The shell model—dead or alive?

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The shell model is the most robust microscopic theory for addressing nuclear structure questions. Unfortunately, it is only as good as the input Hamiltonian and the appropriateness of the selected model space, and both of these elements usually prove to be a significant challenge. There are three basic types of theories: 1) algebraic models, boson and fermion, which focus on symmetries, exact and approximate, of a Hamiltonian and usually use model spaces that are severely truncated; 2) numerically oriented schemes that accommodate larger spaces but rely on special techniques and algorithms for producing convergent results; and 3) models that employ statistical concepts, like statistical spectroscopy of the 70s and 80s and Monte Carlo methods of the 90s, schemes that are not limited by the usual dimensionality considerations. These three approaches, with their pluses and minuses, are reviewed. A new scheme is then suggested that incorporates the best characteristics of these three approaches to yield a symmetry driven theory that is not limited to simplified spaces and Hamiltonians while remaining tractable for large-scale calculations of the type required for testing a theory against experimental data and for predicting new physical phenomena. Special attention is focused on unifying concepts linking the shell model with the much simpler and very successful mean-field and collective-model theories. As an example of a modern shell-model theory, some recent results for M1 (scissors) transitions in deformed nuclei are presented.

Keywords: Shell model; realistic Hamiltonians; M1 transitions; scissors mode

El modelo de capas es la teoría microscópica más sólida capaz de describir la estructura nuclear. Desafortunadamente, solo puede ser tan bueno como el hamiltoniano que se use y como el espacio de Hilbert escogido, y seleccionar ambos elementos es un importante desafío. Existen básicamente tres tipos de teorías: 1) modelos algebraicos, bosónicos y fermiónicos, que se concentran en simetrías (exactas o aproximadas) del hamiltoniano y generalmente usan espacios que están muy truncados; 2) esquemas orientados al cálculo numérico que pueden trabajar en espacios muy grandes pero requieren técnicas especiales y algoritmos que aseguren la convergencia de los resultados; y 3) modelos que emplean conceptos estadísticos, tales como la espectroscopía estadística de los 70s y 80s y los métodos de Monte Carlo de los 90s, en los que no aparecen las limitaciones dimensionales usuales. En este trabajo se hace una revisión de estas tres aproximaciones, con sus ventajas y desventajas. Luego se propone un nuevo esquema que incorpora las mejores características de los tres para dar lugar a una teoría basada en simetrías que no está limitada a espacios y hamiltonianos muy simplificados y que es útil para los cálculos de gran escala necesarios para probar la teoría frente a la información experimental y para predecir nuevos fenómenos físicos. Se presta particular atención a conceptos unificadores que ligan al modelo de capas con teorías más sencillas y muy exitosas como las de campo medio y los modelos colectivos. Como un ejemplo de una teoría moderna de modelo de capas se presentan algunos resultados sobre transiciones M1 (modos "de tijera") en núcleos deformados.

Descripciones: Modelo de capas; hamiltonianos realistas; transiciones M1; modos de tijera

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1. Review of existing shell-model strategies

The shell model remains the most robust microscopic theory for studying the structure of atomic nuclei. First indications of a shell structure in nuclei came with the success of the single-particle model proposed independently by Mayer and Hazen, Jensen, and Suess*. A key feature of this very simple and yet extremely significant model is a strong spin-orbit interaction which is essential to achieve shell closures at the experimentally observed "magic" nucleon numbers (2, 8, 20, 28, ...). Most modern microscopic shell-model theories build on this simple foundation by adding two-body (and sometimes higher particle rank) residual interactions to the single-particle (mean) field picture.

The collective model, which likens the dominant degrees-of-freedom of a nucleus to be those of a viscous liquid drop, is a complementary approach to nuclear structure put for-
Comparing $\text{Sp}(2, R)$ with $\text{Sp}(6, R)$ Calculations

4. Conclusions

We have studied the $\text{Sp}(2, R)$ model of Arickx et al. [2] for light nuclei and its extended version (pseudo-$\text{Sp}(2, R)$) for heavy nuclei. The Hamiltonian of the Refs. 8–10 was used. The basis states have good angular momentum, contrary to those used by Arickx et al. We applied the $\text{Sp}(2, R)$ model with the same Hamiltonian as in Refs. 8 and 9 to all nuclei discussed in Refs. 8 and 10 including also $^{12}$C. We could prove that the energy values do not change significantly when the same parameters are used for the complete description (the deviations between the $\text{Sp}(6, R)$ and $\text{Sp}(2, R)$ calculations are getting larger towards lighter nuclei). The $B(E2)$-transitions within a band were lowered by about 20% while those between bands did change more. When we tried to fit the $B(E2)$-transitions we observed that good agreement can be obtained for the intra-band transitions. If we also wanted to fit the inter-band transitions the agreement in the energies was not as good.

