NUCLEAR STRUCTURE

PROCEEDINGS OF THE CONFERENCE: BOLOGNA 2000
STRUCTURE OF THE NUCLEUS AT THE DAWN OF THE CENTURY

Bologna, Italy 29 May – 3 June 2000

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SHELL MODEL DESCRIPTION OF $^{158}$Gd

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The pseudo-SU(3) model is used to describe the low-energy spectra as well as $E2$ and $M1$ transition strengths in $^{158}$Gd. The Hamiltonian includes spherical single-particle energies, the quadrupole-quadrupole and proton and neutron pairing interactions, plus four rotor-like terms. The parameters of the Hamiltonian were fixed by systematics with the rotor-like terms determined through a least-squares analysis. The basis states are built as linear combinations of SU(3) states which are the direct product of SU(3) proton and neutron states with pseudo-spin zero. The calculated results compare favorably with the available experimental data, which demonstrates the ability of the model to describe such nuclei.

1 Introduction

Recently the pseudo-SU(3) model was used to successfully describe three low-lying bands in well-deformed heavy nuclei. A reasonable reproduction of the fragmentation of the $M1$ strength was also obtained. In these applications the parameters of the interactions were determined through a least-squares fit to energy levels below approximately 2 MeV. In the present work the quadrupole-quadrupole ($Q \cdot Q$) and pairing interaction ($H_{p,q}$) strengths were fixed by systematics while the interaction strengths of the other terms included in the Hamiltonian were allowed to vary to give an overall best fit to the data. As a result of this analysis, a consistent set of parameters has been identified. Using this parameter set in a quadrupole-quadrupole driven truncated model space, excellent agreement with the experimental data is obtain for $^{158}$Gd. The theory gives correct values for the four lowest energy bands, the $E2$ transition probabilities, the sumrule for $M1$ transitions from the ground state, the correct positions of the $1^+$ energies, and a reasonable reproduction of the fragmentation of the $M1$ strength.

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Table 1: Interaction strengths used in the Hamiltonian (1).

<table>
<thead>
<tr>
<th>χ</th>
<th>$G_\pi$</th>
<th>$G_\nu$</th>
<th>a</th>
<th>b</th>
<th>$a_{sym}$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0077</td>
<td>0.133</td>
<td>0.108</td>
<td>-0.003</td>
<td>0.23</td>
<td>0.00154</td>
<td>0.0000760</td>
</tr>
</tbody>
</table>

2 Model Hamiltonian

The Hamiltonian used in the present study consists of the following terms:

$$H = \sum_{\sigma=\pi,\nu} (H_{sp}^\sigma + G_\sigma H_P^\sigma) - \frac{1}{2} \chi \hat{Q} \cdot \hat{Q} + aJ^2 + bK_J^2 + a_3 \tilde{C}_3 + a_{sym} \tilde{C}_2, \quad (1)$$

where the last four preserve the pseudo-SU(3) symmetry, $\tilde{C}_2$ and $\tilde{C}_3$ being the second and third order Casimir invariants of SU(3). The term proportional to $J^2$ represents a small correction to the moment of inertia, $K_J^2$ breaks the SU(3) degeneracy of the different $K$ bands within an SU(3) irreducible representation (irrep), $\tilde{C}_3$ sets the position of the $0^+$ energies relative to one another, and the last term, which is proportional to $\tilde{C}_2$, distinguishes between the A and $B_\alpha$ ($\alpha = 1, 2, 3$) type internal symmetries, pushing the $1^+$ energies which are bandheads of $B_\alpha$-type structures up relative to the A-type symmetries.

Basis states are built by strong coupling proton and neutron SU(3) irreps and eigenstates are a linear combination of these. The most important configurations are those with highest spatial symmetry, indeed, only configurations with pseudo-spin equal to zero were included for the even-even nuclei considered in this study. The results of previous shell-model calculations, in either a standard full pf-shell configuration space or in an SU(3) basis, show that the Hilbert space can be truncated to those irreps that are favored by the quadrupole-quadrupole interaction. Based on these results, from the set of all allowed pseudo-SU(3) irreps only 18 with the largest values for $C_2 = 1/4 Q \cdot Q + 3/4 L^2$, were used in the $^{156}$Gd calculations reported here.

3 Results

The pairing ($G_\pi$ and $G_\nu$) and quadrupole-quadrupole ($\chi$) interaction strengths were taken from systematics: $G_\pi = 21/A$, $G_\nu = 17/A$, and $\chi = 35 A^{-5/3}$. The dependence of the spectra on the strengths of the other terms in the Hamiltonian was analyzed with best fit values given in Table 1. To obtain the correct excitation energies of the second and third $0^+$ states in the chosen model space, the single-particle strengths needed to be reduced by a factor of four from the so-called realistic values.
Figure 1: Energy spectra of $^{158}$Gd obtained using Hamiltonian 1 and parameters given in Table 1. The right-hand side of the figure gives the theoretical and experimental $M1$ transition strength spectra of $^{158}$Gd. Note that the transitions are clustered and some of the clusters appear to be more fragmented than others.

Table 2: $B(E2)$ transition probabilities in $^{158}$Gd.

