Correlations between the quadrupole deformation, \( B(E2; 0_1 \rightarrow 2_1) \) value, and total GT\(^+\) strength

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Abstract

This study investigates the effect of four interactions on the ground-state quadrupole deformation, \((\beta, \gamma); B(E2; 0_1 \rightarrow 2_1)\) value; and total Gamow-Teller transition strength, GT\(^+\). Three of the four terms are typical of a SU(3) theory, namely, the second- and third-order SU(3) Casimir invariants, \(C_2\) and \(C_3\), which act as multiples of the identity within representations of SU(3), and the quadrupole-quadrupole interaction which generates intra-representation splitting. The fourth is monopole pairing which generates inter-representation mixing. It is found that changes in \(\beta\) track those in \(B(E2; 0_1 \rightarrow 2_1)\) while the total Gamow-Teller transition strength follows the \(\gamma\) degree of freedom, with the latter two quantities anti-correlated with the former two. These correlations are representative of what is expected for heavy deformed nuclei when described in the pseudo-SU(3) model.

1. Introduction

During the last few years there have been numerous experimental attempts to observe and measure subtle nuclear-structure features like Gamow-Teller transitions or even nuclear \(\beta\beta\)-decay [1]. To give a proper description of the physics requires a good understanding of the related underlying nuclear-structure effects and theoretical investigations normally use either the shell model [3] or the quasi-particle random phase approximation (QRPA) [4]. However, a description of the properties of heavy deformed nuclei, which are typical of the rare-earth and actinide regions, are difficult in both theories. Conventional shell-model investigations face severe computer related problems because the encountered matrix dimensionalities are associated with CPU and storage requirements that are difficult to accommodate even with present day computers. The QRPA, on the other hand, by construction assumes a dominance of vibrational excitations, whereas
in the rare-earth and actinide regions the deformation-driving, quadrupole-quadrupole interaction prevails.

In view of these circumstances, it is essential to employ symmetries, even if only approximate, to simplify calculations for heavy deformed nuclei. Pseudo-spin [6,7] qualifies because even though it is not an exact symmetry it is known to provide a reasonable description of a number of interesting phenomena [8,9]. Indeed, the pseudo-SU(3) model [8], which takes full advantage of the pseudo-spin concept, seems to be a reasonable theory for structure calculations in the rare-earth and actinide regions. This model has been applied to various low-energy properties of strongly deformed nuclei, such as the scissors mode [10], identical bands [11], and even ββ-decay [12–14].

A common feature of these pseudo-SU(3) shell-model applications is that the configuration space is normally limited to a few (frequently just one) irreducible representations (irreps) of SU(3) and a very schematic Hamiltonian. To describe more subtle nuclear properties requires an extended theory. In particular, it seems that a larger configuration space will contribute to an increased precision of the pseudo-SU(3) model description. As a first step towards a generalization of the pseudo-SU(3) model, in recent studies symmetry-breaking terms were introduced into the theory and their effects on low-lying eigenstates were investigated. In addition, phenomena like the γ dependence of B(E2) values and excitation energies and the effects of pairing on K-band mixing were studied and compared to results obtained in the generalized collective model (GCM) [15].

The purpose of this contribution is twofold:

- To study the effect of different interaction terms in the pseudo-SU(3) Hamiltonian on the total β+ Gamow–Teller (GT+) strength. This will provide important information for anticipated future investigations of ββ-decay rates. Even though the pseudo-SU(3) scheme yields a number of simplifications, it is still not possible to work in a truncation free environment in the rare-earth or actinide regions. This complication is avoided here by switching to a real system that can be studied without imposing major truncations and with physics that is similar to what governs in heavy deformed nuclei. In particular, it is well-known that the simplifications connected with the pseudo-spin emerge from a reduction in the strength of the spin–orbit term in the pseudo representation, and this in turn leads to the pseudo-SU(3) scheme as a good approximation. This situation is similar to what is found in light nuclei where SU(3) has been demonstrated to be a good symmetry many years ago. Therefore, as a starting point for studying the dependence of observables on parameters of the SU(3) Hamiltonian, properties of the strongly deformed 20Ne nucleus, which exhibits a rotational structure typical of a quadrupole–quadrupole dominated interaction that is found in rare-earth and actinide nuclei, are considered here.

