ALPHA PARTICLE SPECTROSCOPIC STRENGTHS FOR LEVELS POPULATED IN THE $^{38,39,40}$Na($^3$Li, d)$^{38,39,40}$Mg REACTIONS:*

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Abstract: The $^{38,39,40}$Na($^3$Li, d)$^{38,39,40}$Mg reactions have been studied at a bombarding energy of 3.0 MeV. Alpha particle spectroscopic strengths have been extracted for several low-lying levels by a finite-range distorted wave analysis. These are compared with theoretical predictions. The agreement is good both for relative strengths to different levels within a nucleus and for ground-state strengths relative to the $^3$Na($^3$Li, d) result. Strengths calculated using eigenfunctions determined in large shell-model computations agree well with pure symmetry S(3) predictions. The $^3$Na($^3$Li, d) angular distributions for transitions to the ground-state band of $^{38}$Mg are characterized by the laws of the two L-transfers allowed for populating each level. For the 3.5 MeV unassigned parity level in $^{38}$Mg, a two-step calculation gives a better account of the data than does a compound nuclear calculation.

NUCLEAR REACTIONS $^{38,39,40}$Na($^3$Li, d), $E = 3.0$ MeV, measured $\gamma(E\gamma, 0)$.

1. Introduction

The study of α-particle transfer among the light sd shell nuclei (16 ≤ A ≤ 28) plays a role of particular importance in testing our understanding of the spectroscopy of multiparticle transfer reactions for the following reasons. First, shell-model calculations for α-particle stripping and pickup strengths are feasible and have been performed; the SU(3)-SU(4) model provides good approximate wave functions for the low-lying levels of these nuclei. Second, experimental studies are straightforward; for example, high resolution ($^3$Li, d) and ($^3$Li, d) data are readily acquired. Third, to the extent that it is a direct one-step process, the transfer of an α-particle is the simplest possible multinucleon transfer reaction because it involves the transfer of a (0n)³, S = T = 0 cluster; the reaction channels are therefore simplified. Fourth, such

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studies allow one to probe the interplay between rotational and clustering degrees of freedom in characteristic of the structure of these nuclei.

In this article, experimental and theoretical results on the $^{24,26,28}$Ne($^6$Li, d) $^{24,26,28}$Mg reactions performed at a bombarding energy of 32 MeV are presented and compared. Although the three nuclei studied are macroscopically similar (being all basically deformed), they are microscopically quite distinct. In particular, whereas the $^{24}$Ne($^6$Li, d) $^{24}$Mg reaction carries one between two well-defined rotational nuclei whose structures have been the focus of numerous theoretical calculations, that is not the case for $^{28}$Mg in the $^{28}$Ne($^6$Li, d) reaction. And in the $^{28}$Ne($^6$Li, d) reactions, the non-zero angular momentum (I) associated with the target ground state allows for multiple L-transfers in the transition to each final state. Differences reflecting characteristic structures should therefore be expected in the results obtained for the three cases.

The results of a previous study of the $^{24}$Ne($^6$Li, d) reaction, at a bombarding energy of 18 MeV, have been reported by Middleton; also a plane-wave analysis of the $^{28}$Ne($^6$Li, d) reaction was reported at 18 MeV has been published. We have published a letter on our $^{28}$Ne($^6$Li, d) results. There is no previously published study of a-particle transfer on $^{28}$Ne.

Theoretical predictions for relative a-particle transfer strengths, assuming the initial and final nuclear states to have good SU(3)-SU(3) quantum numbers, are available for the light sd shell nuclei $^{6,8}$Li. Absolute strength comparisons are feasible by comparing those results with a recent tabulation of four-particle SU(3)-SU(3) coefficients of fractional overlap. Strength predictions using eigenfunctions determined in large shell-model calculations for the initial and final states are also available for the $^{24}$Ne($^6$Li, d) reaction and the $^{28}$Ne($^6$Li, d) reaction. In this article, the theoretical predictions are compared with each other as well as with the experimental findings. As expected (subsect. 4.2), it is found that the simple SU(3)-SU(3) symmetry limit gives a good first approximation for the spectroscopic strengths.

2. Experimental procedure

A 300–500 nA beam of 32.3 MeV $^6$Li ions from the University of Rochester MP tandem accelerator bombarded gas-cell targets of natural $^{24}$Ne (91% $^{24}$Ne), enriched (99.9% $^{24}$Ne for the study of the ($^6$Li, d) reaction on $^{28}$Ne, $^{21}$Ne, and $^{20}$Ne, respectively. Two gas cells were used for each target, one in the angular range of 5° to 15° and the other in the range of 12.5° to 50°. Measurements at the overlap angles of 12.5° and 15° were performed to normalize the two sets of data. The energy loss of $^6$Li ions in the (mica or gold) entrance window of the gas cell and in the thickness of gas preceding the active region was estimated to be 300 keV, so the effective $^6$Li beam energy was 32.0 MeV.

Outgoing deuterons were momentum analyzed by an Enge split-pole magnetic spectrometer and, in the case of the $^{28}$Ne targets, detected in a Kodak NTB-30
Fig. 1: Spectrum of deuterons from the $^{23}$Ne($^3$Li, d) reaction. The excitation energies of the levels in $^{25}$Mg are indicated in MeV.