<table>
<thead>
<tr>
<th>$J_i \rightarrow J_f$</th>
<th>$B(E2)_{\text{EXP}}[e^2 b^2]$</th>
<th>$Th. [e^2 b^2]$</th>
<th>$J_i \rightarrow J_f$</th>
<th>$B(E2)_{\text{EXP}}[e^2 b^2]$</th>
<th>$Th. [e^2 b^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01 $\rightarrow$ 21</td>
<td>5.023</td>
<td>5.031</td>
<td>22 $\rightarrow$ 41</td>
<td>0.00137</td>
<td>0.00604</td>
</tr>
<tr>
<td>21 $\rightarrow$ 41</td>
<td>2.639</td>
<td>2.590</td>
<td>22 $\rightarrow$ 21</td>
<td>0.0299</td>
<td>0.0832</td>
</tr>
<tr>
<td>41 $\rightarrow$ 61</td>
<td>-</td>
<td>2.268</td>
<td>22 $\rightarrow$ 01</td>
<td>0.0177</td>
<td>0.2395</td>
</tr>
<tr>
<td>61 $\rightarrow$ 81</td>
<td>2.123</td>
<td>2.121</td>
<td>24 $\rightarrow$ 41</td>
<td>0.00705</td>
<td>0.00128</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>24 $\rightarrow$ 01</td>
<td>0.00157</td>
<td>0.000196</td>
</tr>
</tbody>
</table>

Figure 1 shows the energy levels for $^{158}$Gd which are in excellent agreement with the experimental values. Results for selected (intraband and interband) transitions along with their experimental counterparts are given in Table 2. The energy levels belonging to one band exhibit almost the same SU(3) structure. It was also found that for the lowest four bands only five SU(3) irreps contribute more than 2% to the eigenstates. The SU(3) content of the ground state band is given in Table 3. Notice that the SU(3) irreps are only of the even-even type. The same basic structure was found for members of the $K = 2$ band. The second and third $K = 0$ bands exhibit different SU(3) content. For example, in Table 3, the second $K = 0$ band shows an approximately equal mixture of five even-even SU(3) irreps. This mixture supports the hypothesis that the second $K = 0$ band is not dominated by a single shape as different SU(3) irreps correspond to different intrinsic shapes.

The right-most spectrum given in Figure 1 shows the theoretical and ex-
Table 3: SU(3) content of calculated eigenstates for members of the ground state and $K = 0^+_2$ bands in $^{158}$Gd. Only irreps that contribute more than 2% to the eigenstates are shown.

<table>
<thead>
<tr>
<th>band</th>
<th>${\lambda, \mu}$</th>
<th>${\lambda_\pi, \mu_\pi}$</th>
<th>${\lambda, \mu, \mu\nu}$</th>
<th>$0^+$</th>
<th>$2^+$</th>
<th>$4^+$</th>
<th>$6^+$</th>
<th>$8^+$</th>
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<tr>
<td></td>
<td>(28, 8)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
<td>78.1</td>
<td>78.8</td>
<td>80.5</td>
<td>83.0</td>
<td>85.8</td>
</tr>
<tr>
<td>$K = 0^+_1$ (ground state)</td>
<td>(30, 4)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
<td>4.3</td>
<td>4.0</td>
<td>3.4</td>
<td>2.4</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(30, 4)</td>
<td>(10, 4)</td>
<td>(20, 0)</td>
<td>5.6</td>
<td>5.4</td>
<td>5.0</td>
<td>4.3</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>(30, 4)</td>
<td>(12, 0)</td>
<td>(18, 4)</td>
<td>8.5</td>
<td>8.3</td>
<td>7.8</td>
<td>7.1</td>
<td>6.2</td>
</tr>
<tr>
<td></td>
<td>(32, 0)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
<td>2.5</td>
<td>2.4</td>
<td>2.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$K = 0^+_2$</td>
<td>(28, 8)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
<td>19.2</td>
<td>18.3</td>
<td>16.0</td>
<td>17.5</td>
<td>21.6</td>
</tr>
<tr>
<td></td>
<td>(30, 4)</td>
<td>(10, 4)</td>
<td>(18, 4)</td>
<td>18.6</td>
<td>20.1</td>
<td>23.3</td>
<td>27.0</td>
<td>30.1</td>
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<td></td>
<td>(30, 4)</td>
<td>(10, 4)</td>
<td>(20, 0)</td>
<td>23.7</td>
<td>24.7</td>
<td>26.2</td>
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<td></td>
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<td>(12, 0)</td>
<td>(18, 4)</td>
<td>23.4</td>
<td>22.0</td>
<td>19.4</td>
<td>16.5</td>
<td>13.7</td>
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<td></td>
<td>(32, 0)</td>
<td>(12, 0)</td>
<td>(20, 0)</td>
<td>15.1</td>
<td>14.8</td>
<td>13.9</td>
<td>12.3</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Experimental M1 transition spectrum for $^{158}$Gd. The calculated $1^+$ energies are in the correct energy interval. The experimental and theoretical M1 transitions peak at the same excitation energies. Similar results were found for $^{156}$Gd and $^{160}$Gd. More calculations of this type are needed to see if the pseudo-SU(3) model can offer a consistent explanation of the low-lying energy band structure and M1 properties of rare earth nuclei.

Acknowledgments

Supported in part by Conacyt (Mexico) and the National Science Foundation (U.S.) through a Cooperative Research grant (INT-9500474). National Science Foundation support from a regular grant (PHY-9970769) and a Cooperative Agreement (EPS-9720652), that includes matching from the Louisiana Board of Regents Support Fund, is also acknowledged. GP thanks the “Charles E. Coates Memorial Award” for travel support.

References