- To explore and extend recent shell-model results by Auerbach et al. [16] who show for several N = Z nuclei that the B(E2; 01 → 21) value and GT+ strength are anti-correlated. Specifically, by changing single-particle energies the calculated GT+ strengths were shown to decrease monotonously with increasing B(E2; 01 → 21) transition probabilities. In the present case this problem is studied within the framework of the SU(3) shell model with a focus on the influence of different effective
interactions that are typical of SU(3) (including pseudo) hamiltonians. Changes in the intrinsic quadrupole deformation as measured by the standard \((\beta, \gamma)\) geometrical parameters, \(B(E2; 0^+_1 \rightarrow 2^+_1)\) value, and \(GT^+\) strength are studied as a function of the relative importance of the various terms in the model hamiltonian. The earliest attempts to correlate electric quadrupole moments and beta decay matrix elements, on one hand, and the pairing and quadrupole–quadrupole terms on the other, are more than thirty years old [17]. Nonetheless, interest in this problem has been revived because of a renewed appreciation for the fundamental importance of \(\beta^+\) decay rates. Of particular interest in this study is the proposition that pairing tends to increase \(\beta^+\) decay rates while a strong proton–neutron interaction seems to generate the opposite effect [1,4].

The underlying assumptions of the model used in this study are presented in the next section. This is followed by a section in which the model is shown to be sufficient for describing the low-energy properties of a strongly deformed nucleus like \(^{20}\text{Ne}\). The \(^{20}\text{Ne}\) results are then used as a starting point for a more general characterized investigation into the dependence of the ground-state deformation as characterized by the \((\beta, \gamma)\) measures, \(B(E2; 0^+_1 \rightarrow 2^+_1)\) value, and total \(GT^+\) strength on the parameters of the theory. How these physical quantities are correlated with one another is also investigated. The results of the study are summarized in a concluding section.

2. Model properties

A description of the SU(3) and pseudo SU(3) models can be found in a number of contributions (see [10,15] and references therein) and therefore only the basic essentials are reviewed here. The quantum numbers that label the basis functions are derived from the eigenvalues of Casimir operators of a physically motivated, group theoretical classification scheme. Additional multiplicity labels are used as necessary to obtain a unique classification scheme for the basis states:

\[
\left\{ m_\pi [ f_\pi \alpha_\pi (\lambda_\pi, \mu_\pi) , m_\nu [ f_\nu \alpha_\nu (\lambda_\nu, \mu_\nu) ] \right\} \rho (\lambda, \mu) \kappa \frac{L}{S_\pi S_\nu} \{ S_\pi, S_\nu \} S; JM, \tag{1}
\]

where the definition and the meaning of all the labels can be found in [10].

The \(^{20}\text{Ne}\) configuration space used in this contribution is specified through two protons \((m_\pi = 2)\) and two neutrons \((m_\nu = 2)\) in the sd-shell where no higher-shell admixtures were considered. All proton and neutron SU(3) irreps contained in the \([ f_\pi ] = [2] \) and \([ f_\nu ] = [2] \) irreps, respectively, were coupled to all possible total SU(3) irreps labeled by \((\lambda, \mu)\) and no additional truncations measures were applied. This \(LS\)-coupled picture relies on the physical assumption of a strong proton–neutron quadrupole–quadrupole interaction which seems to be applicable in the case of \(^{20}\text{Ne}\) as this nucleus is strongly quadrupole deformed.

