New algebraic approach for an exact solution of
the nuclear mean-field plus orbit-dependent pairing hamiltonian

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Abstract

An infinite dimensional algebraic approach is introduced to derive exact particle-number conserving solutions of the nuclear mean-field plus separable orbit-dependent pairing Hamiltonian. As examples, some low-lying levels, spectroscopic factors, and even-odd mass differences of 58–65 Ni isotopes are calculated and compared with experimental values, shell model, and equal strength pairing results. Some limiting cases of the model Hamiltonian are also discussed. © 1998 Elsevier Science B.V. All rights reserved.

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Pairing, in addition to the quadrupole-quadrupole interaction, is an important interaction in nuclear physics. The concept was proposed by Racah as a seniority scheme in atomic physics [1]. Its physical significance was realized first in superconductivity studies [2]. Following the suggestions of Bohr, Mottelson, and Pines [3], the first detailed application of pairing to nuclei was made by Belyaev [4]. The concept has since been applied to other phenomena: high Tc superconductivity [5,6], applications using the Hubbard model [7], and pairing phenomena in liquids [8] and metal clusters [9].

In nuclear physics applications, mean-field approximations are usually supplemented with residual interactions, the short-range pairing interaction being the one most commonly used. In this case, the problem is usually handled approximately using Bardeen-Cooper-Schrieffer (BCS) or Hartree-Fock-Bogolyubov (HFB) methods, sometimes in conjunction with correction terms evaluated within the random-phase approximation. However, when BCS or HFB methods are applied to nuclei there are some serious drawbacks. First of all, not only is the number of nucleons in a nucleus typically small, the number of valence particles (n ~ 10) which dominates the behaviour of low-lying states is too few to support underlying assumptions of the approximations, specifically, δn/n is not negligible. As a

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result, particle-number-nonconservation effects enter and can lead to serious difficulties, such as spurious states, nonorthogonal solutions, etc. Furthermore, an essential feature of pairing correlations are differences between neighboring even and odd mass nuclei, which are driven mainly by Pauli blocking. It is difficult to treat even-odd differences with these methods because different quasi-particle bases must be introduced for different blocked levels. Another problem with approximate treatments of the pairing Hamiltonian is related to the fact that both the BCS and the HFB approximations break down for an important class of physical situations. A remedy in terms of particle number projection techniques complicates the algorithms considerably, and does not help to achieve a better description of the higher-excited part of the spectrum of the pairing Hamiltonian. For these reasons, a particle-number-conserving method for handling the pairing problem, when feasible, even if only approximate, is an appropriate way to probe the true nature of pairing correlations in nuclei. Over the past few years some progress has been made in the development of better algorithms that bypass the Bogolyubov transformation and are thus free from problems related to particle number nonconservation. For instance, a particle-number-conserving method for treating the pairing problem for well-deformed nuclei was put forward in Ref. [10]. The method uses a configuration-energy truncation scheme and takes the strength of the pairing interaction to be the same for all orbitals. Unfortunately, because of the deformation, each orbital can only accommodate a single pair of particles and this complicates the algorithms considerably, and does not help to achieve a better description of the higher-excited part of the spectrum of the pairing Hamiltonian. For these reasons, a particle-number-conserving method for handling the pairing problem, when feasible, even if only approximate, is an appropriate way to probe the true nature of pairing correlations in nuclei. Over the past few years some progress has been made in the development of better algorithms that bypass the Bogolyubov transformation and are thus free from problems related to particle number nonconservation. For instance, a particle-number-conserving method for treating the pairing problem for well-deformed nuclei was put forward in Ref. [10]. The method uses a configuration-energy truncation scheme and takes the strength of the pairing interaction to be the same for all orbitals. Unfortunately, because of the deformation, each orbital can only accommodate a single pair of particles and this limits the applicability of the theory. Very recently, a Fock-space diagonalization of the pairing Hamiltonian, also for deformed nuclei, was proposed that used some symmetry properties and a many-body Fock-space basis cutoff [11]. In this letter, we will outline a new particle-number-conserving method for the pairing problem.

