^N \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\text{max}} = 19\%\). With increasing interaction strength \(\chi\) these ratios become 27\%, 17\%, 23\% for \(\chi = 0.05\ MeV,\) and 27\%, 19\%, 29\% for \(\chi = 0.1\ MeV.\) The introduction of a pairing force in the unique parity subspace barely changes the results as long as \(\chi > 0.03\ MeV,\) but for small \(\chi\) values the pairing correlations affect the expectation value of \(2Q^N \cdot Q^U\) significantly: for \(\chi \rightarrow 0,\) it drops from \(2\langle Q^N \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\text{max}} = 19\%\) for \(G_U = 0\) to zero for \(G_U = 0.12\ MeV\) and \(G_U = 0.24\ MeV.\) We furthermore observe that the contribution from the normal parity space remains approximately constant over the whole range of \(\chi.\) This behavior is to be expected since the normal parity levels of the system under consideration are degenerate, hence the increasing quadrupole–quadrupole force between normal parity nucleons can only affect the energy contribution of the normal parity space, but the expectation value of \(Q^N \cdot Q^N\) will remain saturated at its maximum value \((\langle Q^N \cdot Q^N \rangle = \langle Q^N \cdot Q^N \rangle_{\text{max}} = 4C_2[(4, 2)] = 184).\) Small deviations from the saturation value are due to the fact that for non-zero \(\Delta E_{1/2}\) the normal parity part of the yrast state is not pure \(L_N = 0.\)

For comparative purposes we have also included – in the right margins of Figs. 1a, b, and c – the expectation values of \(Q \cdot Q, Q^N \cdot Q^N, Q^U \cdot Q^U,\) and \(2Q^N \cdot Q^U\) in \(|\xi_0\rangle = |N; J_N \rangle |(f_{7/2})\rangle^2, \nu = 0\rangle \rangle,\) that is, the configuration with four identical nucleons in the degenerate \(ds\)-shell and two nucleons in the \(f_{7/2}\)-orbital coupled to seniority zero, normalized with respect to \(\langle Q \cdot Q \rangle_{\text{max}} = 4C_2[(10, 2)] = 640.\) We find

that \(\langle Q \cdot Q \rangle_{\text{max}} / \langle Q \cdot Q \rangle_{\text{max}} \approx 4\%\) of \(\langle Q^N \cdot Q^N \rangle_{\text{max}} \approx 17\%\), and \(2\langle Q^N \cdot Q^U \rangle_{\text{max}} \approx 17\%\) of \(\langle Q^N \cdot Q^N \rangle_{\text{max}} \approx 17\%\), and 2 parity components of \(|\xi_0\rangle\) and collectivity and, for realistic \(\langle Q^N \cdot Q^N \rangle_{\text{max}} \approx 17\%\), and 2 parity components of \(|\xi_0\rangle\) and collectivity and, for realistic

These results suggest that the normal parity subspace or states coupled to a unique parity sesionic theory compensates explicitly for their interaction with the norm renormalization would imply contributions to the total quadrupole \(\langle Q \cdot Q \rangle_{\text{max}}\) moments and \(B(E2)\) words, when evaluating deformations complete normal–unique coupled Hilbert space, one must scale the from correlations that cannot occur.

Note that the seniority-zero c able amount of deformation, \(\langle Q^N \cdot Q^N \rangle_{\text{max}} = 51\%\), where \(\langle Q^U \cdot Q^U \rangle,\) unique parity component of our \(|(f_{7/2})^2, \nu = 0\rangle \rangle\) yields a significan (and hence deformation) of the rough first approximation to the presented in Fig. 2 where the over the yrast state, \(|\Psi_0\rangle = |N; J_N \rangle \rangle,\) figure is shown. This quantity is tion strength \(\chi\) for \(\Delta E_{1/2} = 1 h\omega\); in the figure, the overlap increase space become stronger, especially strength; as the quadrupole force find that for realistic interaction overlap \(|\Psi_0\rangle \langle \xi_0\rangle\) is between 60\% mixing of the \(j = 7/2\) intruder – major oscillator shell – the state reasonably well.