Fig. 2: Spectrum of deuterons from the $^{25}$Ne($^3$Li, d) reaction. The dots, open circles and triangles indicate members of three different bands in $^{25}$Mg.
nuclear emulsion plates. Graded aluminum absorbers placed in front of the emulsion prevented tritons from striking the emulsion and slowed the deuterons to the energy at which the emulsion was most sensitive. In the case of the $^{20}$Ne target, a sonic spark counter 9) was used to detect the outgoing deuterons in the focal plane of the spectrograph.

Because the deuterons effectively started to diverge from the exit window of the gas cell rather than from the center of the scattering chamber (as would have been the case with a solid target), the spectrograph could not bring them to a perfect focus. The focal plane position for the best possible resolution was determined by trial and error by moving the spark counter at 45° to the focal plane.

Fig. 1 shows a spectrum measured at 15° for the $^{12}$C($^3$Li, d)$^{24}$Mg reaction. Peaks were identified with known states 10) in $^{24}$Mg by comparing measured energies. The resolution obtained was between 70 and 80 keV. A contaminant peak from the ($^3$Li, d) reaction on the 9% $^{22}$Ne present in the natural Ne target used lies near the $^{24}$Mg ground-state peak but is well resolved from it at all angles. Two contaminant peaks from $^{16}$O($^3$Li, d) transitions to the 6° and 2° levels in $^{22}$Ne are also shown; and there is a strong unidentified contaminant peak on the high energy side of the 6° (6.12 MeV) peak. It is presumed that air adsorbed on the inner surfaces of the teflon tubing connecting the gas cell with the neon cylinder might have partially replaced the neon in the cell during the experiment. In order to get the angular distribution for the 0° (6.43 MeV) level, the $^{22}$Ne($^3$Li, d) measurements were repeated with the polyethylene tubing replaced by a copper tubing. With that arrangement, neither the two contaminant peaks from $^{16}$O($^3$Li, d) nor the unidentified peak appeared.

Figs. 2 and 3 show spectra measured for the $^{25}$Ne($^3$Li, d) and $^{22}$Ne($^3$Li, d) reac-
lations, respectively. Peaks marked by dots, open circles and triangles in Fig. 2 correspond to low-lying levels of the ground-state band (\(K^* = 0^+\)), the first excited band (\(K^* = 2^+\)) and the second excited band (\(K^* = 4^+\)), respectively, in \(^{24}\)Mg.

Absolute cross sections were calculated from a knowledge of the integrated charge, the active thickness of gas and the pressure of gas (4.6 cm of Hg) and are believed to be accurate to \(\pm 20\%\).

3. DWBA calculations

The shapes of (\(^{7}\)Li, d) angular distributions measured in this and previous\(^{11,12}\) studies are characteristic of the transferred orbital angular momentum and change little with changes in \(L\)-value or target mass. This we take as firm evidence that the \(^{7}\)Li, d reaction is predominantly direct at bombarding energies high above the Coulomb barrier. Therefore the well developed formalism of the distorted wave Born approximation (DWBA) was used to analyze the data. The present analysis makes the further assumption that the reaction consists in the direct, one-step transfer of an \(\alpha\)-particle cluster. Two-step and successive-transfer processes are ignored. A test calculation was performed to assess the importance of two-step processes and the results will be discussed in sect. 5. As shown in Fig. 1, the unnatural parity 3\(^+\) level at 5.24 MeV excitation in \(^{24}\)Mg is seen but only weakly, a result that is consistent with a one-step \(\alpha\)-particle transfer mechanism.

The shell-model DWBA code LOLA\(^{13}\) was used to calculate the angular distributions. For the bound state in the final nucleus, a simple target plus \(\alpha\)-particle cluster wave function generated in a Woods-Saxon well of radius \(R = 1.30\) fm and diffuseness \(a = 0.65\) fm was used; the well depth was adjusted to reproduce the known binding energy of the \(\alpha\)-particle in the final nuclear state. The amount of radial mode was fixed by the harmonic oscillator energy-conservation relation

\[
2(N^*+1)+L = \sum_{n=0}^{L} 2n(n+1)+\ell^2
\]

which for (\(d,p\)) yields \(2(N^*+1)+L = 8\). For the \(^{7}\)Li, d system, the wave function of Fair et al.\(^{14}\) was used, with the \(\alpha-d\) system in a relative \(2S\) state. The optical model potential used to generate the distorted waves consisted of the \(^{7}\)Li para-

TABLE 1
Optical model parameters used in the distorted wave analysis of the \(^{24}\)Ne(\(^{7}\)Li, d)\(^{24}\)Mg reaction

| Channel | E (MeV) | R (fm) | \(a_0\) (fm) | \(|W| = 4W_{\text{lab}}\) (MeV) | \(\sigma_{\text{tot}}\) (mb) | \(R_{\text{rms}}\) (fm) |
|---------|---------|--------|-------------|-------------------------------|-----------------|------------------|
| \(^{7}\)Li | 72.6 | 3.72 | 0.87 | 8.6 | 9.24 | 0.81 | 6.79 |
| d | 935 | 3.03 | 0.31 | 42.8 | 295 | 0.72 | 5.03 |
| bound state in \(^{24}\)Mg | * | 3.55 | 0.65 | | | |