The general pairing Hamiltonian for spherical nuclei can be written as

\[ \hat{H} = \sum_{jm} \epsilon_j a_j^\dagger a_j a_j a_{-j} - \sum_{jj'} \frac{1}{2} c_{jj'} S^+(j) S^-(j'), \]

(1)

where the \( \epsilon_j \) are single-particle energies and \( S^\pm(j) \) and \( S^0(j) \) are the pairing operators for a single-\( j \) shell defined by

\[ S^+(j) = \sum_{m>0} (-1)^{-m} a_{jm}^\dagger a_{-j-m}, \]

\[ S^-(j) = \sum_{m>0} (-1)^{-m} a_{j-m} a_{jm}, \]

\[ S^0(j) = \frac{1}{2} \sum_{m>0} (a_{jm}^\dagger a_{jm} + a_{j-m}^\dagger a_{j-m} - 1) \]

\[ = \frac{1}{2} \left( \mathcal{N}_j - \Omega_j \right). \] (2)

In (2), \( \Omega_j = j + 1/2 \) is the maximum number of pairs the \( j \)-th shell can accommodate, \( \mathcal{N}_j \) is the \( j \)-th shell particle number operator, and \( c_{jj'} \) is the strength of the pairing interaction between the \( j \) and \( j' \) shells.

In general, for \( k \) pairs, Hamiltonian (1) can be diagonalized in bases that are products of the single-\( j \) shell pairing wave functions:

\[ |k\rangle = \sum_{k_1} B_{k_1} \cdots S^{+k_1}(j_1) S^{+k_2}(j_2) S^{+k_3}(j_3) \cdots \times S^{+k_p}(j_p) |0\rangle, \]

(3)

where the summation is restricted by

\[ \sum_{i=1}^{p} k_i = k, \]

(4)

\( p \) is the total number of orbitals, the \( B_{k_1} \cdots k_p \) are expansion coefficients. Furthermore, the number of single \( j \)-shell basis vectors included in (3) has to be fixed on a case-by-case basis because of the Pauli Principle.

As an approximation to the general theory, we assume a separable strength pairing (SSP) interaction, \( c_{jj'} = c_j c_j^* \). Though strong, this assumption is physically motivated because it links the pair-pair interaction strength to the individual pair formation probability. In this case, \( |c_j|^2 / \Sigma_j |c_j|^2 \) gives the percentage of single-\( j \) shell pairing in the Hamiltonian. Furthermore, it is expected to be better than the equal strength pairing (ESP) approximation because the \( c_{jj'} = |G| \) for all orbitals. The EPS approximation has
been commonly used in many applications because it is much simpler than the general case.

To diagonalize the SSP Hamiltonian, we need to introduce the following two-parameter algebra generated by \( \{ S_{mn}^\mu, \mu = 0, +, -; m, n = 0, 1, 2, \ldots \} \) with

\[
S_{mn}^0 = \sum_j e_j^{mn} |c_j|^2 n c_j S^+(j),
\]

\[
S_{mn}^+ = \sum_j e_j^{mn} |c_j|^2 n c_j S^+(j),
\]

\[
S_{mn}^- = \sum_j e_j^{mn} |c_j|^2 n c_j S^0(j).
\]  \( \text{(6)} \)

It is easy to show that these generators satisfy the following commutation relations:

\[
\left[ S_{mn}^+, S_{n'n''}^- \right] = 2S_{m+n+n'+1, n''}^0,
\]

\[
\left[ S_{mn}^0, S_{n'n''}^\pm \right] = \pm S_{m+n+n'+n''}^\pm.
\]  \( \text{(7)} \)

Therefore, the \( S_{mn}^\mu \) form an infinite-dimensional algebra, which is a Lie algebra of the two-parameter affine type without central extension. The Hamiltonian \( \text{(1)} \) with \( c_{jj} = c_j c_j^* \) can be written in terms of the \( S_{mn}^\mu \) operators as

\[
\hat{H} = \sum_j e_j \Omega_j + 2S_{10}^0 + S_{10}^+ S_{10}^-.
\]  \( \text{(8)} \)

In the following, we assume that the parameters \( e_j \) and \( c_j \) are all different for different \( j \) values, that is, we only consider the non-degenerate case. The situation is different for degenerate and other simpler cases and these will be discussed later. For the nondegenerate case, the unique lowest-weight state is simply the product of the single-j shell pairing vacua with arbitrary seniority quantum numbers. Therefore, it suffices to consider the total seniority zero case. The lowest-weight state satisfies

\[
S_{mn}^0 |0\rangle = 0 \quad \forall \, m, n.
\]  \( \text{(9)} \)