The overall agreement in the energies was better for the heavy nuclei (where the pseudo-symplectic model was applied [9]). The parameters obtained, after applying a fit, do not deviate significantly from the original parameter values of the $\text{Sp}(6, R)$ model. Note, however, that the cases discussed are axially symmetric nuclei (though for $^{24}$Mg there is an appreciable triaxiality) which have a well defined large deformation so that only one symplectic irrep dominates at low energies.

Using more realistic Hamiltonians will mix symplectic irreps, which is relatively easy to deal with in the $\text{Sp}(2, R)$ model space compared to the $\text{Sp}(6, R)$ model space, the latter approaching the complete shell model space when all symplectic irreps are taken into account.

Finally we would like to mention that the fitting routine for diagonalizing the $\text{Sp}(6, R)$ Hamiltonian within the $\text{Sp}(2, R)$ model space is available [21].

We thank the referee for pointing out that the same type of calculations were intended by P. Rochford in his Ph.D. thesis [22]. However, he did not use the traceless quadrupole-quadrupole interaction which led then to a lack of convergence. The Hamiltonian used here does not suffer from this defect. Interesting to note is that, though there were problems of convergence, P. Rochford also obtained the result that the stretched irreps dominate and that for triaxial nuclei this is not the case. We saw that for $^{24}$Mg, which has an appreciable triaxiality, the stretched irreps still dominate, i.e. further investigation in this direction have to be done.

Acknowledgments

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References

card by Bohr and Mottelson. This scheme considers deformation to be the single most characteristic feature of atomic nuclei. Within this setting, even-even nuclear systems rotate, and they have an intrinsic static deformation, and vibrate. For pairs of closed shell systems, vibrations about a spherical equilibrium shape describe the dynamics while away from closed shells the vibrations are superimposed on a non-spherical (either axial or triaxial) distribution that rotates as a quantum liquid droplet. This vibration-rotation picture works surprisingly well, especially considering the fact that the nucleon degrees-of-freedom are completely submerged in the collective motion. A modern version of the theory is the so-called generalized collective model due to the work of Greiner and his students. The latter puts a harmonic oscillator face on the theory and offers solutions in terms of a five-dimensional oscillator structure with the relative importance of anharmonicities related to complexity of the corresponding potential energy surface.

The Nilsson model is a scheme that combines the collective and single-particle schemes into a unified theory for odd-A nuclei. At the heart of this approach is the assumption that the last unpaired particle (proton or neutron) is weakly coupled to an even-even core that can described in terms of a simple collective model picture. The motion of the “last” unpaired particle is assumed to track that of the core in an adiabatic manner, with the particle-core coupling terms represented through scalar products of the core and particle angular momentum operators. This remains the philosophy behind modern mean-field theories which typically take pairing correlations and higher-order deformation of the core configuration into account. The success of this phenomenology, for example in reproducing and even predicting superdeformed phenomena, rivals the very best of modern shell-model theories, and furthermore, it is simple.

Providing a microscopic description of collective phenomena in nuclei remains the number one low-energy nuclear theory challenge. Addressing this matter from a traditional $jj$-coupled shell-model perspective leads to extremely large model spaces and computational requirements that for all but relatively light nuclei are untenable. The limitations of such an approach have been pushed back through the introduction of efficient numerical methods, such as the Lanczos scheme, which can be used to obtain the lowest eigenvalues and eigenvectors of large sparse matrices. Indeed, the Lanczos algorithm is designed to generate solutions in a recursive manner via the action of the system Hamiltonian on a well-chosen starting vector. Typically, with modern computers, this approach renders calculations for $fp$-shell nuclei doable. The most efficient of modern schemes exploits the fermion nature of the system by representing the so-called M-Scheme Slater determinants as bit strings with a “1” denoting an occupied single-particle state and a “0” an unoccupied one. With the Hamiltonian given in a complementary second-quantized representation, its action can be determined using logical rather than arithmetic operations which improves computational efficiency.

