M1 strengths in deformed nuclei

J P Draayer†, T Beuschel† and J G Hirsch‡

† Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803-4001, USA
‡ Departamento de Fisica, CINVESTAV del IPN, AP 14-740, 07000 DF, Mexico

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Abstract. The Elliott SU(3) model, extended via pseudo-spin for heavy nuclei, is used to study low-lying magnetic dipole excitations in deformed nuclei. Proton and neutron degrees of freedom are handled explicitly and the system Hamiltonian includes single-particle energies as well as quadrupole–quadrupole and pairing two-body interactions. The calculated excitation spectra and M1 strength distribution of the strongly deformed even–even $^{156–160}$Gd isotopes are shown to be in good agreement with experimental results. Results for the $\gamma$-unstable (soft rotor) $^{196}$Pt nucleus and the even–odd $^{163}$Dy system are also reported and found to compare favourably with the available experimental data, demonstrating the ability of the model to describe very different types of systems.

1. Introduction

Low-lying M1 transition strength distributions observed in nuclei of the rare earth and actinide regions [1,2] reflect on both the collective and non-collective aspects of the nuclear interaction. A study of this decay mode thus allows one to probe how well a nuclear model can incorporate these complementary features [3–6].

The earliest interpretation of the ‘scissors’ mode by Lo Iudice and Palumbo [7] in 1978—six years before it was detected experimentally—was that of a collective magnetic-dipole state of two spheroids, one representing protons and the other neutrons, exercising relative rotational oscillations. A feature that remains unexplained in such a model with only a single collective degree of freedom is the observed fragmentation of the mode, that is, the break-up of the M1 strength among several levels closely packed and clustered around a few strong transition peaks in the energy region between 2 and 4 MeV. Also, this simple ‘two rotor model’ (TRM) is not able to address the case of nuclei with an odd number of protons or neutrons. The effect of the odd nucleon not only complicates the excitation spectra, it also introduces a far richer and more complex M1 strength distribution.

The Elliott SU(3) model [8], extended via pseudo-spin for applications to heavy nuclei, goes beyond the TRM of Lo Iudice and Palumbo. The pseudo SU(3) scheme—as it is usually called—can be used to give a reasonable description of the complex experimental data. It is a many-particle shell-model theory that accounts for the fermion nature of the nucleons and takes full advantage of pseudo-spin symmetry, which in heavy nuclei is manifest in the near degeneracy of the orbital pairs $(l - 1) j = l+1/2, (l + 1) j = l-1/2$. Since the embedding within the shell model and algebra structure of the pseudo oscillator is the same as for the normal oscillator, the pseudo SU(3) scheme can be used to partition the full space into a direct sum of distinct subspaces. Following its introduction in the late 1960s [9, 10], the pseudo SU(3) scheme has been applied to various properties of heavy deformed nuclei [11–13]. However,
these applications have been limited to the use of schematic nucleon–nucleon interactions because of technical difficulties related to the calculation of $SU(3)$ matrix elements of more general interactions. Recently, however, a code was released [14] that removes these limitations and allows for the introduction of interactions like pairing and single-particle spin–orbit terms into pseudo $SU(3)$ model calculations. As shown below, such terms are vital for an adequate description of M1 strength distributions.

The $SU(3)$ model is an ideal tool for studying M1 strengths since the theory offers an interpretation of structural features in terms of collective (rotational) degrees of freedom. In short, it is a fully microscopic shell-model version of the TRM. The direct connection is accomplished by exploiting a relation between $SU(3)$ and the algebra of a quantum rotor, which is covered in the next section.

2. Geometric interpretation of $1^+$ states

In the TRM the scissors-like relative motion of the proton and neutron distributions is parametrized in terms of an angle $\theta$ between the $z$-axes of the axially symmetric distributions. However, it can be shown that an additional ‘twist’ mode is possible [15] for triaxial shapes because rotations by $\phi_\nu$ and $\phi_\pi$ about the $z$-axes of the proton and neutron distributions emerge as additional degrees of freedom. As for the scissors mode, it is the difference of these two rotations that gives rise to a new M1 mode. This new mode, which together with the scissors motion, determines the basic structure of the M1 transition spectrum, has a very simple interpretation within the framework of the pseudo $SU(3)$ model.

In a pure $SU(3)$ limit of the theory, the coupling of an axially symmetric proton and neutron distributions gives rise to a single scissors mode, in agreement with the TRM model. However, if one of the two distributions is triaxial, an additional scissors + twist mode is possible. Like the scissors mode, this new mode corresponds to a specific $SU(3)$ irreducible representation (irrep) that can be interpreted as an eigenstate of a two-dimensional harmonic oscillator potential $H_{\text{int}}$ describing the proton–neutron interaction in terms of the collective variables $\theta$ and $\phi = \phi_\pi - \phi_\nu$.

$$H_{\text{int}} = \hbar\omega_\theta (n_\theta + \frac{1}{2}) + \hbar\omega_\phi (n_\phi + \frac{1}{2}) + E_0'.$$

Using the mapping between the variables of the joint rotor and $SU(3)$, this oscillator structure can be derived from a general $SU(3)$-preserving nuclear Hamiltonian [16].