Furthermore, it can be proven that in this case the eigenvectors of the Hamiltonian for \( k \)-pair excitations can be written as

\[
|k\rangle = \mathcal{N} S^+(x_1) S^+(x_2) \cdots S^+(x_k) |0\rangle,
\]  \( \text{(10)} \)

where \( \mathcal{N} \) is the normalization constant and

\[
S^+(x_i) = \sum_j \frac{c_j}{1 - e_j x_i} S^+_j.
\]  \( \text{(11)} \)

The wave functions given by \( \text{(10)} \) are similar to the algebraic Bethe ansatz \([12]\) which has proven to be a very useful and powerful tool for solving various spin-chain models and one-dimensional many-body problems \([13]\).

To determine the operators \( S^+(x_i) \) and the \( \epsilon \)-number variables \( \{ x_i, i = 1, 2, \ldots, k \} \), we first expand \( \text{(10)} \) in terms of the \( x_i \) around \( x_i = 0 \),

\[
|k\rangle = \sum_{n_i} \chi_{1}^{n_1} \chi_{2}^{n_2} \cdots \chi_{k}^{n_k} S_{n_0}^+ S_{n_1}^+ \cdots S_{n_k}^+ |0\rangle.
\]  \( \text{(12)} \)

where \( S_{n_0}^+ \) are nothing but Fourier-Laurent coefficients in the expansion of \( S^+(x_i) \), namely

\[
S_{n_0}^+ = \frac{1}{2} \pi i \phi dx_i \chi_i^+ S^+(x_i).
\]  \( \text{(13)} \)

However, using the commutation relations \( \text{(7)} \), one can find that the basis vectors \( \text{(12)} \) are not algebraically closed under the \( \hat{H} \) action. In order to keep them algebraically closed, we need the following auxiliary conditions

\[
\sum_{i=1}^{p} \frac{a_i}{1 - \epsilon_j z_i} = |c_j|^2, \quad j = 1, 2, \ldots, p.
\]  \( \text{(14)} \)

where \( \{ a_i, z_i, i = 1, 2, \ldots, p \} \) are unknown \( \epsilon \)-numbers that need to be determined. Because we have assumed that the \( e_j \) and \( c_j \) are all different for different \( j \) values, \( \text{(15)} \) is always valid in this case. Using \( \text{(12)}, \text{(14)} \), and commutation relations \( \text{(7)} \), one can prove that \( x_i, a_j, \) and \( z_j \) must satisfy the following relations:

\[
E_i = \frac{k}{\sum_{i=1}^{k} x_i},
\]  \( \text{(15a)} \)

\[
\frac{2}{x_i} = \sum_{j \neq i} \frac{\Omega_j |c_j|^2}{e_j x_i - 1} + 2 \sum_{j \neq i} \left( \sum_{m} \frac{a_m}{x_j - z_m} + 1 \right) x_i / (x_j - x_i),
\]  \( \text{(15b)} \)

for \( i = 1, 2, \ldots, k \), where \( E_i \) is the \( k \)-pair excitation energy, under the conditions

\[
\sum_{i=1}^{p} \frac{a_i}{1 - \epsilon_j z_i} \sum_{r>q} \frac{z_i}{x_r - z_q} (x_q - z_i) \quad \sum_{r>q} \frac{1}{(1 - \epsilon_j x_q)(1 - \epsilon_j x_r)}
\]  \( \text{(16)} \)

for \( j = 1, 2, \ldots, p \). Though these relations were de-
derived in the $x_j = 0$ region, they are valid in the entire complex plane. Therefore, the coefficients $x_j$ ($i = 1, 2, \ldots, k$), $z_j$, $a_j$ ($j = 1, 2, \ldots, p$), and eigenvalues of the pairing energy are simultaneously determined by the system of equations (14), (15) and (16).

So far we have assumed that the $e_j$ and $c_j$ are all different for different $j$ values. When the $e_j$ are all the same or the $c_j$ are all the same the situations is greatly simplified [14]. These special cases can be regarded as limiting cases of the Hamiltonian (8), which are summarized in Fig. 1. It should be pointed out that the exact solutions for the equal strength pairing limit case with $c_{ij} = |G_{ij}|$ which is a special case of what is shown in Fig. 1, was studied by Richardson [15,16] in terms of boson mappings including the effect of the Pauli Principle. That result can now be obtained more simply and directly by using the infinite dimensional algebraic approach.

