Pseudo-Spin Symmetry/Cocoyoc-6191

Pseudospin in Perspective

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Abstract

The pseudospin concept, which is a special coupling scheme for carrying out many-particle microscopic shell-model calculations in heavy nuclei, is shown to follow logically from the fact that in the usual single-particle picture the strength of the spin-orbit interaction is approximately four times the strength of the orbit-orbit term and, as a consequence, the single-particle field can be transformed into an equivalent pseudo form with a very small spin-orbit splitting. Nilsson-type level schemes for normal and pseudo forms of the single-particle fields, with values of the deformation parameter extending well into the superdeformed regime, are compared with one another and with results from mean field calculations. From these comparisons it is clear that the pseudo-LS coupling picture, and for strongly deformed systems the pseudo-SU(3) model and its multi-ho pseudo-symplectic extension, are expected to be good schemes for shell-model calculations. The role of intruder levels, and in particular their contribution to the quadrupole collectivity of strongly deformed systems, is also considered. Some results for ^{238}U that explore the build up of quadrupole collectivity through multi-ho configuration mixing are also presented.

1. Introduction

A single-particle Hamiltonian consisting of a three-dimensional isotropic harmonic oscillator, \( H_0 \), augmented with the usual one-body spin-orbit (l-s) and orbit-orbit (l^2) interactions,

\[
H = H_0 + C l s + D l^2,
\]

(1)

is known to be a good starting point for nuclear structure studies.\(^1,2\) The \( l^2 \) term, with \( D < 0 \), pushes high angular momentum states down relative to those with lower \( l \) values, a feature that occurs naturally when a more realistic interaction like a Woods-Saxon form is used for the central potential, while the phenomenological l-s term with \( C < 0 \), which couples space and spin degrees of freedom, is required to achieve shell closures at the magic numbers 2, 8, 20, 50, 82, 126 and 184.

Unfortunately, the required value for \( C \) in (1) is so large that the spin-orbit interaction actually destroys the underlying SU(3) symmetry of the oscillator for all but the lightest (\( A \leq 28 \)) nuclei, and in so doing renders it of little apparent value in attempts at unraveling the structure of heavier (\( A \geq 100 \)) systems. Specifically, for heavy nuclei the \( j = n+1/2 \) orbital of the \( n \)-th oscillator shell, which includes
levels with \( j = l \pm 1/2 \) with \( l = n, n-2, \ldots, 1 \) or 0, is pushed down among the orbitals of the next lower shell. This yields new shells with normal parity \( j = l/2, 3l/2, \ldots, n-1/2 \) orbitals plus a \( j = n+3/2 \) unique parity intruder from the shell above.\(^3\)

In this paper this unfavorable scenario is shown to be convertible into a much more favorable one because \( C = 4D \). Firstly, this condition insures that the pseudo-spin is a good symmetry because the level splitting generated by the \( l\cdot s \) and \( l^2 \) interactions can be duplicated by a pseudo-oscillator hamiltonian plus a pseudo \( l^2 \) interaction and a pseudo \( l\cdot s \) term that is very small.\(^4,5\) Secondly, when deformation is important, each pseudo-spin symmetry has associated with it an yrast band that is dominated by its leading irreducible representation (irrep) of pseudo SU(3), the symmetry group of the pseudo oscillator.\(^6\) And finally, there is a pseudo-symplectic extension of the pseudo SU(3) theory that incorporates major shell admixtures \( (2n\hbar\omega, n = 1, 2, \ldots) \) into the picture.\(^7\)

2. Single-Particle Picture

If the eigenvalues of the single-particle hamiltonian (1) are plotted as a function of the Nilsson parameter \( \mu = 2D/C \) that measures the relative strength of the \( l^2 \) and \( l\cdot s \) terms, for the special value \( \mu = 0.5 \) the orbital pairs with \( j = l + 1/2 \) and \( j = (l+2) - 1/2 \) are found to be degenerate for all \( l \) values. Furthermore, the splitting of these degenerate pairs follows a rule where \( \tilde{l} = \frac{(l+1)}{2} \) and \( \tilde{s} = \frac{l+1}{2} \).