For the general case of coupling of triaxial proton and neutron distributions, there is an additional pure twist mode as well as another twist + scissors mode which is distinguishable from the scissors + twist mode through the so-called $SU(3)$ outer multiplicity labelling of the $SU(3)$ irreps involved. This means there is a maximum of four distinct orbital M1 modes. As an example, the $1^+$ states with non-zero $B(M1)$ transition strength for the even–even $^{156-160}$Gd and $^{196}$Pt isotopes are given in table 1 together with their classification as scissors, twist, or the combination modes.

The summed transition strength for the Gd isotopes in table 1 is close to the experimental results, which validates the underlying $SU(3)$ structure in this case. However, the experimental results suggest a much larger number of $1^+$ states with non-zero M1 transition probabilities to the $0^+$ ground state that are usually clustered around a few strong peaks. This can be understood in terms of a fragmentation of the pure-symmetry states under the influence of $SU(3)$ breaking residual interactions.
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Table 1. \(B(M1)\) transition strengths \([\mu_2^2]\) in the pure-symmetry limit of the pseudo \(SU(3)\) model. The strong coupled pseudo \(SU(3)\) irrep \((\lambda, \mu)_{\gamma}\) for the ground state is given with its proton and neutron sub-irreps and the irreps associated with the \(1^+\) states, \((\lambda', \mu')_{1^+}\). Each transition is labelled as a scissors (s) or twist (t) or a combination mode.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>((\lambda_{\pi}, \mu_{\pi}))</th>
<th>((\lambda_{\nu}, \mu_{\nu}))</th>
<th>((\lambda, \mu)_{\gamma})</th>
<th>((\lambda', \mu')_{1^+})</th>
<th>(B(M1))</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{156}\text{Gd})</td>
<td>(10, 4)</td>
<td>(18, 0)</td>
<td>(28, 4)</td>
<td>(26, 5)</td>
<td>1.91</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>(27, 3)</td>
<td>1.61</td>
<td>s + t</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(26, 9)</td>
<td>1.77</td>
<td>s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(27, 7)</td>
<td>1.82</td>
<td>s + t</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(27, 7)</td>
<td>0.083</td>
<td>t + s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(29, 6)</td>
<td>0.56</td>
<td>t</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^{160}\text{Gd})</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
<td>(28, 8)</td>
<td>(26, 9)</td>
<td>1.77</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>(27, 7)</td>
<td>1.82</td>
<td>s + t</td>
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<td></td>
<td>(29, 6)</td>
<td>0.56</td>
<td>t</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^{196}\text{Pt})</td>
<td>(2, 8)</td>
<td>(4, 18)</td>
<td>(6, 26)</td>
<td>(24, 7)</td>
<td>0.56</td>
<td>t</td>
</tr>
</tbody>
</table>

3. Calculations with a realistic Hamiltonian

To investigate the effect of the symmetry-breaking interaction terms, the following generalization of the \(SU(3)\)-conserving Hamiltonian was used,

\[
H_{PSU(3)} = -(a_2 + a_{\text{sym}})C_2 + a_3 C_3 + b K_2^2 + c J^2 + c \sum_{i_{\pi}} \tilde{l}_{i_{\pi}} \tilde{s}_{i_{\pi}} + C_\nu \sum_{i_{\nu}} \tilde{l}_{i_{\nu}} \tilde{s}_{i_{\nu}} + D_\pi \sum_{i_{\pi}} l_{i_{\pi}}^2 + D_\nu \sum_{i_{\nu}} l_{i_{\nu}}^2 + G_{\pi} H_{\pi}^P - G_{\nu} H_{\nu}^P.
\]

(2)

Here \(C_2\) and \(C_3\) are the second- and third-order invariants of \(SU(3)\), which are related to the axial and triaxial deformation of the nucleus, and \(J^2\) and \(K_2^2\) are the square of the total angular momentum and its projection on the intrinsic body-fixed symmetry axis, which generate rotational bands and \(KJ\)-band splitting, respectively. The parameter \(a_{\text{sym}}\) is introduced to shift \(SU(3)\) irreps with either \(\lambda\) or \(\mu\) odd relative to those with \(\lambda\) and \(\mu\) both even, for which \(a_{\text{sym}}\) is zero, as the former belongs to different symmetry types \((B_\alpha, \alpha = 1, 2, 3, \text{rather than } A)\) of the intrinsic Vierergruppe \((D_2)\) [17]. The one-body proton and neutron spin–orbit and angular momentum terms, together with the two-body pairing terms, \(H_{\pi}^P\) and \(H_{\nu}^P\), are \(SU(3)\) symmetry-breaking interactions.

Since the quadrupole–quadrupole interaction, \(Q \cdot Q = 4C_2 - 3L^2\), dominates for deformed nuclei, only basis states with \(C_2\) larger than a certain value are expected to give a significant contribution in the low-energy region. In the present application, for both proton and neutron distributions, all \(SU(3)\) basis states with \(C_2 \geq C_{2\text{min}}\) were selected with \(C_{2\text{min}}\) set so that all irreps lying below approximately 6 MeV were included in the analysis. Then all possible couplings of these proton and neutron \(SU(3)\) irreps were taken to give coupled \(SU(3)\) irreps that form basis states of the model space. Also, only states with \(J \leq 8\) and \(S = 0\) or \(S = \frac{1}{2}\) were considered.