Bonafide algebraic shell-model schemes, such as the SU(3) model of Elliott, can be used to describe collective phenomena. Indeed, the motivation behind the use of algebraic methods stems from a desire to incorporate symmetries that are responsible for collective behavior into the shell-model picture from the onset. If this is done, the model space can be partitioned into basis states of irreducible representations (irreps) of the corresponding symmetry group and a symmetry dictated truncation of the model space can be invoked. Although conceptually simple, its implementation is usually compromised because it requires one to know the coupling coefficients of the underlying symmetry group. These are available for SU(2) (hence the popularity of $jj$-coupling codes) and SU(3) (some analytic results and general codes for all cases of interest) however not in as simple a form as for SU(2). This accounts for the fact that the number of applications of the latter are rather limited. (As noted below, however, recent developments put SU(3) shell-model calculations within reach of non-expert users.)

No scheme developed to date is capable of embracing both collective and non-collective effects within a simple common framework. The so-called Monte Carlo shell-model code can handle huge spaces, but it does not give detailed results regarding transition strengths. Its limitations are similar to those encountered much earlier in statistical spectroscopy where all observables are expressed in terms of expectation values of the transition operator and system Hamiltonian. This prescription fails because it is not designed to handle dual (two-scale—collective and non-collective; many-particle and single-particle) phenomena. A theory that is to the $jj$-coupling and algebraic models what the Nilsson scheme is to the single-particle and the collective models is needed.

2. Proposed new oblique shell-model scheme

Our group at Louisiana State University (LSU) in Baton Rouge is working on a new shell-model scheme (code name "Red Stick") that seeks to incorporate the best of the currently available shell-model technologies, both algebraic and numeric. The philosophy behind the scheme is relatively simple: since the LSU SU(3) code is based on an M-Scheme logic, it can be merged with the scheme technology that follows from a $jj$-coupling philosophy into a dual-base m-scheme code. The overlap matrix of the two sectors (collective—SU(3) and single-particle—spherical basis) will obviously be nonzero. Leading SU(3) configurations will be combined with leading spherical configurations to form a relatively small but non-orthogonal or oblique basis. The challenge will be to calculate the overlap matrix elements, which couples the single-particle and collective sectors.

This is a nontrivial challenge because experience teaches us that the expansion of collective states in terms a single-particle picture normally requires nearly the full model space. However, it is the overlap, and not an expansion of one set in terms of the other that is required. As a consequence, the
solution depends linearly rather than quadratically on the dimensionality of the model space. Since this is the topic of another talk, we refer the interested reader to that write-up entitled “Basis Generator for M-Scheme SU(3) Shell Model Calculations” by V. Gueorguiev and J.P. Draayer. To repeat, the “Red Stick” project is a work-in-progress. The objective is to extend the successful single-particle Nilsson concept to a many-particle shell-model theory that is conceptually simple and easy to use. The example that follows is an application of the existing SU(3) code. It illustrates the suitability of the SU(3) scheme for describing collective phenomena and at the same time underscores the importance of simultaneously incorporating mixing associated with an underlying single-particle picture.

3. M1 transition strengths in deformed nuclei

The low-lying M1 transition strength distribution observed in nuclei of the rare earth and actinide regions [2, 3] appears to reflect on both the collective and non-collective aspects of the nuclear interaction. A study of this decay mode thus allows one to probe how well a nuclear model can incorporate these complementary features [4–7].

The earliest interpretation of the “scissors” mode by Lo Iudice and Palumbo [8] in 1978—six years before it was detected—was that of a collective magnetic-dipole state of two spheroids, one representing protons and the other for neutrons, exercising rotational oscillations relative to one another. A feature that remains unexplained in such a model with only a single collective degree of freedom is the observed fragmentation of the mode, that is, the break-up of the M1 strength among several levels closely packed and clustered around a few strong transition peaks in the energy region between 2 and 4 MeV.

A scheme that goes beyond the Two Rotor Model (TRM) of Lo Iudice-Palumbo and gives a reasonable description of the complex experimental data is the pseudo SU(3) model. Basic elements of the pseudo SU(3) scheme and some results generated using it are presented in this section.