Taking into account unique par course improve the approximation the upper three curves in Fig. 2 \(|N; J_N \rangle |(f_{7/2})^2, \nu, J_U \rangle \rangle, \) that identical nucleons in the degenerate coupled to seniority \(\nu = 0\) or 2, 4.
of the lowest eigenstate, 0.12 MeV, and 0.24 MeV, results plotted are \( \langle Q \cdot Q \rangle \) with respect to the \( \langle Q \cdot Q \rangle_{\text{max}} = 4 C_2 (10, 2) \) from the normal parity nucleons, \( \langle Q^U \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\text{max}} = 15\% \). As easing \( \chi \) for \( G_U = 0 \) the maximum possible for \( \chi = 0.05 \) MeV, and peak \( G_U = 0.12 \) MeV or quasi-parity space, we find values, but for realistic \( \chi \), the expectation value of \( \chi \) limit, \( \chi / \Delta \epsilon_{7/2} \rightarrow \infty \), we expected.

Activity of \((\tilde{d}s)^4 \otimes (f^2p)^2\)
also show the relative occupancy, the unique orbitals of differing parity as \( \langle f_{7/2} \rangle^2 \), \( \langle f_{7/2} \rangle^2 \) = 0\], \( \langle Q^U \cdot Q^U \rangle \) with quasi-parity space barely any values the pairing significantly; for \( \chi \rightarrow 0 \), it zero for \( G_U = 0.12 \) MeV c contribution from the whole range of \( \chi \) of the system under quasiparticle-quasiparticle force energy contribution of the will remain saturated at \( \chi = 0.2 \) (2) = 184. Small deviation for non-zero \( \Delta \epsilon_{7/2} \) the right margins of Figs. \( U \cdot Q^U \) and \( 2Q^N \cdot Q^U \) in addition with four identical the \( f_{7/2} \) orbital coupled to \( \big| \langle (10, 2) \rangle = 640 \). We find that \( \langle Q \cdot Q \rangle \) \( \epsilon_0 \rangle / \langle Q \cdot Q \rangle_{\text{max}} \approx 46\% \), \( \langle Q^N \cdot Q^N \rangle \) \( \epsilon_0 \rangle / \langle Q \cdot Q \rangle_{\text{max}} \approx 29\% \), \( \langle Q^U \cdot Q^U \rangle \) \( \epsilon_0 \rangle / \langle Q \cdot Q \rangle_{\text{max}} \approx 17\% \), and \( 2 \langle Q^N \cdot Q^U \rangle \) \( \epsilon_0 \rangle / \langle Q \cdot Q \rangle_{\text{max}} = 0 \), that is, the normal parity components of \( \big| \xi_0 \rangle \) and \( \big| \psi_0 \rangle \) exhibit the same amount of quadrupole collectivity and, for realistic interaction parameters, the unique parity component of \( \big| \xi_0 \rangle \) displays nearly as much deformation as that of \( \big| \psi_0 \rangle \). However, the total quadrupole collectivities of \( \big| \xi_0 \rangle \) and \( \big| \psi_0 \rangle \) differ significantly since \( \big| \psi_0 \rangle \) shows a non-negligible contribution from the cross-coupling term \( 2Q^N \cdot Q^U \) to the total deformation, whereas for \( \big| \xi_0 \rangle \) it is exactly zero because of the \( v = 0 \) restriction.

These results suggest that truncating the normal–unique coupled space to its normal parity subspace or to states which consist of a normal parity component coupled to a unique parity seniority-zero configuration is not justified unless the theory compensates explicitly for the influence of the intruder particles and/or their interaction with the normal parity nucleons. To proceed without proper renormalization would imply that roughly two thirds/one third of the contributions to the total quadrupole collectivity (and hence large contributions to the quadrupole moments and BE(2) strengths of the system) are neglected. In other words, when evaluating deformation-dependent quantities one must work with the complete normal–unique coupled Hilbert space, or when forced to truncate the Hilbert space, one must scale the results by a factor that accounts for contributions from correlations that cannot occur in the truncated space.