As is well-known from earlier work [17] on a comparison of exact ESP results and the BCS approximation [18], the excitation energies of low-lying states with seniority one and two calculated within the BCS approximation are only correct to within about 200 keV with ground state energies usually not given to better than about 500 keV. Hence, an exact solution of the pairing Hamiltonian, when feasible, is important for probing the true nature of pairing correlations in nuclei. To demonstrate the effectiveness of the theory as well as the accuracy of the SSP assumption, as examples, we calculated low-lying levels, spectroscopic factors, and even-odd mass differences of the $^{58-65}$Ni isotopes. The results were compared with experimental values and those of shell model (SM) and ESP results. These isotopes have been studied in great detail within a $(2p_{3/2}, 1f_{5/2}, 2p_{1/2})^n$ SM picture, for which a set of

$$\varepsilon_j = e \quad \text{for all } j.$$

$$|k> = S_{y_1}^+ S_{y_2}^+ \cdots S_{y_k}^+ |0>.$$

$$S_{y_i}^+ = \sum_j \frac{c_{ij}}{1-e^{-\xi_i y_i}} S^+_j.$$

$$E_k = \sum_i \frac{2}{y_i} - \sum_j |c_j|^2 \Omega_j.$$

$$\frac{1}{\xi_i} = \frac{1}{2} \sum_j \frac{|c_j|^2}{\xi_j^2 + 1} + \sum_{p \neq i} \frac{1}{\xi_p + \xi_i}.$$

$$c_{jj'} = c_j c_{j'}.$$

Eigenvalues and eigenvectors are given by (15), (16), and (17).

$$\varepsilon_{jj'} = |G| e^{\delta_{jj'}}$$

for all $j, j'$.

$$|k> = S_{\xi_1}^+ S_{\xi_2}^+ \cdots S_{\xi_k}^+ |0>.$$

$$S_{\xi_i}^+ = \sum_j e^{\delta_{ij}} S^+_j.$$

$$E_k = \sum_i \frac{2 |G|}{\xi_i}.$$

$$\frac{1}{\xi_i} = \frac{1}{2} \sum_j \frac{\Omega_j}{\xi_j^2 + 1} + \sum_{p \neq i} \frac{\xi_p}{\xi_p^2 + \xi_i}.$$

Fig. 1. Various limiting cases of the pairing Hamiltonian (1) and the corresponding solutions.
Table 1

<table>
<thead>
<tr>
<th>Ni isotopes</th>
<th>E_{exp}</th>
<th>E_{SM}</th>
<th>E_{SSP}</th>
<th>E_{ESP}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni^{68}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ni^{70}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ni^{66}</td>
<td>0.24</td>
<td>0.21</td>
<td>0.26</td>
<td>0.32</td>
</tr>
<tr>
<td>Ni^{64}</td>
<td>0.04</td>
<td>0.04</td>
<td>0.05</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Effective interaction matrix elements were deduced from the experimental spectrum of Ni^{67} (E_{3/2} = 1.1126 MeV, and E_{5/2} = 0.7685 MeV). The parameters c_{j} (in MeV) were obtained from the effective interaction two-body matrix elements \langle J^2|J = 0\rangle given by [20], which yields c_{1/2} = 0.89, c_{3/2} = 0.46, c_{5/2} = 0.58, c_{7/2} = 0.686, c_{9/2} = 0.323, c_{11/2} = 0.457. While in (8) the single-particle energies are taken to be the same as those of the SM, here the c_{j} parameters were determined as follows: Firstly, we calculated the seniority zero one-pair ground state wavefunction, |N = 1\rangle = \sum c_{j}S_{j}', from the J = 2 two-body pairing Hamiltonian with no single-particle energies. We then constructed, as was done in [14], the SPP two-body pairing Hamiltonian, \mathcal{H}_{SPP} = \sum_{j} c_{j} S_{j}' S_{j}'' where c_{j} = \sqrt{g} c_{j}' and g is a real parameter, using the generalized pairing operator \sum_{j} c_{j} S_{j}''', which should reproduce the seniority zero one-pair ground state energy derived in the SM. From this exercise we determined c_{1/2} = 0.746 MeV, c_{3/2} = 0.683 MeV, and c_{5/2} = 0.652 MeV. The pairing strength in the ESP limit was taken from [17,18], which gives |G| = 0.3331 MeV. The experimental data were taken from [21–26].