The pseudo SU(3) model is a many-particle shell-model theory that takes full advantage of pseudo-spin symmetry [9, 10], which in heavy nuclei is manifest in the near degeneracy of the orbital pairs [(l − 1)j = ± 1/2, (l + 1)j = ± 1/2]. Since the Lie algebra of the pseudo oscillator is the same as for the normal oscillator and therefore the pseudo SU(3) symmetry can be used to partition the full space into distinct subspaces. Following its introduction in the late sixties [11, 12], the pseudo SU(3) model has been applied to various properties of heavy deformed nuclei [13–15], however these have been limited to schematic nucleon-nucleon interactions because of technical difficulties related to the calculation of SU(3) matrix elements of more general interactions. Recently, however, a code was released [16] that removes these limitations and allows for the introduction of interactions like pairing into pseudo SU(3) model calculations. As will be shown below, such terms are important for an adequate description of experimental results [17].

As a microscopic theory that takes full account of the fermion structure of a nucleus, the pseudo SU(3) is able to describe single-particle properties of a nucleus while simultaneously allowing for an interpretation of structural features in terms of collective (rotational) degrees of freedom. This is accomplished by exploiting a relation between SU(3) and the algebra of a quantum rotor, which is covered in the next section.

3.1. Geometric interpretation of 1+ states

The pseudo SU(3) scheme allows for a very elegant generalization of the geometrical picture introduced as part of the TRM. In the TRM the scissors-like relative motion of the proton and neutron distributions is parameterized in terms of an angle θ between the z-axes of the axially symmetric distributions. However, it can be shown that an additional “twist” mode is possible [18] for triaxial shapes because rotations by φx and φϕ about the z-axes of the proton and neutron distributions emerge as additional degrees of freedom. As for the scissors mode, it is the difference of these two rotations that gives rise to a M1 mode. This new mode, which together with the scissors motion determines the basic structure of the M1 transition spectrum, has a simple interpretation within the framework of the pseudo SU(3) model.

In a pure SU(3) limit of the theory, the coupling of an axially symmetric proton and neutron distribution gives rise to a single “scissors” mode, in agreement with the TRM model. If one of the two distributions is triaxial however, an additional “scissors-twist” mode is possible. Like the “scissors” mode, this new mode corresponds to a specific SU(3) irreducible representation (irrep) that can be interpreted as an eigenstate of a two-dimensional harmonic oscillator potential Hint describing the proton-neutron interaction in terms of the collective variables θ and ϕ = φx − φϕ.

\[ H_{\text{int}} = \hbar \omega_{\theta} \left( n_{\theta} + \frac{1}{2} \right) + \hbar \omega_{\phi} \left( n_{\phi} + \frac{1}{2} \right) + E_{0}. \]  

Using the mapping between the variables of the joint rotor and SU(3), this oscillator structure can be derived from a general SU(3) preserving nuclear Hamiltonian [19].

In the most general case for the coupling of triaxial proton and neutron distributions, there is an additional “twist” mode as well as another “twist+scissors” mode which is distinguishable from the “scissors+twist” mode through the so-called SU(3) outer multiplicity labelling of the SU(3) irreps involved. This means there is a maximum of four distinct M1 modes. As an example, the 1+ states with non-zero B(M1) transition strength for the even-even 156–160Gd and 106Pt isotopes are given in Table I together with their classification as scissors, twist, or the combination modes.

The summed transition strength for the Gd isotopes in Table I is very close to the experimental results which validates the underlying SU(3) structure in this case. However, the ex-
perimental results suggest a much larger number of $1^+$ states with non-zero M1 transition probabilities to the $0^+$ ground state that are usually clustered around a few strong peaks. This can be understood in terms of the fragmentation of the pure-symmetry states under the influence of SU(3) breaking residual interactions.