Note that the seniority-zero configuration \( \big| f_{7/2} \rangle^2 \), \( v = 0 \) displays a considerable amount of deformation, \( \langle f_{7/2} \rangle^2 \), \( v = 0 \) \( \langle Q^U \cdot Q^U \rangle_{\text{max}} = 15\% \), where \( \langle Q^U \cdot Q^U \rangle_{\text{max}} = 51\% \), where \( \langle Q^U \cdot Q^U \rangle_{\text{max}} = 4C_2 [\lambda U \cdot \mu U]_{\text{max}} \) = 216; when truncating the unique parity component of our model space to this state only, one finds that \( \langle f_{7/2} \rangle^2 \), \( v = 0 \) yields a significant contribution to the total quadrupole collectivity (and hence deformation) of the system. Indeed, the state \( \big| \xi_0 \rangle \) may serve as a rough first approximation to the calculated yrast state of the model, as is demonstrated in Fig. 2 where the overlap of \( \big| \xi_0 \rangle \rangle = \big| \langle 1 N; J_N \rangle \langle f_{7/2} \rangle^2 \rangle \big| 0 \rangle \rangle \) with the yrast state, \( \big| \psi_0 \rangle = \big| \langle 1 N; J_N \rangle \langle U \rangle \big| 0 \rangle \rangle \) \( \big| 0 \rangle \rangle \) (see the lower three curves in the figure) is shown. This quantity is plotted as a function of the quadrupole interaction strength \( \chi \) for \( \Delta \epsilon_{7/2} = 1 \) h\( \omega \) and \( G_U = 0, 0.12 \) MeV, and 0.24 MeV. As shown in the figure, the overlap increases as the pairing contributions in the unique parity space become stronger, especially for small values of the quadrupole interaction strength; as the quadrupole force grows in strength, the overlap diminishes. We find that for realistic interaction parameters \( \Delta \epsilon_{7/2} = 1 \) h\( \omega \), \( \chi = 0.1 \) MeV the overlap \( \langle \psi_0 \rangle \rangle \) is between 60% and 70%, thus implying that – though there is mixing of the \( j^U = 7/2 \) intruder orbital with its original partners from the same major oscillator shell – the state \( \big| \xi_0 \rangle \) approximates the calculated yrast state reasonably well.

Taking into account unique parity configurations with seniority \( v > 0 \) will of course improve the approximation, as can be seen (for the case in question) from the upper three curves in Fig. 2. Plotted is the overlap of \( \big| \psi_0 \rangle \rangle \) with \( \big| \delta_0 \rangle \rangle = \big| \langle 1 N; J_N \rangle \langle f_{7/2} \rangle^2 \rangle \big| v \rangle \rangle \) \( \big| 0 \rangle \rangle \), that is, the yrast state of the Hilbert space of two identical nucleons in the degenerate \( \tilde{d}s\)-shell and two nucleons in the \( f_{7/2} \) orbital coupled to seniority \( v = 0 \) or 2, for \( \Delta \epsilon_{7/2} = 1 \) h\( \omega \), \( G_U = 0, 0.12 \) MeV, and 0.24
Fig. 2. The lower three curves show the overlap of \( |\Psi_0\rangle = |N; J_0\rangle ((f_{7/2})^2, v = 0) \) with \( |\delta_0\rangle \) for three values of \( G_\perp \): 0, 0.12 MeV, and 0.24 MeV. The upper three curves display the overlap of \( |\Psi_0\rangle = |N; J_0\rangle ((f_{7/2})^2, v, J_0) \) with \( |\delta_0\rangle \) for the same interaction parameters.

MeV, and \( \chi \) ranging from 0 to 0.25 MeV. We find that the overlap \( \langle \Psi_0 | \delta_0 \rangle \), which turns out to be independent of the strength of the pairing correlations in the unique parity \( f_{7/2} \) space, is very close to 100% for small values of \( \chi (\chi \leq 0.03 \) MeV), and lies at about 90% for realistic interaction parameters (\( \chi \approx 0.1 \) MeV), thus indicating that the state \( |\delta_0\rangle \) yields a very good approximation to the calculated ground state of the full \((ds)^4 \otimes (fp)^2\) system. These findings are further underscored by the results shown in Fig. 3. Displayed are the expectation values of \( Q \cdot Q, Q^N \cdot Q^N, Q^U \cdot Q^U \), and \( 2Q^N \cdot Q^U \) in the yrast state of the \((ds)^4 \otimes (f_{7/2})^2\) space, for \( \Delta E_{7/2} = 1 \) h\( \omega \), \( G_U = 0.12 \) MeV and 0.24 MeV, as a function of the quadrupole interaction strength. We see that for \( G_U > 0 \) and small \( \chi \) values (\( \chi \leq 0.03 \) MeV) the quadrupole collectivity of the state \( |\delta_0\rangle \) displays a behavior like that of \(|\Psi_0\rangle\), which is to be expected since \( \langle \Psi_0 | \delta_0 \rangle \approx 100\%\) for these parameter values. As \( \chi \) increases, the quadrupole collectivities saturate (for about \( \chi = 0.05 \) MeV) at \( \langle Q \cdot Q \rangle_{\delta_0} / \langle Q \cdot Q \rangle_{\max} = 61\%\), \( \langle Q^N \cdot Q^N \rangle_{\delta_0} / \langle Q \cdot Q \rangle_{\max} = 27\%\), \( \langle Q^U \cdot Q^U \rangle_{\delta_0} / \langle Q \cdot Q \rangle_{\max} = 15\%\), and \( 2\langle Q^N \cdot Q^U \rangle_{\delta_0} / \langle Q \cdot Q \rangle_{\max} = 19\%\). Note that \( \langle Q^U \cdot Q^U \rangle_{\delta_0} / \langle Q \cdot Q \rangle_{\max} \) is nearly as large as \( \langle Q^N \cdot Q^U \rangle_{\delta_0} / \langle Q \cdot Q \rangle_{\max} \) and that \( 2\langle Q^N \cdot Q^U \rangle_{\delta_0} / \langle Q \cdot Q \rangle_{\max} \) does not – like in the seniority-zero case – vanish, but contributes significantly to the total quadrupole collectivity.