* Adjusted to give the correct binding energy as determined by the separation energy procedure.
### Table 2

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<th>E0 (MeV)</th>
<th>J°</th>
<th>K</th>
<th>J'</th>
<th>S(J°, 0)</th>
<th>S(J°, 0)SU(0)</th>
<th>S(J°, 0)SU(0)ZP</th>
<th>S(J°, 0)SU(0)FW</th>
<th>( S(J°, 0)SU(0)FW' )</th>
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<td>0.03</td>
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*Provided by Conte).*

### Table 3

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<th>E0(MeV)</th>
<th>J°</th>
<th>K</th>
<th>J'</th>
<th>S(J°, 0)</th>
<th>S(J°, 0)SU(0)</th>
<th>S(J°, 0)SU(0)ZP</th>
<th>S(J°, 0)SU(0)FW</th>
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<td>1.29</td>
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<td>1.32</td>
<td>2.78</td>
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<td>2</td>
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<td>0.19</td>
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<tr>
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<td>2</td>
<td>0.42</td>
<td>0.14</td>
<td>0.34</td>
<td>0.40</td>
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*Calculated eigenstates do not correspond to well-developed rotational bands.*

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Spectroscopic strengths for \(^{12}\)C(\(\alpha\), \(d\)) transitions exhibit interference patterns in the relative intensities of the transitions. The spectroscopic strengths were determined by analyzing the angular distributions of the \(^{12}\)C(\(\alpha\), \(d\)) transitions. Absolute strengths were obtained by normalizing the spectroscopic strengths to the known cross sections. The results are summarized in Table 2 and Table 3.

The ratio \(\langle d\alpha/d\Omega\rangle_{\text{SU}}/\langle d\alpha/d\Omega\rangle_{\text{SU,FW}}\) gives the spectroscopic strengths. Absolute strengths thus determined are, however, very unreliable because of the uncertainties in the bound-state and optical-model parameters, the strong dependence of the calculated cross sections on the bound-state parameters, and the possible inadequacy of the cluster-transfer assumption in predicting absolute magnitudes. Relative spectroscopic strengths \(S_{\alpha}\) obtained by normalizing each of the three ground-state to ground-state transition strengths to unity, are much more stable and may be compared reliably with

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meters employed in ref. 14 and the average deuterons parameters of Newman et al. 15). These optical and bound-state parameters have proved adequate to fit most of our \(^{14}\)N(\(\alpha\), \(d\)) angular distributions measured on \(\alpha\) shell targets; they are listed in Table 1.

The ratio \(\langle d\alpha/d\Omega\rangle_{\text{SU}}/\langle d\alpha/d\Omega\rangle_{\text{SU,FW}}\) gives the spectroscopic strengths. Absolute strengths thus determined are, however, very unreliable because of the uncertainties in the bound-state and optical-model parameters, the strong dependence of the calculated cross sections on the bound-state parameters, and the possible inadequacy of the cluster-transfer assumption in predicting absolute magnitudes. Relative spectroscopic strengths \(S_{\alpha}\) obtained by normalizing each of the three ground-state to ground-state transition strengths to unity, are much more stable and may be compared reliably with
Table 4

<table>
<thead>
<tr>
<th>$E$(MeV)</th>
<th>$^a$</th>
<th>$^b$</th>
<th>$^c$</th>
<th>S(Li, d)</th>
<th>S(Io) [UO(I)]</th>
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<tr>
<td>0.0</td>
<td>0°</td>
<td>0</td>
<td>(40)</td>
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<tr>
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<td>(40)</td>
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<td>2.56</td>
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<td>(40)</td>
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<td>3.59</td>
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<td>4°</td>
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<td>(40)</td>
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<td>(50)</td>
<td>1.20 (0.06)</td>
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$^a$ Tentative assignments.

$^b$ Errors determined by the least-squares fitting procedure.

Table 5

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<th>Final nucleus</th>
<th>S(Li, d)</th>
<th>S(Io) [UO(I)]</th>
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<td>$^{28}$Ni</td>
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</tr>
<tr>
<td>$^{27}$Mg</td>
<td>0.99</td>
<td>0.33</td>
</tr>
<tr>
<td>$^{26}$Mg</td>
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<tr>
<td>$^{25}$Mg</td>
<td>0.21</td>
<td>0.18</td>
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</table>

theoretical predictions. They are listed in tables 2.4. An additional comparison with theory is possible, based on the empirical observation $^{15}$) that all the ($^6$Li, d) ground-state transitions in the $\text{sd}$ shell can well fitted by the same optical and bound-state parameters. This is done in table 5, where the $^{28,29,30}$Ne($^6$Li, d) ground-state strengths are compared with each other and with the strength for the $^1$Q($^6$Li, d)$^{28}$Ne(g.s.) transition.