Besides components in the hamiltonian that proved to be essential in previous applications of the pseudo-SU(3) model, there are some additional terms that are important
for an appropriate description of nuclear-structure properties. Specifically, in the present analysis the following are included:

- The single-particle energies $\sum_i \varepsilon_i a_i^\dagger a_i$ for protons ($i = \pi$) and neutrons ($i = \nu$), respectively, which explicitly include the spin–orbit splitting. The numerical values for the single-particle energies $\varepsilon_\pi$ and $\varepsilon_\nu$ were taken from a general set that was tested empirically for many cases across a broad range of nuclei [18]. In this contribution the single-particle energies are kept fixed, which is an essential difference from the contribution of Auerbach et al. [16] where changes in the single-particle energies were utilized to investigate deformation-induced correlations between $B(E2; 0^+ \rightarrow 2^+_1)$ values and total GT+ strengths.

- The algebraic quadrupole–quadrupole interaction, $\mathbf{Q}^a \cdot \mathbf{Q}^a$, where the colons indicate a normal ordering of creation and annihilation operators and $\mathbf{Q}^a$ denotes the algebraic quadrupole operator [10]. The normal ordering of the product means that only the pure two-body part of the quadrupole–quadrupole interaction is considered. This choice is dictated by the fact that one-body terms are taken into full account by the single-particle energies. It is expected that this operator dominates the effective nuclear interaction in the rare-earth and actinide region which is the regime of application of the pseudo-SU(3) model.

- The square of the total angular momentum, $J^2$, which is important to accommodate the rotational excitation energy spectra found in well-deformed nuclei.

- The square of the projection of the $z$-component of the total angular momentum in the intrinsic frame of reference, $K^2$. This operator was first derived from considerations that identified the angular momentum operators of the asymmetric rotor model with a linear combination of SO(3) invariants in the SU(3) algebra [19,20]. The $K^2$ operator is required to accommodate band splitting features found in strongly deformed nuclei.

- A monopole pairing interaction, $H^a$, for protons ($i = \pi$) and neutrons ($i = \nu$) which has been shown to induce deformation through irrep mixing (see [15]).

- There is also the possibility of introducing higher-order terms as motivated by the GCM [21]. This can be appreciated by noting that there is a mapping between the SU(3) irrep labels, $(\lambda, \mu)$, and the intrinsic quadrupole deformation variables, $(\beta, \gamma)$ [22,23]:

$$
\begin{align*}
\beta &= \sqrt{\frac{4\pi}{5}} \frac{1}{\mathcal{A}F^2} (C_2 + 3)^{1/2}, \\
\gamma &= \frac{1}{3} \cos^{-1}\left(\frac{C_3}{2 (C_2 + 3)^{3/2}}\right),
\end{align*}
$$

which implies the relations

$$
C_2 \sim \beta^2, \quad C_3 \sim \beta^3 \cos^2 3\gamma,
$$

where $C_2$ and $C_3$ denote the second- and third-order Casimir invariants of SU(3). Adding linear terms in $C_2$ and $C_3$ introduces the possibility of driving the system
Towards specific \((\beta, \gamma)\) values.

Summarizing, this yields the following form for the model Hamiltonian:

\[
H = \sum_\pi \epsilon_\pi a_\pi^+ a_\pi + \sum_\nu \epsilon_\nu a_\nu^+ a_\nu - \frac{1}{2} \chi \cdot \mathbf{Q}^\alpha \cdot \mathbf{Q}^\alpha:
\]

\[
- G_\pi H_P^\pi - G_\nu H_P^\nu + aK_j^2 + bJ^2 + cC_2 + dC_3
\]

where the real numbers \(\chi, G_\pi, G_\nu, a, b, c,\) and \(d\) are strength parameters of the respective interactions.