We conclude that the quadrupole collectivities are independent of \( \chi \): \( \langle Q \cdot Q \rangle_{\delta_0} / \langle Q \cdot Q \rangle_{\max} \approx 27\%\), \( \langle Q^N \cdot Q^N \rangle_{\delta_0} / \langle Q \cdot Q \rangle_{\max} \approx 27\%\), and \( 2\langle Q^N \cdot Q^U \rangle_{\delta_0} / \langle Q \cdot Q \rangle_{\max} \approx 19\%\), hence conclude that the state \( |\Psi_0\rangle \) is a good eigenstate of the full system, \( |\delta_0\rangle \), and actual structure of \( |\delta_0\rangle \).

Even if one takes into account the nuclear levels and further restricts the configurations only, one is often to be handled with modern computer programs with deformation-dependent objectives, more radical truncation and a coupled Hilbert space by incorporating the zeroth factor to compensate for what will be considered the validity of this.

Truncating the Hilbert space obtained from calculations performed on the physics of the full system is by the truncation scheme. While phenomena like backbending might appear like unique parity levels, it may be necessary to superpose nuclear levels in the calculation of nuclear properties like transition rates.

For the truncation and subspaces, the ratio of the contributions from the quadrupoles \( (Q^U \cdot Q^U) \) and their couplings to have to vary smoothly with the parameters to remain more or less independent. Our study shows that the first ratio will be well approximated as \( 2\langle Q^N \cdot Q^U \rangle \) and its derivative address the issue of the angular-momentum ratio \( t = (Q^N \cdot Q^U) + 2\langle Q^N \cdot Q^U \rangle \) as a function of \( \chi \) for non-vanishing \( \Delta E_{7/2} = 1 \) h\( \omega \), and for the low momentum \( J = 0, 2, 4 \) (see Fig. 4). The graph, was averaged over the nearly degenerate for most \( \chi \) values.

We find that the \( t \) measures slightly are approximately equal over the range of \( J = 6 \), slightly smaller than the range as the above-mentioned level crossings.
significantly to the total quadrupole collectivity of the system. (For \( G_U = 0 \) we find that the quadrupole collectivities of the subspaces as well as the total collectivity are independent of \( \chi \): \( \langle Q^2 \rangle_{\Omega_0} / \langle D_{Q} \rangle_{\Omega_0} = 51 \%, \langle Q^N \cdot Q^N \rangle_{\Omega_0} / \langle D_{Q} \rangle_{\Omega_0} = 27\%, \langle Q^U \cdot Q^U \rangle_{\Omega_0} / \langle D_{Q} \rangle_{\Omega_0} = 15\%, \) and \( 2 \langle Q^N \cdot Q^U \rangle_{\Omega_0} / \langle D_{Q} \rangle_{\Omega_0} = 19\%. \) We hence conclude that the state \(| \Omega_0 \rangle \) gives a reasonable approximation to the lowest eigenstate of the full system, \(| \Psi_0 \rangle \); and furthermore, \(| \Omega_0 \rangle \) is very close to the actual structure of \(| \Psi_0 \rangle \).