4. Results

41. The $^{28}$Ne($^6$Li, d)$^{25}$Mg reaction

Fig. 4 shows the experimental and calculated ($^6$Li, d) angular distributions for transitions to the 0°, 2°, 4° and 6° members of the ground-state band of $^{25}$Mg. Good fits are obtained for the 0° and 2° levels. But the angular distribution for the 4° level is not well reproduced by the calculation. Moreover, the experimental distribution for this level has a shape which is quite different from the one characteristic of the one observed for $L = 4$ transfer at (nearly) the same bombarding energy for other doubly even sd shell targets ($^{14}$, $^{16}$O, $^{24}$Mg) studied. That characteristic $L = 4$ shape is
Fig. 4. Angular distribution of the $^{28}\text{Ne}(^3\text{Li},d)$ transitions to members of the ground-state band in $^{29}\text{Mg}$. The solid curves are DWBA calculations described in the text.

Fig. 5. Angular distribution of the $^{28}\text{Ne}(^3\text{Li},d)$ transitions to members of an excited $K^+ = 2^+$ band in $^{29}\text{Mg}$. The solid curve is as in Fig. 4. The dashed line represents a smooth curve drawn through the experimental points for the $L = 2$ transition to the 1.37 MeV level of $^{29}\text{Mg}$. 
exhibited by the angular distribution for the $4^+$ (6.01 MeV) level in $^{24}$Mg shown in fig. 5.

It must be noted that the spectroscopic strength predicted and observed for the transition to the $4^+$ (4.12 MeV) level is very small compared to the strengths for populating other levels in $^{24}$Mg. When the strength is so small, interference effects from higher order processes are expected. This may account for the uncharacteristic shape of the $L = 4$ angular distribution for the 4.12 MeV level.

The general experience with multinucleon transfer reactions, including ($^7$Li, d), is that angular distributions depend primarily on the $L$-transfer and not on the details of the microscopic structure of the transferred nucleons. As an example, note the similarity of the dashed curve in fig. 5, which is a smooth curve drawn through the experimental points for the transition to the $2^+$ (1.37 MeV) level shown in fig. 4, to the measured angular distribution for the $2^+$ (4.24 MeV) level. The weak ($^7$Li, d) transition to the $4^+$ (4.12 MeV) level is an exception to this statement.

As stated before, the ($^7$Li, d) reaction on an unidentified contaminant obscured the peak corresponding to the $6^+$ (8.12 MeV) level in $^{24}$Mg in spectra taken at angles forward of 25°. The $6^+$ angular distribution shown in fig. 4 includes the contribution from this contaminant. Inspection of the spectra showed that the $6^+$ peak, was essentially free of the contaminant at angles beyond 25°. Thus it was possible to extract a spectroscopic strength for this level with an uncertainty of about 50%.

Fig. 6. Angular distribution of the $^{24}$Mg($^7$Li, d) transitions to three excited levels in $^{24}$Mg. The solid curves are as in Fig. 4.
Fig. 5 shows angular distributions for transitions to the 2\(^+\) and 4\(^+\) members of a \(K^* = 2^+\) band in \(^{24}\)Mg. Fig. 6 shows the distributions for the 0\(^+\) (6.63 MeV) and 2\(^+\) (7.35 MeV) members of an excited \(K^* = 0^+\) band and for another 2\(^+\) level at 8.65 MeV excitation. All the angular distributions except the one for the \(L = 4\) transition to the 6.0 MeV level are well fitted by DWBA calculations using the optical and bound-state parameters given in the last section. For the 4\(^+\) (6.63 MeV) level, however, it has not been possible to fit the measured shape using the same set of optical and bound-state parameters as the set that fits the lower \(L\) transfers. This is in fact the case for all the \(L = 4\) \((^6\text{Li}, d)\) angular distributions measured in the sd shell, which is understandable since they all have nearly the same shape. The calculated \(L = 4\) curve in fig. 5 has been obtained with the optical parameters of table 2 but with a bound-state radius of 1.0 \(a_0\) fm rather than the 1.3 \(a_0\) fm used for fitting all the other \(L\) transfers. Use of the smaller radius produces a much improved fit for all the \(L = 4\) angular distributions measured in the sd shell. [The one exception is in the case of the 4\(^+\) (4.12 MeV) level, as already discussed.] Use of \(R = 1.3 \ a_0\) fm produces a curve similar in shape to that shown in fig. 4 for the transition to the 4\(^+\) (4.12 MeV) level. The cross section calculated with the smaller radius was four times smaller than the one calculated with the larger radius. Therefore, for the sake of consistency, the spectroscopic strengths quoted in tables 2–4 were obtained using a constant bound-state radius of 1.3 \(a_0\) fm for all the transitions. Several other \(^6\text{Li}\) optical potentials were found to give reasonable fits to the \(L = 0\) and \(L = 2\) angular distributions with comparable relative spectroscopic strengths but none of them gave a good fit for the \(L = 4\) distribution, no matter what bound-state radius was used.

Table 2 lists the relative spectroscopic strengths (normalized to unity for the ground-state transition) obtained for several low-lying levels in \(^{24}\)Mg and compares them with theoretical predictions based on wave functions for target and final nuclear states calculated in the SU(3) basis. The numbers in the column headed \(S_{np}[SU(3)]\) are those calculated \(^{1}\) assuming a (0s)\(^n\) p-particle transfer to final states which are assumed to have pure SU(3)-SU(4) symmetry. The \(\langle \mu \rangle\) values assigned to the final states in \(^{24}\)Mg are shown in the fourth column. For the target ground state \(\langle \mu \rangle\) is (80). Since \(\langle \mu \rangle\) for the relative motion of an \(n\)-particle cluster transferred into the sd shell is (80), as is determined by oscillator energy conservation, it follows that the SU(3) couplings considered for \(S_{np}[SU(3)]\) are (80) \(\times\) (80) = \(\langle \mu \rangle\).