3. Results

Before proceeding with the proposed study, it is necessary to determine physically reasonable ranges for the parameters of the Hamiltonian. These ranges can be determined by applying the theory to \(^{20}\text{Ne}\) and asking it to reproduce the low-energy properties (excitation energies and \(B(E2)\) values) of this light ds-shell nucleus. The Hamiltonian of Eq. (3) is certainly not general enough to describe the properties of all ds-shell nuclei, nevertheless, for \(^{20}\text{Ne}\), which is strongly deformed, it is sufficient to give a reasonable description of the experimental energies and \(B(E2)\) values\(^1\). A question that

\(^1\) The parameters of the theory were actually fitted to eleven energy levels and four \(B(E2)\) values.
still needs to be answered is whether or not this hamiltonian will also allow for a proper description of more subtle nuclear features like those found in $\beta\beta$-decay.

Fig. 1 depicts the experimental positive-parity energy levels for $^{20}\text{Ne}$ up to about 10 MeV in comparison to results obtained by Wildenthal [24] and the present analysis in which the following hamiltonian was used:

$$H_{\text{Ne}} = \sum_{\pi} \epsilon_{\pi} a_{\pi}^+ a_{\pi} + \sum_{\nu} \epsilon_{\nu} a_{\nu}^+ a_{\nu} - \frac{1}{2} \chi :Q^a \cdot Q^{a'}:$$

$$- G (H_P^2 + H_P^r) + aK_j^2 + bJ^2 .$$

(4)

The four parameters of this hamiltonian (the single-particle energies, $\epsilon_{\pi}$ and $\epsilon_{\nu}$, were taken from [18]) were determined by a least-squares, best-fit procedure aimed at giving optimal agreement with the experimental excitation energies and $B(E2)$ values for the positive-parity states below about 10 MeV. The best-fit values for the parameter were found to be $\chi = 83.28$ keV, $G = 400$ keV, $a = 2.07$ MeV, and $b = 42.6$ keV. The values for $\chi$ and $G$ are in good agreement with general, empirically derived expressions for these parameters [18]. Note that while the SU(3) prediction for the energy of the second $0^+$ state is slightly low, the third is fitted well and the others appear in the correct energy region.

In Table 1 the experimental $B(E2)$ values are compared with the calculated results in which the quadrupole transition operator was taken to be

$$Q = e_{\pi} Q^\pi + e_{\nu} Q^\nu$$

with the definitions $e_{\pi} = (1 + e_{\text{eff}}) e$ and $e_{\nu} = e_{\text{eff}} e$ for proton and neutron charges, respectively. The numerical value for the effective charge, $e_{\text{eff}}$, that was found to yield the best agreement to the experimental data is $e_{\text{eff}} = 0.558$. Again the SU(3) shell-model results are in reasonable agreement with the experimental data.

Using the parameter values of $^{20}\text{Ne}$ as a starting point, the influence of the various operators in Eq. (3) on the $B(E2; 0_1^+ \rightarrow 2_1^+)$ value and total $GT_+^+$ strength was studied. For completeness note that the Gamow–Teller operator for $\beta_+ (\beta_-)$ decay, $\Gamma_{m^+}^+ (\Gamma_{m^-}^-)$, is defined as

$$\Gamma_{m^+}^+ = \sum_k \sigma_{m,k} t_k^+, \quad \Gamma_{m^-}^- = \sum_k \sigma_{m,k} t_k^- ,$$

Table 1
A comparison of available experimental $B(E2)$ values of $^{20}\text{Ne}$, $B(E2)_{\text{exp}}$, with theoretical results of this contribution, $B(E2)_{\text{th}}$ (units: $10^{-2} e^2 b^2$)