Even if one takes into account only the largest-\( j \) orbital of the unique parity levels and further restricts the unique parity subspace to low (or zero) seniority configurations only, one is often faced with a Hilbert space which is still too large to be handled with modern computers. Since many applications are only concerned with deformation-dependent observables, it is worthwhile to focus on an alternative, more radical, truncation scheme, namely one which replaces the total, coupled Hilbert space by its normal parity subspace only and introduces a rescaling factor to compensate for what is left out in such an analysis. In what follows we will consider the validity of this approach.

Truncating the Hilbert space of a system – and rescaling the results that are obtained from calculations performed in the restricted space – is only justified if the physics of the full system is properly represented in the subspace determined by the truncation scheme. While it is not possible, for example, to account for phenomena like backbending without explicitly including the particles in the unique parity levels, it may be admissible to reduce the valence space of the nucleus in question to the normal parity orbitals when determining collective nuclear properties like transition strengths and multipole moments.

For the truncation and subsequent rescaling to be physically meaningful in our case, the ratio of the contributions from the normal parity space \( \langle Q^N \cdot Q^N \rangle \) to those that arise from the quadrupole correlations among the unique parity nucleons \( \langle Q^U \cdot Q^U \rangle \) and their couplings to the normal parity particles \( 2 \langle Q^N \cdot Q^U \rangle \) has to vary smoothly with the interaction parameters, and furthermore, they must remain more or less independent of the total angular momentum of the system. Our study shows that the first requirement is met, since \( \langle Q^N \cdot Q^N \rangle, \langle Q^U \cdot Q^U \rangle \) as well as \( 2 \langle Q^N \cdot Q^U \rangle \) vary smoothly with \( \chi \) as can be seen in Fig. 1. In order to address the issue of the angular-momentum dependence of the results, we plotted the ratio

\[
t = \frac{\langle Q^U \cdot Q^U \rangle + 2 \langle Q^N \cdot Q^U \rangle}{\langle Q^N \cdot Q^N \rangle}
\]

as a function of \( \chi \) for non-vanishing spin–orbit and orbit–orbit interactions, \( \Delta \chi_j = 1 \hbar \omega \), and for the lowest eigenstate of the system with total angular momentum \( J = 0, 2, 4 \) (see Fig. 4). For \( J = 6 \) the \( t \) ratio, which is also indicated in the graph, was averaged over the three lowest eigenstates, since these states are nearly degenerate for most \( \chi \) values and, as a result, exhibit some level crossings.

We find that the \( t \) measures show a very smooth behavior, and that their values are approximately equal over the whole range of \( \chi \) values. (The fact that \( t \) for \( J = 6 \) is slightly smaller than the relevant ratios for lower angular momenta, as well as the above-mentioned level crossings can be understood from the restricted size
Fig. 3. Quadrupole collectivities for the yrast state |0\rangle of the (\bar{d}d)^*\otimes(f_{7/2})^2 system, J = 0, as a function of \chi for \Delta\varepsilon_{7/2} = 1 \hbar\omega and for (a) G_U = 0.12 MeV and (b) G_U = 0.24 MeV. Plotted are \langle Q \cdot Q \rangle / \langle Q \cdot Q \rangle_{\text{max}}, the total quadrupole collectivity normalized with respect to \langle Q \cdot Q \rangle_{\text{max}} \equiv 4C_m(10,2) = 640, as well as the contributions arising from the normal parity nucleons, \langle Q^N \cdot Q^N \rangle / \langle Q \cdot Q \rangle_{\text{max}}, the unique parity nucleons, \langle Q^U \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\text{max}}, and the correlations between particles which occupy orbitals of opposite parities, 2\langle Q^N \cdot Q^U \rangle / \langle Q \cdot Q \rangle_{\text{max}}.

Fig. 4. Ratio of the contributions arising for normal parity particles and their couplings to the normal parity subspace only. The ratio of the contribution of the state of (\bar{d}d)^*\otimes(f_{7/2})^2 as a function of \chi for G_U.

