Antisoliton model for fission modes

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Received 1 October 2000; accepted 31 October 2000

Abstract

Antisolitons traveling on the surface of a nucleus are shown to generate highly deformed shapes. The dynamics is based on solutions of the non-linear Korteweg-de Vries (KdV) equation. The theory is used to model the onset of nuclear fission. Aspects of the dynamics, its relation to fluid mechanics, and some conceptual problems arising from the application of the theory are also discussed. © 2001 IMACS. Published by Elsevier Science B.V. All rights reserved.

Keywords: Antisoliton model; Fission modes; Korteweg-de Vries equation; Soliton solutions; Nonlinear systems; Cluster emission

Fission is a division of a nucleus into two parts. This division occurs as a result of extreme deformation, driven by competition between the Coulomb repulsion of the constituent protons and mutual attraction of the system’s nucleons due to the nucleon–nucleon interaction. The dynamics of fission is often simulated using the concept of a liquid drop model (LDM) which parameterizes the nucleus in terms of its surface expanded in terms of spherical harmonics [1]. The liquid is considered to be a nearly incompressible fluid of almost uniform density with the deformation energy of the system given by a sum of the potential and the kinetic energies of the nucleons of the system. However, such a simple model cannot be used to characterize fission without additional considerations because a simple droplet configuration fissions into equal fragments, whereas real nuclei most often fission into unequal mass fragments (asymmetric fission). Indeed, the usual approaches use different parametrizations to describe highly deformed shapes, for example, the two-center shell model or a model that exploits Cassinian ovals [2]. Although such parametrizations are convenient for carrying out calculations, within such frameworks the parameter that controls the neck is only indirectly related to the shape of the system [3]. The purpose of this letter is to offer a more physical interpretation for the parameter controlling the neck. The model that is proposed is consistent with experimental results which show that most often it is asymmetric modes that lead to fission [3–6], fusion [5,6], or cluster emission.

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PII: S0378-4754(00)00290-1
In this letter, highly deformed shapes are associated with solutions of non-linear equations of motion used to describe the nuclear dynamics. The connection between parameters of this non-linear theory and parameters used in the two-center shell model is discussed. The approach describes large amplitude collective motion in nuclei in terms of antisoliton solutions of the KdV equation. As reported, for example, by Sandulescu and co-workers in a review article on this subject [5,6], traveling solutions exist as cnoidal waves on the surface of a liquid drop. The model has been used to investigated more detailed phenomena as well [7,8]. These solitary waves define a new type of excitation that derives from small oscillations on the surface that grow into solitary waves under a dynamics that is governed by non-linear hydrodynamic equations of motion. However, these solutions cannot generate the highly deformed shapes that are of interest in fission processes, especially the formation of a neck between fragments.

The dynamics of the model is based on [5–7] where the usual series expansion of the flow potential \( \Phi \) in terms of spherical coordinates is replaced by non-linear, localized densities that are solutions of the system

\[
\Phi = \sum_{n=0}^{\infty} \left( \frac{r}{R_0} - 1 \right)^n f_n(\theta, \phi, t). 
\]

In this expression \( r \) is the sum of the radius \( R_0 \) of the undeformed nucleus and the deformed shape \( \xi(\theta, \phi, t) \) introduced by a traveling wave in the \( \phi \) direction \( \eta(\phi - Vt) \), and a function \( g(\theta) \) that describes its profile off the equatorial plane

\[
\xi(\theta, \phi, t) = \eta(\phi - Vt) g(\theta). 
\]

The profile \( g(\theta) \) is proportional to \( P_2^2(\cos \theta) \) which is in agreement with the constant velocity of the bump. The recursion relations for the functions \( f_n \) in spherical coordinates are obtained using Laplace’s equation.

The dynamical behavior of the physical system is based on Euler’s equation

\[
\frac{\partial V}{\partial t} + (V \cdot \nabla) V = -\frac{1}{\rho} \nabla P, 
\]

where \( P \) denotes the pressure, \( V \) the velocity, and \( \rho \) the density of the fluid. As experiments [9–12] concerning traveling surface patterns show, the amplitude of the deformation is assumed to be smaller than the angular half-width \( L \). The potential energy that describes a transformation of the initial core to the deformed shape is given as a sum of the surface, centrifugal, and Coulomb energies as well as shell effects

\[
P = E_s + E_{cent} + E_c + E_{sh}, 
\]

Eq. (2) in conjunction with boundary conditions typical of non-linear systems yield

\[
\Phi|_{\Sigma_1} + \frac{V^2 R_0^4 \sin^2 \theta}{2h^2} \xi^2 = \frac{\sigma}{\rho R_0}(2\xi + 4\xi^2 - \Delta \Omega \xi - 3\xi^2 \xi_\phi \cot \theta), 
\]

where \( \Delta \Omega \) is the angular part of the Laplacian. Differentiation of this non-linear equation with respect to \( \phi \) gives the dynamical equation for the evolution of the shape function \( \eta(\phi - Vt) \)

\[
A(\theta) \eta_t + B(\theta) \eta_\phi + C(\theta) g(\theta) \eta_\phi + D(\theta) \eta_{\phi \phi} = 0, 
\]

which is the Korteweg-de Vries equation with coefficients depending parametrically on \( \theta \).
Eq. (6) has soliton and antisoliton solutions

$$\eta_{\text{sol/anti}} = \pm \eta_0 \text{sech}^2 \left[ \frac{(\phi - Vl)}{L} \right],$$

where $V$ is the velocity, $L$ the half-width and $\eta_0$ the amplitude. These two solutions are stable formations in time and move in opposite directions along the axis. The soliton has a positive amplitude whereas the antisoliton has a negative amplitude. These waves also have a special shape-motion dependence: the larger the amplitude the narrower the width and the greater the velocity. This shape-motion dependence is indicated by a connection between the coefficients in KdV equation and the constraints $L$, $\eta_0$ and $V$

$$L = \sqrt{\frac{12D}{C \eta_0}}, \quad V = \frac{g(\theta)C + 3A}{3B}.$$  

These expressions can also be used to experimentally distinguish solitary waves from other linear modes [13]. Two identical solitons or antisolitons shifted by $\pi$ are also solutions to Eq. (6).