3.2. Calculations with a realistic Hamiltonian

To investigate the effect of the symmetry breaking terms the following generalization of SU(3) conserving Hamiltonian has been used,

$$H_{PSU(3)} = -(a_2 + a_{\text{sym}})C_2 + a_3C_3 + bK_2J^2 + cJ^2 + D_\pi \sum_{i_v} t^2_{i_v} + D_\nu \sum_{i_v} t^2_{i_v} - G_{\pi} \frac{\sin \gamma}{\sqrt{2}} \left( Y_2^2 + Y_2^2 \right)$$

(2)

where $C_2$ and $C_3$ are the second and third order invariants of SU(3), which are related to the axial and triaxial deformation of the nucleus, and $J^2$ and $K^2$ are the square of the total angular momentum and its projection on the intrinsic body-fixed symmetry axis, which generate rotational bands and $K_J$-band splitting, respectively. The parameter $a_{\text{sym}}$ is introduced to shift SU(3) irreps with either $\lambda$ or $\mu$ odd relative to those with $\lambda$ and $\mu$ both even, for which $a_{\text{sym}}$ is zero, as the former belongs to different symmetry types ($B_0, \alpha = 1, 2, 3$, rather than $A$) of the intrinsic Vierergruppe ($D_2$) [20]. The one-body proton and neutron angular momentum terms, $t^2_{i_v}$ and $t^2_{i_v}$, together with the two-body pairing terms, $H_{\pi}^p$ and $H_{\nu}^p$, are SU(3) symmetry breaking interactions.

A suitable set of SU(3) basis functions, one first determines the proton and neutron occupancies by filling pair-wise from below the single-particle levels of the generalized Nilsson Hamiltonian [21],

$$h_0 = h_{\text{occ}} + C \cdot 1 \cdot s + D \cdot s^2$$

$$- m \omega^2 r^2 \beta \left( Y_2^2 \right)$$

(3)

for values of $\beta$ and $\gamma$ that give the lowest total energy of the combined proton and neutron systems. One then determines the number of valence-space nucleons in the normal and unique parity levels, the latter being intruder states that are pushed down into the valence space from the next higher shell by the strong spin-orbit interaction. An overall simplifying assumption made in most pseudo SU(3) model calculations is that the relevant dynamics can be described by taking into account the nucleons in the normal parity sector only [22]; the nucleons in intrusion states (unique parity sector) are assumed to follow in an adiabatic manner the motion of the nucleons in the normal parity sector with their effect represented through a reparameterization of the theory.

For the nuclei investigated here, the occupation numbers and the corresponding deformations $\beta$ and $\gamma$ are given in Table II. In accordance with experimental results, the Gd isotopes show a relatively large axial deformation and $^{196}$Pt is triaxial and gamma-soft. Whereas the pseudo SU(3) model is expected to work well for the Gd case, $^{196}$Pt has been included to explore the limits of the theory.

Since the quadrupole-quadrupole interaction, $Q \cdot Q = 4C_2 - 3L^2$, dominates for deformed nuclei, only basis states with $C_2$ larger than a certain value are expected to give a significant contribution in the low-energy region. In the present application, for both proton and neutron distributions, all SU(3) basis states with $C_2 \geq C_{2\text{min}}$, were selected with $C_{2\text{min}}$ set so that all irreps lying below approximately 6 MeV were included in the analysis. Then all possible couplings of these proton and neutron SU(3) irreps were taken to give coupled SU(3) irreps that form basis states of the model space. Also, only states with $J \leq 8$ and $S = 0$ were considered.

The parameters for the Hamiltonian given in Eq. (2) and the effective charges $e_{\pi} = 1 + q_{\text{eff}}$ and $e_{\nu} = q_{\text{eff}}$ used in the E2 transition operator

$$T_{E2} = A^{1/3} \sum_{\sigma \leq \pi, \nu} \sum_i e_{\pi,i}^2 Y_{2M}[F_{\sigma}(i)]$$

(4)

were determined through a fitting procedure that included as input all known levels with $J \leq 8$ up through 2 MeV in ener-
gy and selected B(E2) transition strengths. This procedure gave, in general, very good agreement between the experimental and theoretical numbers (Figs. 1 and 2, and Table III), which gives an indication of the goodness of the pseudo SU(3) model in this mass and energy region, even for the $^{190}$Pt case.

After determining eigenstates of the system, the M1 transition [15] operator

$$T_{\mu}^{1}(M1) = \sqrt{3/4} \mu_N \sum_{\sigma} \left( g_\sigma^L L_{\mu}^{\sigma} + g_\sigma^S S_{\mu}^{\sigma} \right)$$

(5)

with orbit $g$-factors

$$g_\sigma^L = 1 \quad g_\sigma^S = 0$$

(6)

(i.e., with no effective $g$-factors) was used to determine transition strengths between the $0^+$ ground state and $1^+$ states.