Note that there exists a maximal L value for the system. This is due to the fact that SU(3) is a consequence, all rotational bands in the rigid-rotor model predict a simple L(L + 1) with angular momentum, whereas the SU(3) LLL(1) behavior of the energies and a fall greater than about \frac{1}{2} L_{\text{max}}. It is this L \leq \frac{1}{2} L_{\text{max}} that is regarded as "low": In our example, we have L values up to L = 4 may be considered low-energy structure of the system we are dealing with. Hence it translates to J \leq 4. And indeed the states with J > 4 deviates from the behavior of the model space.7.) From this we see that the collectivity quantities reflect the behavior of the model.
of the model space\(^7\). From this we conclude that the nucleons in the normal parity orbitals reflect the behavior of the complete normal–unique coupled system, as far as collective properties are concerned. It thus follows that when evaluating collective quantities such as \(\langle Q \cdot Q \rangle\), it suffices to truncate the Hilbert space to its normal parity subspace, provided an appropriate scaling factor is introduced in order to account for the contributions that arise due to the presence of the unique parity nucleons.

\(^7\)Note that there exists a maximal \(L\) value within each SU(3) irrep \((\lambda \mu)\), namely \(L_{\text{max}} = (\lambda + \mu)\). This is due to the fact that SU(3) is a compact group with finite-dimensional representations. As a consequence, all rotational bands in the model terminate, a result that stands in contrast to the collective model in which the bands extend indefinitely. Furthermore, for prolate nuclei the collective rigid-rotor model predicts a simple \(L(L+1)\) energy rule and \(B(E2)\) values that increase monotonously with angular momentum, whereas the SU(3) description is known to show deviations from the simple \(L(L+1)\) behavior of the energies and a fall-off in predicted \(B(E2)\) transition strengths for \(L\) values greater than about \( \frac{1}{2} L_{\text{max}}\). It is this \(L \leq \frac{3}{2} L_{\text{max}}\) condition that defines which angular momenta can be regarded as “low”: in our example, we have \((\lambda \mu)_{\text{max}} = (10, 2)\) for the normal–unique coupled space, hence \(L\) values up to \(L = 4\) may be considered sufficiently small. Since we are interested in the low-energy structure of the system we are primarily concerned with \(S = 0\) states, thus the \(L \leq 4\) condition translates into \(J \leq 4\). And indeed we see that the behavior of the quadrupole collectivity for states with \(J > 4\) deviates from the behavior of the low-angular-momentum configurations.
For practical purposes one can obtain an estimate for the scaling factor by considering for the \( J = 0 \) case the asymptotic limit \((x \to \infty)\) for the normal parity subspace, where the leading SU(3) representation \((\lambda_N \mu_N)_{\text{max}}\) yields good quantum numbers, and for the complete, SU(3)-coupled Hilbert space, where the leading irrep \((\lambda \mu)_{\text{max}}\) gives the proper quantum labels. The “asymptotic” scaling factor for \( J = L = 0 \) can be written as

\[
s^n_{QQ} = \frac{\langle Q \cdot Q \rangle_{\text{max}}}{\langle Q^N \cdot Q^N \rangle_{\text{max}}} = \frac{C_2[(\lambda \mu)_{\text{max}}]}{C_2[(\lambda_N \mu_N)_{\text{max}}]}.
\]

(Note: Since we are interested in the low-lying states of even–even nuclei, we consider \( S = 0 \), that is, \( J = L \) configurations only.) As we have seen, the introduction of a non-zero spin–orbit force in the unique parity space lowers the expectation values of \( \langle Q \cdot Q \rangle \) to \( p \langle Q \cdot Q \rangle_{\text{max}} \), where \( p \approx 0.70 \) \((\langle Q \cdot Q \rangle)\) is \( 70\% \) of \( \langle Q \cdot Q \rangle_{\text{max}} \) for realistic interaction parameters (and for \( J = 0, 2, 4, \) and 6), whereas \( \langle Q^N \cdot Q^N \rangle \) remains saturated at \( \langle Q^N \cdot Q^N \rangle_{\text{max}} \). (Small deviations from the saturation value are due to the fact that for non-zero \( \Delta \varepsilon_{7/2} \) the normal parity part of the yrast state is not a pure \( L_N = 0 \) state.) Hence the scaling factor that has to be applied to collective quantities that are obtained from calculations performed in a Hilbert space which is restricted to its normal parity levels is given as \( s_{QQ} = p s^0_{QQ} \), with \( p \approx 0.70 \). For example, for the case of \((ds)^8 \otimes (fp)^2\) we have \( (\lambda_N \mu_N)_{\text{max}} = (4, 2), \) \( (\lambda_L \mu_L)_{\text{max}} = (6, 0), \) and \( (\lambda \mu)_{\text{max}} = (10, 2) \), hence \( C_2[(\lambda \mu)_{\text{max}}] = 160 \) and \( C_2[(\lambda_N \mu_N)_{\text{max}}] = 46 \). Therefore \( s_{QQ}^0 = 3.48 \), and hence \( s_{QQ} = 2.4 \).