\[
|\tilde{n}(\tilde{l}, \tilde{s}) \tilde{j} \tilde{m}\rangle = U_{n,j,m}(\tilde{l}, \tilde{m}) |n(l, s) j m\rangle,
\]

\[
U_{n,j,m}(\tilde{l}, \tilde{m}) = \delta_{n-1,\tilde{n}} \delta_{j,\tilde{j}} \delta_{m,\tilde{m}} \delta_{l+1/2, \tilde{l}+1/2}. \tag{2}
\]

From the structure of \( U_{n,j,m}(\tilde{l}, \tilde{m}) \) it is clear that the transformation is simply a relabeling of the basis states that associates all levels of the \( n \)-th shell, except the defector with \( j = n+1/2 \), with levels of the \( \tilde{n} \)-th shell of a "pseudo" oscillator with the same algebraic properties as the "normal" oscillator, where \( \tilde{n} = n - 1 \).

Under this "normal ↔ pseudo" mapping the single-particle hamiltonian can be shown to transform as follows:

\[
H_0 + Cl\cdot s + Dl^2 \quad \xrightarrow{\text{normal}} \quad \tilde{H}_0 + (4D-C)\tilde{l}\cdot \tilde{s} + D\tilde{l}^2 + (\hbar\omega+2D-C). \quad \tag{3}
\]

Since the \((\hbar\omega+2D-C)\) term is an additive constant, the pseudo form for the interaction, \( \tilde{H} = \tilde{H}_0 + \tilde{C}\tilde{l}\cdot \tilde{s} + D\tilde{l}^2 \), has the same excitation spectrum as the normal one, \( H = H_0 + Cl\cdot s + Dl^2 \), when \( \hbar\tilde{\omega} = \hbar \omega, \quad \tilde{C} = (4D-C) \) and \( \tilde{D} = D \). This transformation is meaningful because for real \((A \geq 100)\) systems \( C = 4D \) so \( \tilde{C} = 0 \).
NILSSON LEVEL SCHEMES

(μ = 2D/C = 0.5 & κ = -C/2ħω = 0.05)

A) PSEUDO + T² Y_{20}

B) NORMAL

C) PSEUDO + T² Y_{20}

Fig. 1 A comparison of Nilsson-type single-particle energy level diagrams. Inset (B), which is the exact pseudospin limit of the theory, μ = 2D/C = 0.5, shows that the degeneracy of the spin-orbit partners remains unchanged when the deformation inducing r²Y_{20} interaction is turned on. Inset (A) is the pseudo-spin realization of the case (B) results, that is, the r²Y_{20} interaction is replaced by r²Y_{20} and β by β = [(n+3/2)/(n+1/2)]β where the quantity in the bracket multiplying β is an overall renormalization factor that compensates for average differences in the diagonal matrix elements of the r² term in the n and n-1 shells. Inset (C) is the same as (B) but with the βr²Y_{20} matrix elements replaced by the actual matrix elements of βr²Y_{20}. The highest j orbital, which for the n = 5 shell is the h_{11/2}, must be excluded from consideration since in the "normal ↔ pseudo" mapping such levels are considered to have defected out of the valence space to the shell below.
Specifically, in the neutron (ν) and proton (π) spaces, \( \mu_\nu = 0.4 \) and \( \mu_\pi = 0.6 \). This places heavy nuclei very close to the exact pseudo-spin limit (\( \mu = 0.5 \)) of the theory.\(^8\) The familiar single-particle shell-model hamiltonian for heavy nuclei can therefore be replaced by a less familiar but equivalent pseudo form which is inherently simpler to handle because it has a much smaller spin-orbit term.

3. Nilsson Level Schemes

These results and the effect of deformation on them are shown in the series of three Nilsson diagrams given in Figure 1. Specifically, in the figures a plot of the eigenvalues of the usual Nilsson hamiltonian with \( \mu = 2D/C \) and \( \kappa = -C/2\hbar \omega \),

\[
\begin{align*}
H/\hbar \omega &= N_0 - \kappa(2I^2 + \mu I^2) - \beta r^2 Y_{20},
\end{align*}
\]

are given for the \( n = 5 \) shell and a range of the deformation parameter, \( 0 \leq \beta \leq 1 \), that extends well into the superdeformed regime. Inset (B) of Figure 1, which is the \( \mu = 0.5 \) case, shows that the degeneracy of the spin-orbit partners remains unchanged by the deformation inducing \( r^2 Y_{20} \) interaction which breaks rotational symmetry by mixing spherical orbitals and splitting the spectrum up into one which has only the projection of the angular momentum as good quantum number. This follows because \( r^2 Y_{20} \) is a purely spatial and hence spin independent interaction.