Results derived using detailed shell-model calculations have recently become available through the work of Cooper \(^\dagger\). The Darenstadt SU(3) shell-model code was used to generate the initial and final nuclear eigenstates. For 0\(^+\) states in both \(^{20}\)Ne and \(^{24}\)Mg, the basis included all Pauli-allowed sd shell couplings, yielding matrices of dimensionality 21 and 325, respectively. Based on an SU(3) percentage analysis of the low-lying 0\(^+\) states in \(^{24}\)Mg, the \(J^p > 0^+\) calculations for that nucleus were truncated to comparable dimensionality, e.g. see ref. \(^\dagger\). The \(n\)-particle spectroscopic strengths derived using wave functions so calculated for the interaction of Proedem
and Wildenthal 4) are given under the column labeled $S_{0+}(\text{FW})$ in table 2, normalized to unity for the transition to the ground state of $^{24}\text{Mg}$.

The results of the detailed calculation are seen to be in excellent agreement with the results of the simple pure-symmetry calculation. Moreover, the predicted results are generally in very good agreement with the experimental spectroscopic strengths. Note that, as predicted, the $2^+$ level of the ground-state band is much more strongly populated than the $4^+$ level, while the opposite is true for the excited $K^* = 2^+$ band. Despite the difficulty in extracting the angular distribution from the data for the $6^+$ (8.12 MeV) level (as discussed above), it is clear that the level is weakly populated, as predicted.

There is, however, a big discrepancy in the strength for the second $0^+$ level. The experimental strength is too large compared to the prediction by a factor of about four. Similar enhancements are observed in the transitions to the first excited $0^+$ levels in the other light sd shell nuclei that we have studied 1). The results are presented in fig. 7. In this figure, $S_{0+}^{(0^+ \text{ exc.})}$ and $S_{0+}^{(g.s.)}$ denote the spectroscopic strength for ($^3\text{Li}, d$) transitions to the first excited $0^+$ state and to the ground state, respectively, in the nucleus $A_i$ (except for $A_i = 28$), where $A_i = 20, 22, 24, 26$ and 28 refer respectively to the nuclei $^{20}\text{Ne}, ^{22}\text{Ne}, ^{24}\text{Mg}, ^{26}\text{Mg}$ and $^{28}\text{Si}$. For $^{28}\text{Si}$, $S_{0+}^{(0^+ \text{ exc.})}$ denotes the strength for the second excited $0^+$ level, which is at 6.69 MeV excitation. The

![Graph showing the ratio of spectroscopic strength for excited states vs. $A_i$.](image)

**Fig. 7.** Experimental and theoretical values of the ratio of the spectroscopic strength for an excited $0^+$ state to that for the ground state plotted as a function of the mass of the final nucleus populated in ($^3\text{Li}, d$) reactions. The excited $0^+$ state is the second $0^+$ state in the isotope, except for $A_i = 28$, where it is the third.
first excited 0+ level in 28Si was not considered for this figure because it is known to contain strong particle-hole admixtures; no theoretical prediction for the (Li, d) transition to this state is available.

The enhanced strengths observed for the excited 0+ levels point to the probable importance of particle-hole admixed configurations. Indeed, if the second 0+ level in 28Ne were truly (sd)2 with (3p) = (42) and if the (Li, d) reaction transferred an S = T = 0 cluster with 2(N-1)+L = 8, one would expect zero strength for this level, quite contrary to what is observed. The effect of such admixtures (which have not been taken into account when calculating the theoretically predicted strengths) could also be important in the ground-state band. Our findings suggest, however, that for relative strengths it may be of little consequence, particularly if the transfer is strong.

The results of a 25Ne(14Li, l) reaction at 14 MeV reported by Middleton 1 have been analyzed by a plane-wave model by Neogy et al. 2. They obtain a ratio of spectroscopic strengths of 0.30:0.04 for the transitions to the 6.43 and 7.35 MeV levels, which is in good agreement with the ratio from the present experiments given in table 2.

4.2. THE 25Mg(14Li, l)28Mg REACTION

Since the ground-state spin of 25Mg is 3/2+, two L-values can contribute to the transition to each J = 1 final state in 28Mg. By using the characteristic shapes of pure-L angular distributions obtained from the (14Li, d) reaction as a neighboring doubly even nucleus 25Ne, one can separate the contributions of the two L-values and extract the relevant L-dependent spectroscopic strengths. The results obtained in this way have already been published 2 and are included here only for the sake of completeness. They are given in table 3.

It is seen from table 3 that the simple predictions S25Mg(SU(3)) are in good agreement with results from more detailed shell-model calculations using two different effective interactions given in the last two columns and with the observed strengths. In this reaction, as in the previously studied 23Mg(14Li, d) reaction 23, transitions to members of the ground-state band proceed via the lowest allowed L-transfer. But no similar empirical result has emerged from the systematic study of transitions to members of excited bands.