Two solitons placed $180^\circ$ out of phase with one another represent two opposing bumps on the surface of the spherical core, whereas two-antisolitons likewise positioned represent two opposing holes in the core. As indicated in Fig. 1, these disturbances move in the same direction with the same angular velocity. In the model introduced here, we are interested in the two-antisoliton scenario because, as can be seen from Fig. 1(b–d), they can be used to describe highly deformed shapes as produced in the fission process. These antisolitons disturbances on the surface of the nucleus are envisioned to be uniformly distributed around their common circular path. They can be used to describe symmetrical shapes, typical of the irrotational LDM, as well as antisymmetrical shapes which arise from shell effects. These waves arise as an excitation on the surface of a nucleus and the faster they travel the faster they form a neck between the fragments.

Fig. 1. Solitons (a) and antisolitons (b–d) solutions of the KdV (MKdV) equation on the surface of nuclei.
We consider them to represent holes below (antisolitons) or particles above (solitons) the core. There is no problem with spurious excitations of the center of mass — it is the center of the coordinate system of the nucleus. Fig. 1 illustrates how this model can reproduce highly deformed shapes that characterize fission; it cannot, however, describe the break-up of a nucleus. The antisoliton solutions move with the same velocity in the same direction. The “final” shape they can describe is shown in Fig. 1(d). After reaching the velocity that produces this shape, the antisolitons continue to travel on the surface in that configuration with conservation of total volume

\[ V = \iiint r^3(\theta, \phi) \sin \theta \, d\theta \, d\phi, \]  

(10)

where the volume of the antisoliton is

\[ V_{\text{anti}} = \frac{4\pi}{3} R_0^3 - \frac{4\pi}{3} \iiint (R_0 + \eta)^3 \sin \theta \, d\theta \, d\phi. \]  

(11)

The fission process occurs at relatively low energies whereas the break-up of a compound system requires high energies. In the two-center shell model the break-up is sensitive to the single-particle motion in the neck region at the scission point: if the single-particle effects generate a depletion from this region the two fragments can separate. In our model, it depends on the energy of the antisoliton pair. The antisoliton pairs travel on the surface of the nucleus many times before forming a neck, which is in agreement with the calculation demonstrated by Hill [14].

Also, we can relate the non-linear parameters in our model \((L, V, \text{and } \eta_0)\) to the parameters required for a description of the fission modes in the two-center shell model. The elongation coordinate, which approaches the distance between the fragments on the way to separation, can be related to the half-width of the antisolitons. The fragmentation coordinate, which measures the deviation from symmetry in the mass distribution, can be connected to a combination of the half-width and the amplitude. The neck coordinate, which gives the thickness of the neck, can be related to the amplitude (velocity) of the antisolitons. These three coordinates are enough for a full description of the fission shapes of the two-center shell model as well as for the two-antisoliton model.

One of the main characteristics of the fission process is the probability of barrier penetration. With initial and final states for a given parent nucleus, this expression for the spontaneous fission is

\[ \exp \left[ (-2^{3/2}h) \times \iiint \left( \text{potential energy} \times \text{reduced mass} \right)^{1/2} \, d\text{distance} \right]. \]  

(12)

In terms of the present model, Eq. (12) can be written as

\[ \exp \left[ (-2^{3/2}h) \times \int_0^{\eta_0} \sqrt{\left(A_{\text{anti}} \times P[\eta] \right)} \, d\eta \right]. \]  

(13)

where \(P[\eta]\) is the energy of the soliton given by Eq. (4) [5–8]. The potential valley along the path in the two-antisoliton model should have an additional minimum with respect to the same path in the liquid drop, as indicated in Fig. 2. The first minimum is related to the harmonic oscillations on the surface and the second follows from the soliton solution.

In this report, we discussed fission modes in terms of non-linear phenomena and introduced a model which can produce highly deformed nuclear shapes. We used the well-known Euler–Lagrange formalism and a knowledge of analytical solutions of the KdV equation on the surface of a liquid drop. As the
velocity and amplitude of the non-linear disturbance increases, the non-linear contributions grow and the solution develops into a stable and localized soliton or antisoliton wave. The model introduced here uses two traveling antisolitons on the nuclear surface. These antisolitons describe shapes very similar to those encountered in fission modes. Three parameters: the half-width, velocity, and amplitude of the antisolitons serve to characterize the formation of the neck on the way to fission. While the model describes shapes on the way to fission, it cannot generate a break-up configuration. A description of the same model in three dimensions will be the topic of a follow-on paper.

Acknowledgements

This work was supported in part by the US National Science Foundation under Grant No. 9603006 and Cooperative Agreement No. 9720652 that includes matching from the Louisiana Board of Regents Support Fund.

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