Having shown that the relative contributions from correlations among the normal and unique parity particles as well as from the coupling between the two subspaces varies smoothly with the interaction parameters of our theory and that the \( t \) measure defined in Eq. (14) is nearly independent of the total angular momentum of the system, we conclude that the normal parity particles are sufficient to represent the full set of valence nucleons when describing “collective” properties of low-lying states like electromagnetic transitions and multipole moments in the pseudo-SU(3) scheme, provided the results are renormalized appropriately. For realistic heavy deformed nuclei, we expect the quadrupole correlations, and hence the sum over the \( B(E2) \) transition strengths (see Eq. (10)) to scale like \( s_{QQ}^0 = p s^0_{QQ} \), with \( p \approx 0.70 \). The \( B(E2) \) strength of the lowest transition occurring in this summation is known to dominate the sum (typically contributing more than 75\% for reasonably well-deformed nuclei), thus a calculation of the strength of this particular transition based on the normal parity subspace only will require rescaling of the \( B(E2) \) result by the same simple factor. Quadrupole moments may be adjusted similarly by employing a renormalization factor that is roughly the square root of \( s_{QQ}^0 \). If we utilize these results in the symplectic/pseudo-symplectic model, thereby taking into account multi-shell correlations as well as intruder levels, a reproduction of \( B(E2) \) values can be achieved without the use of effective charges [35].

7. Summary and conclusions

The purpose of the present study is to explore the intruder levels of heavy nuclei, and in particular, to account for their effect through moments and electromagnetic transitions. The extension, was constructed to include the function of deformation the core and orbit–orbit forces on the quadrupole interaction on the interaction on the results.

Our study was designed to investigate the role of the proton and neutron valence particles in the relative sizes and fraction of the spaces, while at the same time not neglect the space that was selected consistently as a unique parity complement. In order to avoid truncating the model particles in low oscillator shells of each subspace (4 and 2 respective interaction hamiltonian) by coupling angular momentum \( (J = 0) \) which was done for \( J = 2, 4 \), and the angular-momentum independent collectivity of the combined system orbit–orbit, quadrupole–quadrupole interactions.

The study shows that there are specific patterns in the system arising from the presence of particles in the unique parity space addition, interactions between \( t \) contributions to the total quadrupole moments we conclude that it is incorrect to consider the minor role in the dynamics of the system. It is appropriate to assign to the intrinsic deformation of a nucleus. Our study and the unique parity subspaces up the collectivity (in roughly equal proportions) in the system. It is not normal and unique, as well as in important in determining the final results.

The good news is that a proper correlations allows one to make...
7. Summary and conclusions

The purpose of the present study was to examine the extent to which particles in the intruder levels of heavy deformed nuclei contribute to the dynamics of the nucleus, and in particular, to see whether or not a truncated space allows one to account for their effect through scaling when calculating quantities like quadrupole moments and electromagnetic transition strengths. Towards that end, a model Hilbert space, based on the Elliott SU(3)-coupling scheme and its pseudo-SU(3) extension, was constructed to investigate in a truncation-free environment and as a function of deformation the competition between the symmetry-breaking spin–orbit and orbit–orbit forces on the one hand and the symmetry-preserving quadrupole–quadrupole interaction on the other, as well as the influence of the pairing interaction on the results.

Our study was designed to replicate the most important characteristic features of the proton and neutron valence spaces of typical heavy deformed nuclei, such as the relative sizes and fractional occupancies of the normal and unique parity spaces, while at the same time remaining a completely solvable theory. The Hilbert space that was selected consists of one normal parity shell and an associated unique parity complement. In order to be able to perform a complete calculation without truncating the model space we restricted our consideration to a few particles in low oscillator shells $((\tilde{d}s)^4 \otimes (\tilde{f}p)^2)$, fixed the number of particles in each subspace (4 and 2, respectively, with no pair-scattering terms included in the interaction Hamiltonian), coupled the many-particle wavefunctions to low total angular momentum ($J = 0$ for everything but a consideration of scaling factors, which was done for $J = 2, 4,$ and 6 in addition to $J = 0$ to demonstrate explicitly the angular-momentum independence of the results), and studied the quadrupole collectivity of the combined system as a function of the strengths of the spin–orbit, orbit–orbit, quadrupole–quadrupole, and (in the $J = 0$ case) the pairing interactions.