Results for some of the low-lying seniority zero, one, and two levels calculated within the SM, SSP, and exact ESP frameworks are recorded in Tables 1 and 2. As can be seen, the SM does a good job of reproducing the 0_{1}^{+} and 0_{2}^{+} energies. Furthermore, the SSP results are very close to the SM results; indeed, they are much better than the ESP predictions. Because only the J = 0 pairing interaction is considered, even the SM results show deviations from the experimental data, especially for the seniority one and two states. This is due to missing residual interactions which could, presumably, be treated...
by perturbation theory [18]. The real test is how well SSP reproduces the SM results. The observed systematic difference in the binding energies of odd-A and even-even isotopes is a direct consequence of the pairing force. The residual interaction, though strong, shows little even-odd structure. Fig. 2 shows the even-odd mass differences of these isotopes calculated in the SM, SSP, and ESP, respectively, and compared with the experimental results. In short, it is very clear that for all measures the SSP results are very close to those of the SM, much closer than those of the ESP limit.

To calculate overlaps, matrix elements, etc. in the exactly solvable SSP theory, one encounters a new infinite dimensional non-linear algebra generated by

\[ A_m^0(x_1, \ldots, x_\mu; \nu) \]

which satisfy the following commutation relations

\[
\begin{align*}
[A_m^0(x_1, \ldots, x_\mu), A_n^0(y_1, \ldots, y_\nu)] &= -2A_{m+n+1}^0(x_1, \ldots, x_\mu, y_1, \ldots, y_\nu), \\
[A_m^0(x_1, \ldots, x_\mu), A_n^\pm(y_1, \ldots, y_\nu)] &= \pm A_{m+n}^\pm(x_1, \ldots, x_\mu, y_1, \ldots, y_\nu). 
\end{align*}
\]  

(18)

Using the commutation relations (18), for example, one can derive the following matrix elements:

\[
\begin{align*}
\langle 0 | A_m^0 (x^+) A_n^0(x) | 0 \rangle &= (-2) A_1(x^+, x; \nu = 0), \\
\langle J | A_0^0 (x^+) A_0^0(x) | J \rangle &= (-2) A_1(x^+, x; \nu = 0), \\
\langle 0 | A_0^0(x^+) A_0^0(x) A_0^0(x_2) | 0 \rangle &= (-2)^2 (A_2(x_1^+, x_2^+, x_1, \nu = 0) \\
+ A_1(x_2^+, x_1; \nu = 0) A_1(x_1^+, \nu = 0) \\
+ A_1(x_1^+, x_1; \nu = 0) A_1(x_2^+, \nu = 0)) \\
&= \sum_j \left(1 - \epsilon_j x_j\right) \left(1 - \epsilon_j x_j\right) \cdots \left(1 - \epsilon_j x_j\right) \times \left(\nu - \Omega_j\right). 
\end{align*}
\]  

(19)

Using this technique, we calculated spectroscopic factors for pickup and stripping reactions. The results are listed in Tables 3 and 4. Again the results show that the SSP are very close to those of the SM.

<table>
<thead>
<tr>
<th>Residual nucleus</th>
<th>l, J</th>
<th>Exp.</th>
<th>SM</th>
<th>SSP</th>
<th>ESP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{57}$Ni</td>
<td>1, 1</td>
<td>1.04, 1.13</td>
<td>1.211</td>
<td>1.306</td>
<td>1.514</td>
</tr>
<tr>
<td></td>
<td>3, 2</td>
<td>1.05, 1.14</td>
<td>0.405</td>
<td>0.363</td>
<td>0.296</td>
</tr>
<tr>
<td></td>
<td>1, 1</td>
<td>0.21, 0.22</td>
<td>0.384</td>
<td>0.329</td>
<td>0.189</td>
</tr>
<tr>
<td>$^{59}$Ni</td>
<td>1, 1</td>
<td>1.82</td>
<td>1.489</td>
<td>1.499</td>
<td>1.615</td>
</tr>
<tr>
<td></td>
<td>3, 2</td>
<td>2.02</td>
<td>0.960</td>
<td>0.849</td>
<td>0.697</td>
</tr>
<tr>
<td></td>
<td>1, 1</td>
<td>0.20</td>
<td>0.722</td>
<td>0.682</td>
<td>0.420</td>
</tr>
<tr>
<td>$^{61}$Ni</td>
<td>1, 1</td>
<td>1.8</td>
<td>1.264</td>
<td>1.218</td>
<td>0.959</td>
</tr>
<tr>
<td></td>
<td>3, 2</td>
<td>2.5</td>
<td>1.641</td>
<td>1.533</td>
<td>1.562</td>
</tr>
<tr>
<td></td>
<td>1, 1</td>
<td>0.49</td>
<td>0.769</td>
<td>0.793</td>
<td>0.717</td>
</tr>
<tr>
<td>$^{63}$Ni</td>
<td>1, 1</td>
<td>0.47, 0.4</td>
<td>1.723</td>
<td>1.606</td>
<td>1.440</td>
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<tr>
<td></td>
<td>3, 2</td>
<td>3.43, 2.88</td>
<td>1.834</td>
<td>1.989</td>
<td>1.991</td>
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<tr>
<td></td>
<td>1, 1</td>
<td>2.42, 1.80</td>
<td>1.766</td>
<td>1.825</td>
<td>1.872</td>
</tr>
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</table>
Table 4
Spectroscopic factors (2J_f + 1)S from stripping experiments for some low-lying states in the odd-A nickel isotopes 57–63Ni and those calculated by SM, SSP, and ESP, respectively.