Inset (A) of Figure 1 is the pseudo-spin realization of the case (B) results. Specifically, eigenvalues in the \( n = 5 \) shell of the single-particle hamiltonian, with spherical part \( H_0 + CI^2 + Dl^2 \), are compared with its \( \tilde{n} = 4 \) pseudo-spin realization, with spherical part \( \tilde{H}_0 + (4D-C)\tilde{I}^2 + D\tilde{l}^2 + (2D-C+C\hbar \omega) \). In case (A) the \( r^2 Y_{20} \) interaction was replaced by \( \tilde{r}^2 Y_{20} \) and \( \beta \) by \( \tilde{\beta} = [(n+3/2)/(n+1/2)] \beta \) where the quantity in the bracket multiplying \( \beta \) is an overall renormalization factor that compensates for average differences in the diagonal matrix elements of the \( r^2 \) term in the \( n \) and \( \tilde{n} = n - 1 \) shells. Inset (C) is the same as (B) but with the \( \tilde{r}^2 Y_{20} \) matrix elements replaced by the actual matrix elements of \( \beta r^2 Y_{20} \). Notice how closely (C) tracks (A) and (A) tracks (B) and therefore (C) also tracks (B). In making these comparisons, the dashed levels that derive from the highest \( j \) orbital, which in this case is the \( h_{11/2} \), must be excluded from consideration since in the "normal ↔ pseudo" mapping they are considered to have defected to the shell below. The deformation does not change this, even though the highest \( m_j \) members of the defector orbital are shown to penetrate through the other levels, because these levels remain widely separated from others of the same \( m_j \) with which they can mix. Recall that \( H \) commutes with \( J_j^2 \) so the \( |m_j| \)'s are a good quantum numbers. Notice also that even though the degeneracy of the spin-orbit partners is broken in (C) because the \( \beta r^2 Y_{20} \) interaction is not a pseudo-spin scalar, this breaking is very small compared to the other splittings.

The effect of moving off the exact pseudo-spin limit of the theory, which is obtained for \( \mu = 0.5 \), is shown in the Figure 2 where the normal and pseudo results are again compared over the same deformation parameter range, but in this case for \( \mu = 0.4 \), which is appropriate for neutrons in heavy nuclei, and \( \mu = 0.6 \), which
Fig. 2 Nilsson-type single-particle level schemes showing the effect of moving off the exact pseudo-spin limit of the theory which is obtained for \( \mu = 2D/C = 0.5 \). The results on the left are for a normal oscillator calculation and those on the right are for the corresponding pseudo oscillator approximation. The splitting of the pseudo spin-orbit partners, which is small compared to the separation generated by the orbit-orbit interaction, decreases with increasing deformation and takes on a near constant value throughout the superdeformed region, \( \beta > 0.4 \).
applies for protons. As in Figure 1, the effect of the symmetry breaking interaction, which is now the \((4D-C)\tilde{T}\cdot\tilde{s}\) term as \(C\) is not equal to \(4D\), is small compared to the separations generated by the \(D\tilde{T}^2\) and \(\beta r^2 Y_{20}\) terms, where the latter was again replaced by \(\beta T^2 Y_{20}\) since it is simpler to deal with and differences between the two are very small. A particularly interesting feature is the fact that the \(\beta r^2 Y_{20}\) term actually moderates the splitting induced by the \((4D-C)\tilde{T}\cdot\tilde{s}\) interaction. This can be seen by noting that the pseudo-spin partners, which derive from the spherical orbitals with \(j = \tilde{l} \pm 1/2\) and are split by the \((4D-C)\tilde{T}\cdot\tilde{s}\) term, actually converge as a function of increasing strength of the deformation. The results show that the asymptotic limit is achieved around or just slightly beyond a value for \(\beta\) that is characteristic of normally deformed nuclei, namely, \(\beta = 0.3\). It is important to note, however, that the splitting, though small, remains finite and nearly constant throughout the superdeformed regime, a feature that consistent with regularities found in superdeformed structures.