There are basically two reasons for the good agreement between the simple and the more detailed ("realistic") predictions on S25Mg, as shown in tables 2 and 3, for the (14Li, d) reaction on both 25Ne and 28Si. First, detailed shell-model calculations 7,8,11,12 for 25Ne and 28Si show that the space symmetry quantum number fJ and the SU(3) quantum numbers (ij) are approximately good for the low-lying states of these nuclei (typically, c. 70% parity by intensity). Second, the n-particle is an SU(4) scalar and has fJ = 4. Therefore the fJ couplings in the transfer process are all uniquely determined. A dominant space symmetry is the target couples only to the same (modular 4) large component in the residual nucleus; similarly, a small component couples only with the same (modulo 4) small component.
Fig. 8. Angular distributions of the $^{24}$Ne($^{12}$C,$d$) reaction to some levels in $^{26}$Mg. The solid curves are as in Fig. 4.

Fig. 9. Angular distributions of the $^{24}$Ne($^{12}$C,$d$) reaction to the 3.9 MeV level and to two sets of unassigned levels in $^{26}$Mg. The solid curve is as in Fig. 4. The dashed lines are least-squares fitted curves for the unassigned levels, as described in the text.

Fig. 10. Angular distributions of the $^{24}$Ne($^{12}$C,$d$) reaction to two levels in $^{26}$Mg. The solid curve is as in Fig. 4. The 5.69 MeV level is listed as $^2$ in Table 1, but in angular distribution indicates that it probably has a $J^p$ of $3^-$ or $4^-$. 
Cross-symmetry [1] considerations vanish identically. This is true only for the transfer of an $\alpha$-particle. For the transfer of other clusters, cross-symmetry considerations will in general contribute, enhancing the possibility of small components in the wave functions introducing strong coherence effects in the transfer amplitudes. Though SU(3) couplings provide no comparable rigorous selection rule, there is a strong tendency towards reinforcing this behavior because of the strong correlation between the Hamiltonian and $Q^2$, essentially the Cassini invariant of SU(3).

4.3. THE $^{24}\text{Ne} + ^{1}d\to ^{25}\text{Mg}$ REACTION

The angular distributions measured in the $^{24}\text{Ne} + ^{1}d\to ^{25}\text{Mg}$ reaction are presented in figs. 5-10, where they are arranged in the order of increasing excitation energy. They are not collected into groups belonging to different rotational bands, as was done in the case of the $^{24}\text{Ne} + ^{1}d\to ^{24}\text{Mg}$ reaction, because the band structure in $^{24}\text{Mg}$ is unclear. The solid curves in these figures are DWBA curves calculated using the parameters of table 1, except for the $4^+ (5.47 \text{MeV})$ level for which, as for the $4^+ (6.0 \text{MeV})$ level of $^{24}\text{Mg}$ discussed above, a bound-state radius of $1.0 \text{fm}$ was employed.

The experimental resolution was adequate to resolve the members of the two-triplet at excitation of 4.32-4.35-4.38 MeV and 4.64-4.80-4.97 MeV in $^{24}\text{Mg}$. The spin-parity assignment listed for these levels are $4^+ - 2^+ - 3^+$ and $2^+ - 4^+ - 4^+$, respectively. Since the three levels of each triplet are populated by three different $L$-transfers and the shapes of the angular distributions for the different $L$-transfers are distinct, it was possible to extract the separate contributions of the individual levels by a least-squares fitting procedure. In this procedure, which is similar to the one adapted for obtaining the contributions of individual $L$-transfers in the $^{24}\text{Ne} + ^{1}d$ reaction, the angular distributions measured for isolated levels in $^{24}\text{Mg}$ and $^{25}\text{Mg}$ were taken to give the characteristic pure-$L$ shapes.

Because the known isolated, unnatural parity levels like the $3^+ (5.24 \text{MeV})$ level in $^{24}\text{Mg}$ and the $3^+ (3.94 \text{MeV})$ level in $^{25}\text{Mg}$ are only weakly excited, as can be seen in figs. 1 and 3, it was assumed that the contribution of the $3^+$ level to the first triplet in $^{24}\text{Mg}$ was negligible. Thus, the contributions to the first triplet were taken to be only from the $4^+$ and $2^+$ levels, of which the latter was taken to be a $4^+$ level since (i) the angular distribution measured for this triplet was not pure $L = 4$ in character, as it would have been had the triplet consisted of $4^+ - 2^+ - 3^+$ and (ii) it is unlikely that a negative parity level would occur in $^{24}\text{Mg}$ at as low an excitation as 4.33 MeV.

In fact, when the angular distribution for the first triplet was fitted by a mixture of experimentally measured $L = 2$ and $4$ curves, a very good fit was obtained, as is shown by the upper dashed curve in fig. 9. The $L = 2$ curve measured for the $1.81 \text{MeV}$ level in $^{24}\text{Mg}$ and $L = 4$ curve measured for the $6.01 \text{MeV}$ level in $^{25}\text{Mg}$ were used in this least-squares fitting procedure. The least-squares fitting for the second triplet in $^{25}\text{Mg}$ involved using three $L = 2$ and $4$ curves, and the $L = 4$ curve measured for the ground state of $^{25}\text{Mg}$. The fit is shown as the lower dashed curve in fig. 9.