The study shows that there are indeed significant contributions to the dynamics of the system arising from the presence of the unique parity nucleons. Correlations among particles in the unique parity orbitals turn out to be non-negligible and, in addition, interactions between the normal and unique parity spaces yield large contributions to the total quadrupole collectivity of the system. From these results we conclude that it is incorrect to assume that the unique parity particles play a minor role in the dynamics of a deformed nuclear system; neither is it appropriate to assign to the intruder nucleons the leading role for inducing deformation in a nucleus. Our study suggests that correlations within the normal and the unique parity subspaces, together with couplings between the two, build up the collectivity (in roughly equal proportions) and all contribute to the deformation of the system. It is not a matter of which is the most significant—the normal and unique, as well as the normal–unique cross-coupling terms, are all important in determining the final deformation.

The good news is that a proper understanding of the importance of these correlations allows one to make realistic estimates for their relative influence on...
measurable quantities. And as shown by the results, these estimates can, in turn, be used to rescale reliably collective quantities like the quadrupole moments and $B(\text{E}2)$ strengths that are obtained from calculations based solely on a truncated Hilbert space. We have seen that for weak quadrupole interaction strengths, truncating the unique parity subspace to the largest-$j$ orbital results in an yrast state, $|\varphi_0\rangle = (|N; j_n\rangle |(f_{7/2})^2, v, l)\rangle_{YM^*}$, which is very close to the lowest eigenstate of the full space, $|\Psi_0\rangle = (|N; j_n\rangle |U; l)\rangle_{YM}$, and even for stronger and therefore more realistic quadrupole interaction strengths gives a good approximation. If we furthermore truncate the unique parity subspace to seniority-zero configurations only we obtain the state $|\xi_0\rangle = (|N; j_n\rangle |(f_{7/2})^2, v = 0)\rangle$ in $\nu_n^*$, which likewise yields a very reasonable approximation to the calculated yrast state. Although its unique parity component is well deformed, this state lacks those contributions to the total quadrupole collectivity that, in a full space, arise from the correlations between normal and unique parity nucleons. Hence a model which truncates the unique parity states to seniority-zero configurations needs to include this feature through rescaling of all calculated deformation-dependent observables. Similarly, the most radical truncation of the total, coupled Hilbert space to its normal parity subspace requires a scaling factor that accounts for the contributions which arise from the correlations among the unique parity nucleons as well as their couplings to the normal parity particles. Focusing on the latter truncation scheme, we have demonstrated that the nucleons in the normal parity states suffice to represent the physics of the full system. Specifically, we have shown that the $B(\text{E}2)$ scaling factor, for example, varies only weakly with the interaction parameters of the model and is nearly independent of the angular momentum of the system for low-lying rotational states. In particular, for these low-lying states the influence of the unique parity particles can be accounted for by simply applying the ratio of the Casimir invariants of the leading irreps in the total coupled space to that in the normal parity subspace: $C_{\lambda\mu}(\lambda\mu)_{\text{nuc}}/C_{\lambda\mu}(\lambda\lambda\mu\mu)_{\text{nuc}}$.

The use of scaling factors is limited to those situations where the nucleons in the unique parity subspace follow in an adiabatic way the collective motion of their partners in the normal parity orbitals, that is, the unique parity configurations that couple to their corresponding normal parity structures track the behavior of those configurations as one moves up the yrast band. This assumption obviously breaks down when backbending occurs, and an extended version of the present theory then becomes necessary for a proper description of higher-lying rotational states. The latter requires the development of a new truncation scheme for the unique parity subspace of heavy deformed nuclei, which, in turn, will entail a more detailed study of the role of pairing correlations in a full major oscillator shell. An exploration of this challenging matter is underway [36]. Furthermore, for a fully microscopic description of heavy deformed nuclei it is important to investigate the effects of the proton–neutron interaction, which is known to be enhanced relative to the proton–proton and neutron–neutron correlations [37,38]. We have shown, however, that for most practical applications a fairly radical truncation, as long as it is accompanied by an appropriate renormalization procedure, is acceptable for determining the properties of deformation-dependent observables for low-lying collective states.
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References

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