<table>
<thead>
<tr>
<th>Transition</th>
<th>$B(E2)_{\text{exp}} \ [10^{-2} e^2 b^2]$</th>
<th>$B(E2)_{\text{th}} \ [10^{-2} e^2 b^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0_1 \rightarrow 2_1$</td>
<td>$3.27 \pm 0.16$</td>
<td>$2.67$</td>
</tr>
<tr>
<td>$2_1 \rightarrow 4_1$</td>
<td>$1.28 \pm 0.11$</td>
<td>$1.26$</td>
</tr>
<tr>
<td>$0_2 \rightarrow 2_1$</td>
<td>$0.116$</td>
<td>$0.137$</td>
</tr>
<tr>
<td>$0_3 \rightarrow 2_1$</td>
<td>$0.010 \pm 0.002$</td>
<td>$0.0033$</td>
</tr>
</tbody>
</table>
where $\sigma_{m,k}$ denotes the $m$-component of the spin operator, $t_k^\pm$ the isospin raising (+) and lowering (−) operator, and the sum runs over all nucleons $k$. The total GT strength, $S_{\pm}$, is defined as

$$S_{\pm} = \sum f \langle f | T^\pm | g \rangle^2,$$

and is subject to the sum rule

$$S_- - S_+ = 3(N - Z),$$

where $N$ and $Z$ denote the neutron and proton number of the initial nucleus. As mentioned in the introduction, Auerbach et al. [16] established a correlation between the quenching of the total GT+ strength and an increase of the $B(E2; 0^+_1 \rightarrow 2^+_1)$ value by systematically varying the single-particle energies. It was pointed out in [16] that these quenching effects, which are the focus of this contribution, influence $S_-$ and $S_+$ in the same additive way and do not affect the Gamow–Teller sum rule.

Recalling the list of interactions in Eq. (3), it is important to note that although the terms $K^2$ and $J^2$ are essential for a description of the energies of members of excited bands in rotational nuclei, their effect not only on the $B(E2; 0^+_1 \rightarrow 2^+_1)$, but on $B(E2; I_1 \rightarrow (I + 2)_{11})$ values in general, is almost negligible. This is demonstrated for $K^2$ (see Fig. 2) only, because the effect of $J^2$ is even less pronounced.

Fig. 2 depicts the upward $B(E2; I_1 \rightarrow (I + 2)_{11})$ value (in units of $10^{-2} e^2b^2$) for yrast states of even angular momentum as a function of the interaction strength, $\alpha$, of the operator $K^2$ in Eq. (3). For each transition $I \rightarrow I + 2$ there are two curves plotted corresponding to the pairing strength parameters, $G = 0.0$ MeV and $G = 0.4$ MeV, respectively, where for simplicity the choice $G_\pi = G_\rho = G$ was made. Although the numerical values for the pairing strength are rather extreme and cover more than the physically reasonable range, the maximum induced change in the $B(E2; I_1 \rightarrow (I + 2)_{11})$ value is less than 10% for all $\alpha$ values considered. Also, $K^2$ does not have any effect on the total GT strength for transitions out of the $L = 0$ ground state since all $K^2$ matrix elements between angular momentum zero states vanish. The same result holds for the $J^2$ operator.

The effect of the SU(3) invariants $C_2$ and $C_3$ on the correlation between $B(E2; 0^+_1 \rightarrow 2^+_1)$ value and total GT+ strength will be considered next. As suggested in the previous section there is a connection between these two operators and the nuclear collective potential which can be easily understood from a GCM point of view.

To illustrate this connection, consider Fig. 3 where the $C_2$ and $C_3$ dependence of the calculated intrinsic quadrupole deformation variables $\beta$ (left) and $\gamma$ (right) for ground states are shown in three-dimensional plots. To understand these plots, recall that the algebraic quadrupole–quadrupole interaction, $Q^a \cdot Q^a$, which contains both one-body and two-body terms, is diagonal in the SU(3) basis with eigenvalue $4C_2 - 3L^2$. This means that in $L = 0$ states the strength parameter $\frac{1}{2} \chi$ (with $\chi > 0$) multiplies $C_2$ which drives the system towards quadrupole deformed configurations with large $\beta$ values. In this contribution the normal-ordered expression $：Q^a \cdot Q^a ：$ is used, but as can
Fig. 2. Upward \( B(E2; I \rightarrow I + 2) \) values (in units of \( e^2b^2 \)) for yrast states of even angular momentum are depicted as a function of the interaction strength, \( a \), of the operator \( K_j^2 \). The dotted line denotes \( B(E2; I \rightarrow (I + 2)_I \) values for \( G_\pi = G_r = G = 0.4 \) MeV and the dashed line gives the data for \( G_\pi = G_r = G = 0.0 \) MeV. The transition angular momenta are listed on the right-hand side.