New information on the spins and parities of these levels in $^{25}\text{Mg}$ may be
inferred from the results of the present experiment. It is to be emphasized that the following are only suggested and are not definite assignments. They depend on the observation that the known unnatural parity levels are only weakly excited in the \((^3\text{Li}, d)\) reaction. This result is consistent with a one-step \(p\) particle transfer mechanism, but it is not a quantitative rule for distinguishing between natural and unnatural parity levels. (i) The 4.33 MeV level, listed \(^{19}\) as having a spin of (2), is probably a natural parity level because it is strongly populated in the reaction. Hence it most likely has \(J^p = 2^+\). (ii) The 5.29 MeV level, listed as being \(^{20}\), is very weakly populated in the reaction and hence is probably an unnatural parity level. (iii) The 5.69 MeV level, listed as having a spin of (1), is likely strongly excited and in all probability is a natural parity level. The angular distribution measured for this level is shown in fig. 10. It is definitely not characteristic of an \(L = 1\) transfer and is consistent with \(L = 3\) or 4 angular distribution, which suggests a \(J^p\) of \(3^+\) or \(4^+\) for this level.

Table 4 lists the relative spectroscopic strengths measured and calculated for the \(^{22}\text{Ne}(^3\text{Li}, d)\) reaction. The calculated results are the simple SU(3) prediction \(^{19}\), in which good SU(3)-SU(4) wave functions are assumed for the initial and final states. Five of the experimental strengths have been given with errors; these are the errors determined by the least-squares fitting procedure for the corresponding (unresolved) levels. To these errors must be added in quadrature the \(\pm 20\%\) uncertainty that all the spectroscopic strengths given in tables 2-5 have.

Since the structure of levels is \(^{22}\text{Mg}\) is unclear, the \((\alpha)\) assignments given in table 4 are open to challenge. However, additional support for the present assignments is provided by results from the \(^{24}\text{Mg}(t, p)^{24}\text{Mg}\) reaction studied by Hinds \textit{et al.} \(^{21}\). In that study the 2\(^{+}\)(2.58 MeV) level was observed to be more strongly populated than the 2\(^{+}\)(1.81 MeV) level by about a factor of four. If the 2.94 MeV level is dominated by \((\alpha)K^* = (40)^+\) and the 1.81 MeV level by \((\alpha)K^* = (40)^0\), the expected favoring of the upper of the two levels in the \((t, p)\) reaction is by about a factor of two. A similar interpretation, but with \((\alpha) = (0, 2)\), gives a prediction favoring the lower of two levels by a factor of three, consistency with what is observed.

4.4. ABSOLUTE GROUND-STATE STRENGTHS

In tables 2-4, relative \(p\)-particle spectroscopic strengths have been given for \((^3\text{Li}, d)\) reactions on \(^{19},^{21,^{22}}\text{Ne}\), respectively, each normalized relative to unity for the ground state transition. In table 5, the absolute ground-state to ground-state transition strengths for the three reactions, normalized relative to unity for the \(^{19}\text{O}(^4\text{Li}, d)^{20}\text{Ne}\) (g.s.) strength, are presented. The experimental strengths are given in the second column and the strengths predicted using pure-symmetry wave functions for the initial and final nuclear states are given in the third column. For the transitions to the ground states of \(^{24}\text{Mg}\) and \(^{24}\text{Mg}\), strength predictions calculated using wave functions from detailed shell-model calculations with the Predomin-Wildenthal interaction are also available. These differ from the pure-symmetry predictions by less than \(10\%\).

Except for the \(^{22}\text{Ne}(^3\text{Li}, d)\) transition, there is very good agreement between the
predicted and observed absolute strengths. In view of the fact that each experimental number in this table involves the measurement of an absolute cross section, the dispersion of a factor of two between theory and experiment found for the $^{18}$Ne($^7$Li, d) case may not be significant.

5. Higher order processes

The conclusions that have been based on the assumption of a direct, one-step α-particle cluster transfer mechanism. Most of the data on the population of natural parity levels can be understood within this framework. But the shapes of angular distributions for higher L-transfers ($L = 4, 6$) are poorly fitted; as pointed out in subsect. 4.1, a reduced bound-state radius is needed for fitting the $L = 4$ shape. This may signify some problems with the reaction mechanism involved.

The effect of higher order processes, and in particular two-step processes, in the population of both natural and unnatural parity levels must therefore be investigated.

A finite-range coupled channels Born approximation (CCBA) code which can perform this calculation is currently being developed by Nagel and Kühnel [15], who have started an analysis of the $^{18}$O($^7$Li, d) reaction with that code [16]. But only the zero-range CCBA code CHUCK [17] was available to us and it is limited in the number of channels it can handle. Therefore our study of higher order processes was restricted to a consideration of the mechanism of population of just one unnatural parity level in the $^{28}$Si($^7$Li, d) reaction. Contributions from two-step and compound nuclear (CN) processes to the population of this level were investigated. Because of program limitations, the important channels could not all be included in the two-step calculation and thus its results are incomplete.

Fig. 11 shows the angular distribution measured for the 3+$ level at 5.24 MeV excitation in $^{22}$Mg. The level presumably belongs to the $(4d)^2\pi = (642)^1$ band.