In mean field applications, which have been successful in predicting regions of superdeformed and even the most likely \(Z\) and \(N\) combinations to give rise to superdeformation, the harmonic oscillator potential is replaced by a more realistic Woods-Saxon single-particle field. It is therefore instructive to compare the single-particle energy level diagrams shown above with those obtained in mean field work. This is done in Figure 3 where the results of Dudek and coworkers\(^9\,10\) have been assembled into a collage of Nilsson-type plots that starts with the harmonic oscillator. The upper left-hand inset shows how a deformed field transforms the spherical shell closures at nucleon numbers 20, 40, 70, ... into new subshell closures at 26, 34, 46, 54, 66, ... and 28, 40, 60, 80, ... for \(\beta = 0.4\) and 0.6, respectively. The inset on the upper right-hand side is the same except with the highest \(j\) intruder levels shifted down by the amount that is required for realistic applications of the theory. This produces a different set of magic numbers, namely, 20, 28, 40, 50, ... in the spherical limit of the theory and others like 46, 64, and 86 that survive even in the most sophisticated theory of this type. The results shown in the bottom two insets are single-particle levels of the best Woods-Saxon form for neutrons in the \(n = 3\) and \(n = 4\) shells. In the \(n = 3\) case the levels indicated by heavy dots can be identified as members of a pseudo \(\tilde{n} = 2\) shell and are labelled with the corresponding pseudo-asymptotic Nilsson quantum numbers. Levels indicated by heavy solid curves for \(n = 4\) are those associated with \(\tilde{n} = 3\). It is interesting to note that in the asymptotic regime the pseudo structure can be identified as the major feature that is responsible for where the shell gaps occur. In each case there are groups of 1, 2, 3, ..., \(\tilde{n} + 1\) levels that follow rather closely the the predictions of an exact \((\mu = 0.5)\) pseudo-Nilsson model hamiltonian.

4. Second Quantization Picture

Regarding the "normal ↔ pseudo" transformation defined by \((2)\), it should be noted that a special unitary operator that transforms \(Cl\cdot s + Dl^2\) into \((4D-C)\tilde{T}\cdot\tilde{s} + D\tilde{T}^2 + (2D-C)\) has been identified, namely, \(U = \exp(\imath\tilde{h}) = 2\imath h\) where \(h = \tilde{s} \cdot \tilde{r}\) is the helicity.\(^{11}\) However, unlike the \(U_{njm} \tilde{\alpha} \tilde{\mathbf{j}} \tilde{m}(l, \tilde{l})\) of \((2)\), this helicity operator form
Oscillator Symmetry

Harmonic Oscillator

Oscillator & Intruders

Pseudo-Oscillator

N=3 Shell

N=4 Shell

Quadrupole Deformation

Fig. 3 A collage of Nilsson-type single-particle energy level diagrams from Dudek and coworkers that emphasizes similarities between the harmonic oscillator scheme, including its pseudo-spin generalization, and mean-field results based on a realistic Woods-Saxon form for the potential. Note that in each case there are groups of 1, 2, 3, ..., \( \bar{r} + 1 \) levels that follow rather closely the predictions of an exact (\( \mu = 0.5 \)) pseudo-Nilsson model Hamiltonian.
does not carry $H_0$ into $\tilde{H}_0 + \tilde{h}_0$ and therefore is not the "normal $\leftrightarrow$ pseudo" transformation introduced by the original proposers of the pseudo-spin concept. In particular, the helicity transformation does not lead to the pseudo-LS coupling scheme nor to the pseudo SU(3) model and its pseudo symplectic extension that, as is shown below, are essential for the development of a complete yet tractable many-particle shell-model theory for heavy nuclei.