### Table III. Total B(M1) transition strengths $[\mu_\alpha^2]$ from experiment [2, 24] and the present calculation. Experimental and theoretical values for the ground band B(E2,0$^+\rightarrow 2^n_\pi$) transition strengths $[e^2 b^2]$ are also given.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>(\sum B(M1) ) ( [\mu_\alpha^2] )</th>
<th>( B(E2,0^+\rightarrow 2^n_\pi) ) ( [e^2 b^2] )</th>
<th>( \text{Exp.} )</th>
<th>( \text{Thy.} )</th>
<th>( \text{Exp.} )</th>
<th>( \text{Thy.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{156}$Gd</td>
<td>3.40</td>
<td>2.91</td>
<td>4.66</td>
<td>4.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{158}$Gd</td>
<td>4.32</td>
<td>3.02</td>
<td>5.02</td>
<td>5.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{160}$Gd</td>
<td>4.21</td>
<td>3.29</td>
<td>5.19</td>
<td>5.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{190}$Pt</td>
<td>0.69</td>
<td>1.27</td>
<td>1.40</td>
<td>1.56</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Because of the SU(3) symmetry breaking terms $I_{D_{xy}}$ and $H_{D_{xy}}$, the simple picture of the pure SU(3) case with a maximum of four transitions gives way to a more complex transition spectrum which is in much better agreement with experimental results. One finds a number of transitions that are usually close to the observed ones, varying, for example, from five for $^{196}$Pt to eleven for $^{156}$Gd. Also, the centroid of the experimental and theoretical M1 transition strength distribution are usually found to lie at approximately the same energy, so that good overall agreement with experiment is obtained. This is especially remarkable for the nucleus $^{196}$Pt, which from its low-energy spectrum (Fig. 2) is seen to be gamma soft and not a good rotor. In this case a relatively strong pairing interaction was found to mix different SU(3) irreps and produce the required soft rotor results with its weaker M1 transition strengths.

The total M1 strength, which for the full Hamiltonian is a bit lower than for its pure SU(3) limit due to destructive interference associated with the mixing (see Tables I and III), shows reasonable agreement with the experimental results, underestimating them slightly for the Gd isotopes. One possible reason for this discrepancy is missing spin $\frac{1}{2}$ admixtures in the wavefunctions which are known to add M1 strength to the system [23] and are essential at higher energies.

In the case of $^{196}$Pt, the total M1 strength is almost a factor of two too large. Keeping in mind that for this case the total M1 strength is only about one sixth of that for good rotors like the Gd isotopes, this is not unreasonable because the starting point of the theory is the assumption of a reasonably well-deformed system which $^{196}$Pt is not. Nevertheless, it is important to emphasize that in this case the theory predicts a significant reduction (by a factor of 1/3) in the M1 strength, which is certainly in agreement with what is observed experimentally.

To summarize, the collective properties of strongly deformed Gd isotopes, as seen through their rotational spectra and enhanced E2 transitions, and the structure of their M1 transition strength distributions are modified, but not destroyed, by including non-collective one-body and two-body parts in the interaction. In particular, the observed fragmentation of the M1 strength seems to demand pairing, even though the amount required does not wipe out the collective rotational features of these nuclei.

In the case of the gamma-soft $^{196}$Pt, a stronger pairing interaction serves to "soften" the rotor picture by introducing stronger mixing of SU(3) irreps. The results for the low-energy and the M1 transition spectrum suggest that the structure of the pseudo SU(3) model is rich enough to work even at its limits while still using a relatively small configuration space.

In conclusion, note that the pseudo SU(3) model can also be applied to odd-A nuclei (half-integer spin). The odd-A case is part of the thesis project of one of the authors (TB). Initial considerations suggest that a more realistic Hamiltonian is required to achieve the same level for the odd-A case as for the even-A results.

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1. n sabatical leave from Departamento de Física, Centro de Investigación y Estudios Avanzados del Instituto Politécnico Nacional, Apartado postal 14-740, México 07000 D.F., Mexico.

2. References to background articles are not cited in the interest of brevity. Typically, these can be found either in textbooks or review articles dealing with early developments of the shell model. The reader is asked to refer to Ref 1 and references therein to recent publications.


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