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**Fig. 11.** The angular distribution measured in the $^{24}$Mg($^7$Li, d) reaction for the 3+ (5.24 MeV) level in $^{22}$Mg in comparison with two-step (solid line) and compound nuclear (dashed line) predictions.
natural parity members of which are much more strongly populated, as shown in fig. 5. Shown in fig. 11 are the absolute magnitude and the shape of the angular distribution calculated assuming the $3^+$ level to be populated by a two-step process (solid line) and by a CN process (dashed line).

The zero-range coupled channels code CHUCK was used to perform the two-step calculation. Two reaction paths were considered: (i) inelastic excitation of the $2^+$ (1.63 MeV) level of $^{32}$Ne followed by $E = 4$ e-particle transfer to the $3^+$ level of $^{34}$Mg, and (ii) e-particle transfer to the $4^+$ (6.0 MeV) level of $^{32}$Mg followed by an inelastic transition. The optical model and bound-state parameters of table 1, which (as described in sect. 4) give good fits to $(^6$Li, d) angular distributions when used in finite-range DWBA calculations, do not do so when used in zero-range DWBA calculations. For the zero-range two-step calculation, therefore, the set III parameters of Fairbrother et al. 32 were used. The details of the calculation are similar to those described by Gunn et al. 29 in their analysis of the $^{16}$O$(^6$Li, d) reaction to unnatural parity levels in $^{36}$Ne.

The CN calculation was performed with the code STATIE 10. Five open channels, those for protons, neutrons, deuterons, e-particles and $^6$Li, were included. The optical model and level density parameters used were those employed by Hunter et al. 30 in their analysis of the $^{16}$O$(^4$He, $^6$Li)$^{32}$Ne reaction. The compound nucleus formed in this reaction is the same as in the $^{28}$Ne$(^6$Li, d) reaction and the dominant decay channels are the same in the two cases. Because of program limitations, only the two most important reaction paths could be included in the two-step calculating; accordingly, its predictions cannot be considered conclusive. But in the CN calculation, all the important channels were included and its predictions are correspondingly more reliable, particularly as the parameters used were the same as those which accounted well 29 for the absolute cross sections measured in the $^{16}$O$(^4$He, $^6$Li) reaction.

A comparison of the calculated and observed angular distributions shown in fig. 11 clearly indicates that the CN mechanism fails to explain both the magnitude and the shape of the measured angular distribution. The two-step process appears to be a more plausible mechanism for explaining the data. Inclusion of more reaction paths in the calculation may result in an improved fit to the data.

As mentioned before, a more systematic investigation of higher order processes for populating natural as well as unnatural parity levels may be necessary to understand the reaction mechanism. Unfortunately, a CCBA code which would take into account all the important channel couplings is presently unavailable to us.

6. Conclusion

The $(^6$Li, d) reaction on Ne targets at 32 MeV populates levels in Mg nuclei predominantly by direct transitions, except for unnatural parity levels in the even-A nuclei. The angular distributions for low-$L$ transfers are generally well fitted by
SPECTROSCOPIC STRENGTHS

DWBA calculations which assume that the reaction consists in the direct transfer of an α-particle cluster. An unsatisfactory feature is that L = 4 angular distributions are not well fitted by the same bound-state parameters as those that fit lower L-transfers.

In favorable cases, the (4Li, d) reaction can be used to assign or limit the range of L-values of levels. Two general features of the reaction are useful for this. First, the reaction populates unnatural parity levels only weakly and, second, the angular distributions are characteristic of the transferred L-value. The results obtained suggest a J° of 2° for the 4.33 MeV level in 25Mg and a J° of 3° or 4° for the 5.69 MeV level.

The spectroscopic strengths extracted by the DWBA analysis are in good agreement with theoretical predictions. Comparison between experiment and theory has been made both for the relative α-particle strengths within a nucleus and for the absolute ground-state strengths for different nuclei. For the 27Ne(4Li, d) transitions to members of the ground-state band of 25Mg, the lower of the two L-transfers allowed in each case is observed and predicted to dominate.

On the other hand, the measured ratios of α-widths for the first excited 0° states in 24Mg and 26Mg compared with the ground-state widths are higher than predicted theoretically assuming good SU(3)-SU(4) wave functions, and the same is true for several other even nuclei between A = 20 and 28. In the case of the 6.72 MeV 0° state in 26Ne, Fortune et al. 31) have explained the enhanced α-width as due to mixing with a higher 0° state at 8.3 MeV for which it is likely that (sd)³(fp)² or (fp)⁴ is the dominant configuration. A similar explanation may apply to the other example shown in fig. 7.

Theoretical strength predictions which assume good SU(3)-SU(4) wave functions for the initial and final nuclear states agree well with those which use wave functions generated in large shell-model calculations. This is due to two reasons: the SU(3)-SU(4) coupling scheme affords good quantum numbers for low-lying levels of the light to shell nuclei and the α-particle is an SU(4) scalar.

A two-step calculation accounts better for the population of the unnatural parity 3° (5.24 MeV) level in 25Mg than does a compound nuclear calculation, but the fit to the angular distribution is far from satisfactory. More reaction paths must be included in the calculation to determine whether the fit can be improved.

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