To clarify this matter still further, and give the general shell-model result, consider a $(0+1+2)$-body hamiltonian expressed in second-quantized form:

$$H_0 + H_1 + H_2 = H = \tilde{H}_0 + \tilde{H}_1 + \tilde{H}_2$$  \hspace{1cm} (5)

where

$$H_1 = \sum_{\alpha,\beta} \langle \alpha | h | \beta \rangle a_{\alpha}^\dagger a_{\beta} = \sum_{\alpha,\beta} \langle \alpha | h | \beta \rangle a_{\alpha}^\dagger a_{\beta}^\dagger = \tilde{H}_1$$

$$H_2 = \sum_{\rho} C_1^\rho T_1^{\rho} = \sum_{\rho} C_1^\rho T_1^{\rho} = \tilde{H}_2$$

(6)

$$H_2 = \sum_{\rho} C_2^\rho T_2^{\rho} = \tilde{H}_2$$

(7)

The pseudo form given on the right of each of these equations follows from the normal one given on the left by making the substitutions indicated by (2), that is, $a_{\alpha}^\dagger \rightarrow a_{\gamma}^\dagger$, $a_{\beta} \rightarrow a_{\gamma}$, etc., without modifying the matrix elements that multiply the creation and annihilation operator combinations that enter into the sums. This means that the normal and pseudo forms for $H$ carry the same information, namely, that contained in the original set of $(0+1+2)$-body matrix elements.

Simplifications result when the expansion of $H$ in terms of the complete set of pseudo tensors $T_2^\rho$ puts more strength into the symmetry preserving subset of these operators than into the symmetry breaking ones, as compared with the corresponding expansion in terms of the complete set of normal tensors $T_2^\rho$. An example of this is (3) which effects the "normal $\leftrightarrow$ pseudo" transformation for the usual $(0+1)$-body hamiltonian given by (1). According to the arguments given and results shown, the transformation to the pseudo scheme means that $I^2$, which does not break the pseudo space-spin symmetry, suffices to propagate most of the one-body structure information whereas for the normal scheme both $l^2$ and $l$-$s$, which couples different space-spin symmetries, are required. The results given in Figures 1 and 2 show that the introduction of deformation via a single-particle field of the Nilsson type also preserves the pseudo space-spin symmetry. Since the quadrupole field derives from a quadrupole-quadrupole interaction among the nucleons, at least a part of the two-body interaction is pseudo space-spin symmetry preserving. It remains to be shown that the same is true for even more general two-body interactions.
5. Pseudo Space-Spin Symmetry

To grasp the full significance of "normal ↔ pseudo" mapping, recall that the symmetry group for the usual many-particle generalization of the single-particle theory, with particles distributed among the lowest available single-particle levels, is U(NM), the unitary group in (N by M) dimensions, where \( N = (n+1)(n+2)/2 \) is the spatial degeneracy of the \( n \)-th oscillator shell and M is 2 or 4 for a spin or spin-isospin formulation of the theory. The U(N) \( \otimes \) U(M) direct product subgroup of this U(NM) group separates the full (N by M) dimensional space into its spatial and spin (M=2) or spin-isospin (M=4) parts, see Figure 4. Irreps of U(N), which are labeled by a Young pattern \( [f] = [f_1, f_2, ..., f_N] \), specify the space symmetry while irreps \( [f^\pi] = [f_1^\pi, f_2^\pi, ..., f_N^\pi] \) of U(M), which must be related to the \( [f] \) of U(N) by row-column interchange to insure overall antisymmetry in U(NM) as required by the exclusion principle, label the complementary spin or spin-isospin symmetry.\(^{12}\)

![Diagram of Shell Model Symmetries](image)

