Shell Model Foundation for Collective Model Phenomenology

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Abstract

An analytic mapping between the hamiltonian of a triaxial quantum rotor and a shell-model interaction built out of generators of the SU(3) - SO(3) algebra has been established. Under the mapping invariants of the rotor are carried into invariants of SU(3):

\[ g^2 - \langle C_2 \rangle \langle \lambda, \mu \rangle^2 \quad \text{and} \quad g^3 \cos 3 \gamma - \langle C_3 \rangle \langle \lambda, \mu \rangle \]

Here \( g \) and \( \gamma \) are the collective model shape parameters and \( C_2 \) and \( C_3 \) are the second and third order Casimir operators of SU(3), respectively. The \( \lambda \) and \( \mu \) are representation labels of SU(3). Rotor and algebraic results for spectra and sumrules, E2 transitions and quadrupole moments, and other measures will be used to show the uniqueness and quality of the theory.

The mapping establishes a simple one-to-one relationship between \( (\alpha, \gamma) \) and \( (\lambda, \mu) \). Since the shell-model space has a unique decomposition into representations of SU(3), only certain \( (\alpha, \gamma) \) values are allowed. This means, for example, that the potential energy surface work initiated by Gneuss and Greiner and brought to fruition in the work of Hess, can be given a shell-model interpretation. In particular, the Hess \( (\alpha, \gamma) \) potential yields the binding energies of a lattice of allowed shell-model configurations. The two differ on one point: Hess had to go to a sixth order theory (quadratic in \( g^3 \cos 3 \gamma \)) to gain stable triaxial shapes \( (\gamma = 0) \) whereas for the interpretation we are suggesting this is not necessary because only certain \( (\lambda, \mu) - (\alpha, \gamma) \) values are allowed. One cannot slide off the lattice into a region that is Pauli forbidden! This connection between the shell model and the potential energy surface concept will be discussed in detail.

In summary, the SU(3) algebra yields a shell-model realization of rotational dynamics and, as well, gives an algebraic interpretation to the potential energy surface concept. So both the kinetic and potential energy parts of the simplest and most successful collective model theories in nuclear physics can be given an elegant and equally simple shell-model interpretation.

*Supported in part by the U. S. National Science Foundation.