Table 2

<table>
<thead>
<tr>
<th>Multiplicity (( \lambda, \mu ))</th>
<th>( \beta ) [a.u.]</th>
<th>( C_2 )</th>
<th>( \gamma ) [degree]</th>
<th>( C_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (8, 0)</td>
<td>15.1</td>
<td>88</td>
<td>5.2^o</td>
<td>1672</td>
</tr>
<tr>
<td>3 (4, 2)</td>
<td>11.1</td>
<td>46</td>
<td>21.8^o</td>
<td>286</td>
</tr>
<tr>
<td>2 (0, 4)</td>
<td>8.3</td>
<td>28</td>
<td>51.1^o</td>
<td>-308</td>
</tr>
<tr>
<td>3 (2, 0)</td>
<td>5.7</td>
<td>10</td>
<td>13.9^o</td>
<td>70</td>
</tr>
</tbody>
</table>

be seen from Fig. 3a, the deformation driving feature still applies because \( \beta \) remains close to its maximum value. As the \( C_3 \) strength parameter, \( d \), assumes larger positive (negative) values the system is driven toward larger (smaller) \( \gamma \) values. This feature can be understood from the result \( C_3 \sim \beta^3 \cos(3\gamma) \) (see Fig. 3b).

Fig. 3a shows that increasing the strength \( d \) of the \( C_3 \) term induces a reduction in the \( \beta \) deformation. This phenomenon, which cannot be understood from a collective model point of view, follows from the limitations the Exclusion Principle places upon the
The expectation value of the intrinsic quadrupole variables \( \beta \) (left, Fig. 3a) and \( \gamma \) (right, Fig. 3b) are depicted as a function of the \( C_2 \) and \( C_3 \) interaction strength parameters, \( c \) and \( d \) (units: MeV), respectively, for the ground-state \( L = S = 0 \) wave function.

configuration space. As mentioned above, there is a one-to-one mapping that associates each SU(3) irrep \((\lambda, \mu)\) with an intrinsic \((\beta, \gamma)\) deformation. For \(^{20}\text{Ne}\) which has two protons and two neutrons in the \( N = 2 \) shell, there are four SU(3) irreps (some occurring more than once) for \( L = S = 0 \). These are given in Table 2 along with the corresponding \((\beta, \gamma)\), \( C_2 \), and \( C_3 \) values. With the information contained in Table 2 it is easy to understand the mechanism responsible for the aforementioned phenomenon. If \( d \) is large the ground-state wave function is forced to assume larger \( \gamma \) values. This in turn implies that the less symmetric SU(3) irreps contribute more and for \(^{20}\text{Ne}\) these have smaller \( \beta \) values. As a consequence the expectation value of \( \beta \) decreases with increasing \( \gamma \) and this then explains why large (small) values of \( \beta \) track small (large) values of \( \gamma \), as is illustrated in Figs. 3a and b. Specifically, Fig. 3b demonstrates that a large positive \( d \) value makes the \((0, 4)\) SU(3) irrep dominant in the ground state, indeed, for \( d \to \infty \) one finds that \( \gamma \to 51.1^\circ \) while for \( d \to 0 \) the \((8, 0)\) dominates and \( \gamma \to 5.2^\circ \).

To study the correlation between the \( B(E2; 0_1 \to 2_1) \) value and \( \beta \), consider Fig. 4. Values for \( B(E2; 0_1 \to 2_1) \) (in units of \( e^2 b^2 \)) are plotted as a function of the \( C_2 \) and \( C_3 \) interaction strength parameters \( c \) and \( d \), respectively. The functional dependence is
The $B(E2; 0_1 \rightarrow 2_1)$ value (in units of $e^2b^2$) is depicted as a function of the $c$ and $d$ interaction strength parameters, respectively.