Fig. 4 Schematic diagram showing the normal and pseudo symmetry group decompositions for identical nucleons in the \( n \)-th major shell of a three-dimensional isotropic harmonic oscillator potential. For the pseudo decomposition the full \( \Sigma_j(2j+1) = (n+1)(n+2) \) dimensional space is broken up into two parts, a \( (2j_{\text{max}}+1) = 2(n+1) \) dimensional subspace that is frozen out because the \( j_{\text{max}} \) level dips below the Fermi level and the remaining \( \Sigma_j(2j+1) = (n+1)(n+2) - 2(n+1) = n(n+1) \) dimensional valence space that can be mapped onto an pseudo oscillator shell of one less quanta, \( \tilde{n} = n-1 \).
An important difference between the one-body interactions is that $l$-$s$ couples different space-spin ([$[T]$] and [T$^C$]) symmetries whereas $I^2$ does not. Since under the "normal ↔ pseudo" mapping the strength of the spin-orbit interaction is significantly reduced, the transformation really effects a decoupling of the pseudo space-spin ([$\tilde{T}$] and [T$^C$]) symmetries. In addition, it is known that typical two-body interaction, like the surface delta\textsuperscript{4} and quadrupole-quadrupole forms, see below, further reenforce the goodness of the pseudo space-spin symmetry.\textsuperscript{4} A practical outcome of this scenario is that the [T$^C$] irreps of the space-spin $\tilde{U}(\tilde{N}) \otimes \tilde{U}(\tilde{M})$ group are only weakly coupled and can therefore be used to truncate the model space down to a tractable and yet physically meaningful size.

For heavy nuclei the valence protons and neutrons occupy different shells so an identical particle ($\tilde{M}=2$) formulation must be applied to each. Specifying the [T$^C$] and [$\tilde{T}$] labels is then equivalent to specifying the total number of normal parity particles and their pseudo spin, $\tilde{m} = \tilde{T}^1 + \tilde{T}^2$ and $\tilde{S} = (\tilde{T}^C_1 - \tilde{T}^C_2)/2$. Each normal parity $\tilde{m}$-particle space, with $\tilde{m} = \tilde{m}_\pi$ for protons and $\tilde{m} = \tilde{m}_\nu$ for neutrons, is partitioned into subspaces with $\tilde{S} = 0, 1, 2, ..., \tilde{S}_{\max}$ for $\tilde{m}$ even and $\tilde{S} = 1/2, 3/2, 5/2, ..., \tilde{S}_{\max}$ for $\tilde{m}$ odd, where $\tilde{S}_{\max}$ is the minimum of $\tilde{m}/2$ and $\tilde{N}-\tilde{m}/2$. This means that $\Delta \tilde{S}$ is always an integer. Also, to each $\tilde{S}$ there is a complementary set of spatial configurations. To the extent the pseudo-spin symmetry is good, one therefore expects to observe sets of states, like rotational sequences, that differ in total angular momenta ($I = \tilde{L} + \tilde{S}$) by integer (even-A compared with even-A) or half-integer (odd-A compared with even-A) amounts.\textsuperscript{13}

6. Pseudo SU(3) Symmetry

The importance of the SU(3) model for light nuclei follows from the dominance of the quadrupole-quadrupole interaction, Q-Q, over the one-body $l$-$s$ and $I^2$ terms as well as its dominance over all other two-body forms.\textsuperscript{14,15} So even though the spin-orbit interaction is strong, yrast states of nuclei like $^{20}$Ne and $^{24}$Mg are typically dominated to the 60-80% level by their leading SU(3) representations.\textsuperscript{16} How this comes about follows from the fact that Q-Q = 4C$_2$ - 3L$^2$, where C$_2$ is the quadratic invariant operator of SU(3) which is itself a subgroup of U(N). That is, Q-Q conserves the U(N) symmetry, and in addition, since the expectation value of Q-Q is proportional to the square of the deformation, it further subdivides each U(N) irrep [f] into (λ,μ) irreps of SU(3) with the most deformed of these lying lowest and the least deformed highest.

It will now be argued that the pseudo SU(3) scheme,\textsuperscript{6} with $\tilde{SU}(3)$ standing in the same relationship to $\tilde{U}(\tilde{N})$ as SU(3) does to U(N), see Figure 4, provides a similar explanation for observed quadrupole collectivity in heavy deformed nuclei. There are of course some major differences: 1) the valence neutrons and protons occupy different major shells, 2) the normal Q-Q interaction is not the quadratic invariant of $\tilde{SU}(3)$, 3) whereas for the ds shell the coefficient $D$ of $I^2$ is positive, for heavy nuclei it is always negative, and 4) intruder levels penetrate through the neutron and proton shells. The intruder level issue is considered in the next section. But regarding the first, a very simple calculation has been done which
shows that the protons and neutrons being in different shells makes very little
difference so long as they interact, albeit even weakly, through their quadrupole
fields. Specifically, even if the separate proton and neutron interactions are
dominated by pairing, a small $Q_\pi Q_\nu$ interaction between the two suffices to drive
the whole system towards the strong coupled pseudo SU(3) limit of the theory.\(^\text{17}\)