The parameters for the Hamiltonian given in equation (2) and the effective charges \(e_\pi = 1 + q_{\text{eff}}\) and \(e_\nu = q_{\text{eff}}\) used in the \(E2\) transition operator

\[
T_{M(E2)}^2 = A^{1/3} \sum_{\sigma = \pi, \nu} \sum_i e_\sigma r_\sigma^2(i) Y_{2M}(\hat{r}_\sigma(i))
\]

(3)

were determined through a fitting procedure that included as input all known levels with \(J \leq 8\) up through 2 MeV in energy and selected \(B(E2)\) transition strengths. This procedure gave, in general, very good agreement between the experimental and theoretical numbers (figures 1–3),
Figure 1. Energy and M1 transition spectra for the even–even 156–160 Gd isotopes. Experimental excitation energies and E2 transition strengths were used as input into a fitting routine to determine parameters of the Hamiltonian for each system. These were then used to calculate the theoretical spectrum and corresponding M1 transition strengths.

Figure 2. The experimental ground-band energy spectra for 196 Pt does not show typical rotor characteristics. As for the Gd isotopes, the experimentally determined energy spectra and known E2 values were used in a fitting routine to determine the parameters of the Hamiltonian. The M1 transition spectrum derived using these results is in reasonable agreement with experimental with values that are much lower than those for the Gd isotopes.

which gives an indication of the effectiveness of the pseudo $SU(3)$ model in this mass and energy region. Even for the 196 Pt case, which is a soft ($\gamma$-unstable) rotor, the observed strength and its fragmentation are reasonably well reproduced by the model. For the odd-\(A\) 163 Dy case, the $\frac{5}{2}^-$ ground state that is also predicted by the single-particle Nilsson Hamiltonian [18] cannot be reproduced by a pure $SU(3)$ Hamiltonian. However, the single-particle spin–orbit
The experimental ground-band energy spectra for odd–even $^{163}\text{Dy}$ is characterized by a combination of single-particle and rotor features. As for the even–even examples studied, the experimentally determined energy spectra and known $E2$ values were used to determine the parameters of the Hamiltonian. The calculated $M1$ transition spectrum is in reasonable agreement with the relatively large number of experimentally observed low-lying transition strengths.

After determining the eigenstates of the system, the $M1$ transition operator

$$ T^1_\mu(M1) = \sqrt{\frac{3}{4}} \mu_N \sum_\sigma \left[ g_\sigma^o L^\sigma_\mu + g_\sigma^s S^\sigma_\mu \right] $$

with orbit and spin $g$-factors

$$ g_\sigma^o = 1 \quad g_\nu^o = 0 \quad g_\sigma^s = 5.5857 \quad g_\nu^s = -3.8263 $$

(i.e., with no effective $g$-factors) was used to determine transition strengths between the ground state and excited states. Because of the $SU(3)$ symmetry-breaking one-body and pairing terms, the pure $SU(3)$ case with a maximum of four orbital transitions gives way to a more complex transition spectrum which is in much better agreement with experimental results. One finds a number of transitions that are usually close to the observed ones, varying, for example, from five for $^{196}\text{Pt}$ to 11 for $^{158}\text{Gd}$. Also, the centroid of the experimental and theoretical $M1$ transition strength distribution are usually found to lie at approximately the same energy, so that good overall agreement with experiment is obtained. This is especially remarkable for the nucleus $^{196}\text{Pt}$, which from its low-energy spectrum (figure 2) is seen to be a $\gamma$-soft nucleus and not a good rotor. In this case a relatively strong pairing interaction was found to mix different $SU(3)$ irreps and produce the required soft rotor results with its weaker $M1$ transition strengths. Also, for $^{163}\text{Dy}$ (figure 3) the $M1$ transition strength is weaker than for the even–even Gd isotopes smaller smaller $M1$ transition strength could be reproduced.

4. Conclusion

To summarize, the collective properties of the strongly deformed Gd isotopes, as seen through their rotational spectra and enhanced $E2$ transitions, as well as the structure of their $M1$ strengths are modified, but not destroyed, by including non-collective one-body and two-body parts in the interaction. In particular, the observed fragmentation of the $M1$ strength seems to demand pairing, even though the amount required does not wipe out the collective rotational features of these nuclei.

In the case of the gamma-soft $^{196}\text{Pt}$, a stronger pairing interaction serves to ’soften’ the rotor picture by introducing stronger mixing of $SU(3)$ irreps. The results for the low-energy
and M1 transition spectrum suggest that the structure of the pseudo SU(3) model is rich enough to work well even at the limits of the configuration space. The pseudo SU(3) model can also be applied to odd-A nuclei (half-integer spin) where, as expected, the single-particle spin–orbit term is crucial.

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