<table>
<thead>
<tr>
<th>Residual nucleus</th>
<th>J, l</th>
<th>Exp.</th>
<th>SM</th>
<th>SSP</th>
<th>ESP</th>
</tr>
</thead>
<tbody>
<tr>
<td>59Ni</td>
<td>1, 1</td>
<td>3.263</td>
<td>2.616</td>
<td>2.491</td>
<td>2.284</td>
</tr>
<tr>
<td></td>
<td>3, 1</td>
<td>4.060</td>
<td>5.561</td>
<td>5.616</td>
<td>5.689</td>
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<tr>
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<td>1.23</td>
<td>1.274</td>
<td>1.310</td>
<td>1.572</td>
</tr>
<tr>
<td>61Ni</td>
<td>1, 1</td>
<td>0.75</td>
<td>0.896</td>
<td>0.884</td>
<td>1.124</td>
</tr>
<tr>
<td></td>
<td>3, 1</td>
<td>2.39</td>
<td>3.945</td>
<td>4.095</td>
<td>4.112</td>
</tr>
<tr>
<td></td>
<td>1, 1</td>
<td>1.07</td>
<td>1.080</td>
<td>0.906</td>
<td>0.646</td>
</tr>
<tr>
<td>63Ni</td>
<td>1, 1</td>
<td>0.75</td>
<td>0.896</td>
<td>0.884</td>
<td>1.124</td>
</tr>
<tr>
<td></td>
<td>3, 1</td>
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<td>3.945</td>
<td>4.095</td>
<td>4.112</td>
</tr>
<tr>
<td></td>
<td>1, 1</td>
<td>1.07</td>
<td>1.080</td>
<td>0.906</td>
<td>0.646</td>
</tr>
<tr>
<td>65Ni</td>
<td>1, 1</td>
<td>1.24</td>
<td>0.251</td>
<td>0.381</td>
<td>0.547</td>
</tr>
<tr>
<td></td>
<td>3, 1</td>
<td>1.07</td>
<td>0.896</td>
<td>0.884</td>
<td>1.124</td>
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<td>3, 1</td>
<td>1.07</td>
<td>0.896</td>
<td>0.884</td>
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In summary, an exact particle-number-conserving solution to the SSP Hamiltonian has been derived with the help of a two-parameter SU(2) affine Lie algebra without central extension. A comparison of realistic shell-model results and those of the ESP in the description of Ni isotopes shows that the SSP assumption is a very good approximation, much better than ESP. In order to calculate matrix elements etc. in the SSP, one encounters a new infinite-dimensional non-linear algebra. A more detailed study of this algebra will be given separately. The present solution, together with those given in [14], implies the general pairing Hamiltonian \( \mathcal{H} = c_j e^{i\delta_j} c_j^\dagger \), with non-degenerate single-particle energies; the SSP limit, \( c_{j\prime} = c_j c_j^\dagger \), with degenerate single-particle energies; and the ESP case, \( c_{j\prime} = [G] e^{i(\delta_j - \delta_j')} \), where \( \delta_j \) is an orbital dependent phase. Additional applications of the techniques introduced in this article are in progress.

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