To dispense with the second matter, recall that the pseudo-spin scheme is an
excellent starting point for a many-particle description of heavy nuclei, whether
they are deformed or not, see (3) and the discussion associated with (4)-(6). In
particular, the $\tilde{T}\tilde{s}$ interaction is weak relative to $T^2$ so the pseudo-space-spin
symmetry is expected to be good. Also, since the surface delta interaction, which
is known to be a reasonably good effective interaction for many applications, is a
pseudo-spin scalar operator, the residual two-body interaction is not expected to
change this picture very much by inducing additional mixing among different pseudo
space symmetries.\(^\text{4}\) And even more importantly, although Q-Q is not an invariant
of SU(3), under the "normal ↔ pseudo" mapping it transforms into its pseudo
counterpart plus small corrections,\(^\text{18}\)

$$Q\cdot Q = \kappa \tilde{Q} \cdot \tilde{Q} + \ldots.$$ \(\text{(8)}\)

Indeed, within the leading pseudo-space symmetry, the sum of all correction terms,
denoted by the $\ldots$ in (8), have been shown to induce less than a 1\% change in
eigenenergies of calculated yrast states.\(^\text{19}\)

To explore the third question, which concerns the relevance of the pseudo
SU(3) scheme for strongly deformed nuclei for which $D < 0$, which is a condition
that helps to destroy SU(3) in the fp-shell, the many-body problem (tildes now
suppressed for simplicity) with Hamiltonian,

$$H = H_0 + C \sum_i l_i s_i + D \sum_i l_i^2 - \frac{1}{2} \chi Q \cdot Q = H_0 + C \sum_i l_i s_i + D \sum_i l_i^2 - \frac{1}{2} \chi (4C_2 - 3L^2), \text{(9)}$$

was diagonalized in a (ds)\(^4\), T=0 space with the C and D parameters chosen to be
representative of a pseudo-spin application, $C \to \tilde{C} = -0.2$ and $D \to \tilde{D} = -0.2$
(Mev), rather than for a normal ds-shell calculation with $C = -2.0$ and $D = +0.2
(Mev). Note that the last form for H in (9) follows from the first because within a
major shell of the oscillator Q-Q = 4C_2 - 3L^2. Intensities of the various SU(3)
irreps in calculated 0\textsuperscript{+} eigenstates, plotted as a function of the strength of $\chi$, are
shown in Figure 5. The symmetry breaking decreases sharply as the strength of the
deformation inducing quadrupole-quadrupole interaction increases. For $\chi \approx 0.1$,
which is a reasonable choice for normal and pseudo applications, the yrast states
with $J^\pi = 0^+, 2^+, \ldots$ were found to be better than 95\% pure $(\lambda, \mu) = (8,0)$, which is
the leading SU(3) symmetry. By analogy, yrast states of heavy deformed nuclei
are expected to be dominated by the leading pseudo SU(3) symmetry.

The leading pseudo SU(3) symmetry is the most deformed configuration
available in the $0\hbar\omega$ space. To go beyond normally deformed to superdeformed and
perhaps even hyperdeformed shapes requires shifting particles into higher-lying
configurations. For example, in the (ds)\(^4\), T=0 case a superdeformed band can be
obtained by lifting two particles out of the p shell into the fp shell. This leads to SU(3) irreps in the product (8,0) x (0,2) x (6,0) with (λ,μ)=(14,2) the leading one. Since in L=0 states the square of the deformation is proportional to the expectation value of C₂, under the action of the very same hamiltonian the (14,2) is nearly twice as deformed as the 0h0 leading (8,0) irrep: \( \langle C_2(14,2) \rangle / \langle C_2(8,0) \rangle = 3.14 \) which yields \( \beta(14,2)/\beta(8,0) = 1.77 \). Because of the dominance of the Q-Q term, such configurations are expected to display even less representation mixing than the 0h0 ones display. This suggests, for example, that superdeformed bands should be expected to be better normal or pseudo SU(3) nuclei, as appropriate, than normally deformed ones.²⁰,²¹