The total CT+ strength is depicted as a function of the $c$ and $d$ interaction strength parameters, respectively.

Very similar to the one shown in Fig. 3a, namely, the $B(E2; 0_1 \rightarrow 2_1)$ value decreases with increasing $C_3$ strength while it is near its maximum value for negative $C_2$ strengths which corresponds to large $\beta$ deformations. This well-known correlation between $\beta^2$ (see scaling) and the $B(E2; 0_1 \rightarrow 2_1)$ value provides a very nice confirmation of the physical correctness of the mapping procedure between $\beta, \gamma$ and $\lambda, \mu$ given in Eq. (2) because the calculation of the expectation value of $\beta$ on the one hand and of $B(E2; 0_1 \rightarrow 2_1)$ on the other hand are based on quite different procedures.

Specifically, the expectation value of $\beta$ is calculated by means of Eq. (2) while the quadrupole operator necessary for the calculation of the $B(E2; 0_1 \rightarrow 2_1)$ value is determined from its quantum mechanical definition using second quantization techniques (see [10]).

Similar to Fig. 4, Fig. 5 depicts the total Gamow–Teller strength $GT_+$ as a function of $c$ and $d$. While $GT_+$ is small for negative values of the $C_3$ strength parameter (corresponding to small $\gamma$ values and large $\beta$ deformation, see Fig. 3) it first increases to reach a maximum at moderately positive $d$ values and decreases for very large $C_3$. 

Fig. 6. Similar to Fig. 3, this figure depicts the expectation value of the intrinsic quadrupole variables $\beta$ (left, Fig. 6a) and $\gamma$ (right, Fig. 6b) as a function of the strength parameters of the quadrupole–quadrupole interaction and the pairing interaction, $\chi$ and $G$ (units: MeV), respectively, where for simplicity $G_p = G_n = G$.

strength corresponding to oblate shapes and small $\beta$ deformations. Comparing these results with the previous ones shows that the functional dependence of the $\Gamma_T^+$ strength is similar to that of $\gamma$ and the opposite to that of the $\beta$ and $B(E2; \Omega_1 \rightarrow 2^1)$ values.

The effect of pairing and the quadrupole–quadrupole interaction on a correlation between the calculated $B(E2; \Omega_1 \rightarrow 2^1)$ value and $\Gamma_T^+$ strength will be considered next. First, however, consider the influence of the interaction parameters $\chi$ and $G$ (for simplicity, $G_p - G_n = G$) on the intrinsic quadrupole deformation variables.

Fig. 6 depicts the functional dependence of $\beta$ (left-hand side) and $\gamma$ (right-hand side) on the strength parameters $\chi$ and $G$ for the $L = S = 0$ ground-state wave function. As expected, $\beta$ increases both with increasing quadrupole–quadrupole interaction strength parameter $\chi$ and decreasing pairing strength parameter $G$ (see Fig. 6a). More interesting are the results shown as part b of Fig. 6 which illustrates that an increasing $\chi$ drives the system towards more prolate shapes while increasing $G$ induces enhanced triaxiality. For reasons similar to the ones discussed in connection with Fig. 3, the functional dependence of $\beta$ and $\gamma$ are opposite in the sense that $\beta$ assumes large values where $\gamma$ is small and vice versa.

Fig. 7 depicts the behavior of the $B(E2; \Omega_1 \rightarrow 2^1)$ transition as a function of changes in the interaction strengths $\chi$ and $G$. Note that the $B(E2; \Omega_1 \rightarrow 2^1)$ value increases with
Fig. 7. Similar to Fig. 3, this figure depicts the $B(E2; 0_1 \rightarrow 2_1)$ (in units of $e^2 b^2$) as a function of the $C_2$ and $C_3$ interaction strength parameters, $c$ and $d$ (units: MeV), respectively.

Fig. 8. Similar to Fig. 5, this figure depicts the total $GT_+$ strength as a function of $\chi$ and $G$ (units: MeV).

ingcreasing quadrupole strength and decreasing pairing interaction strength. It is again evident that the $B(E2; 0_1 \rightarrow 2_1)$ value is closely correlated with the $\beta$ deformation.

Fig. 8 depicts the changes in the total Gamow-Teller transition strength as a function of $\chi$ and $G$. $GT_+$ increases with decreasing $\chi$, and except for $\chi = 0$, with increasing pairing strength. Note that the $\gamma$ degree of freedom shows a very similar functional dependence.

4. Summary

A systematic study of interactions that will be used in future pseudo-SU(3) shell-model calculations, and in particular, in $GT$ transition or even in $\beta\beta$-decay studies, has been presented. This was done by applying the SU(3) shell-model formalism to the strongly deformed ds-shell nucleus $^{20}$Ne. The latter is a good testing ground because what applies in this case carries over to pseudo-SU(3) shell-model analyses.

A reasonable description for the experimental excitation energies and $B(E2)$ values of $^{20}$Ne was obtained with interaction terms that are typically used in the pseudo-SU(3) model. Starting with the best-fit parameter set obtained for $^{20}$Ne, the effects induced
by changing the strength of the various interactions was studied. In particular, how these changes affect the intrinsic quadrupole deformation of the ground state, \((\beta, \gamma)\), the \(B(E2; 0_1 \rightarrow 2_1)\) value, and the total GT\(_+\) strength was considered. Since the results for \(K^2\) and \(J^2\) are very simple, the study concentrated on the effects of the second- and third-order Casimir operators of SU(3), \(C_2\) and \(C_3\), the two-body part of the quadrupole-quadrupole interaction, and a short-range interaction of the monopole pairing type. Although \(C_2\) and \(C_3\) emerge from purely algebraic considerations, they can be interpreted in the framework of the GCM as the lowest-order terms of a collective potential that is expressed solely in terms of quadrupole shape variables and the results obtained in this contribution confirm the relations \(C_2 \sim \beta^2\) and \(C_3 \sim \beta^3 \cos 3\gamma\).

A strong similarity was found between the functional dependence of the ground-state \(\beta^2\) deformation and the \(B(E2; 0_1 \rightarrow 2_1)\) value when the strength parameters of \(C_2\) and \(C_3\) were varied, and also between the \(\gamma\) and GT\(_+\) strength. In addition the results show a very clear anti-correlation between the \(\beta\) and the \(B(E2; 0_1 \rightarrow 2_1)\) value on the one hand and the \(\gamma\) and the GT\(_+\) strength on the other hand, in the sense that the \(\beta\) and \(B(E2; 0_1 \rightarrow 2_1)\) measures assume large values for those \(C_2\) and \(C_3\) parameters which result in small values for the \(\gamma\) and GT\(_+\) strengths and vice versa. Very similar and clearly pronounced correlations were found when the deformation changes were induced by varying the interactions strengths of the pairing and the quadrupole-quadrupole interactions.

These results, obtained in the SU(3) picture, complement and confirm those presented by Auerbach et al. [16] who changed single-particle energies to find a deformation-induced quenching of the total GT\(_+\) strength with increasing a \(B(E2; 0_1 \rightarrow 2_1)\) value. In this contribution both of the intrinsic quadrupole deformation degrees of freedom were considered as well as the relation between \(\gamma\) and total GT\(_+\) transition strength.

The results reported in this contribution were obtained using interactions that are typical of pseudo-SU(3) shell-model applications and they are of particular importance for the ongoing attempts at describing properly the \(\beta\beta\)-decay of heavy deformed nuclei. In these applications it will be necessary to work in a strongly truncated environment, which is why an understanding of the effects of the various parts of the pseudo-SU(3) hamiltonian is of fundamental importance for further applications of the theory.

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**References**