![Diagram](image.png)

**Fig. 5** Intensity of SU(3) representations in \( L^T = 0^+ \) states of the rotation hamiltonian (9) in the model space \((ds)^4[f] = [4]\) with \((λ,μ)\)'s = (8,0), (4,2), (0,4), and (2,0). The parameters C and D were set at 0.0 and -0.2 (Mev), respectively, while \( χ \) was assigned values between 0.0 and 0.1 (Mev). These parameter values for a pseudo ²⁰Ne system with no spin-orbit splitting, which differ significantly from the values required for a real ²⁰Ne application (C = -2.0 and D = +0.2 for a μ = -0.2), simulate an application of the theory to deformed nuclei of the rare earth and actinide regions. A full breakdown of the intensity is given for the 0_1^+ yrast state while results for the main SU(3) component only are shown for each of the remaining states, \( 0^+_2, α = 2, 3, \& 4. \) A realistic value for \( χ \) is around 0.06 - 0.08. The dashed curve just below the \( O_{1}^{+}(8,0) \) solid line is the result with a spin-orbit interaction with strength C = -0.2, which is also representative as it is down by a factor of ten from the actual value.
7. Pseudo Symplectic Symmetry

Beyond SU(3) is the symplectic model, which can be used to describe both low-lying and giant resonance states of light (A ≤ 28) nuclei that are strongly deformed. It extends the SU(3) theory by including multiple $2\hbar\omega$ shell-model excitations of the monopole ($l=0$) and quadrupole ($l=2$) type. This feature means the action of the real quadrupole operator, not just its intra-shell $0\hbar\omega$ preserving part, can be properly accommodated. The symmetry algebra of this extended model generates a realization of the noncompact symplectic group Sp(3,R), which has the Elliott SU(3) as its maximal compact subgroup. Associated with each SU(3) shell-model irrep is an infinite dimensional Sp(3,R) representation. These symplectic spaces are complete with respect to the action of the real quadrupole operator which means intraband and interband E2 transition strengths between low-lying as well as giant resonance configurations can be reproduced within the framework of the model without the use of proton and neutron effective charges.

The pseudo symplectic model extends the pseudo SU(3) picture by allowing for inter-shell $2\hbar\omega$ excitations of the monopole and quadrupole type among the normal parity orbitals just like the symplectic scheme does for the normal SU(3) model. In this case, however, the underlying symmetry group is pseudo Sp(3,R) with pseudo SU(3) as its maximal compact subgroup. Since the symplectic extension means that multiple $2\hbar\omega$ excitations in the proton and neutron normal parity spaces are included, it would seem that unique parity excitations of this type should also be considered, as well as mixing between the two. In first applications of the theory, however, these couplings were not included. The usual argument given in support of this unequal treatment of the normal and unique parity subspaces is that for states in even-even nuclei that lie below the backbending region the nucleons in unique parity intruder levels are dominated by pairing correlations which differ from quadrupole correlations in that they do not require couplings to higher shells for strength enhancement. That is, the nucleons in the unique parity intruder levels pair off into J=0 configurations and therefore contribute nothing to the dynamics of the system.

However, in light of mean field results which suggest exactly the opposite, namely that the intruders are very important for understanding of superdeformed phenomena, the validity of this untested assumption needs to be reexamined. Preliminary results suggest that the coupling of the intruder level to its like-parity partners, even though these may be as much as a full major shell ($1h\omega$) away, is strong and leads to a sizable contribution to the quadrupole moment and therefore E2 strengths of the many-particle system. In particular, for the same test case considered above, namely Hamiltonian (9) in the (ds)$^4$,T=0 space with $D = -0.2$ and large C values so as to simulate the separation of the $d_{5/2}$ orbital from its $(d_{3/2},s_{1/2})$ partners, the expectation value of Q-Q was found even in the worst case scenario of a full $1h\omega$ separation to develop as much as 60-70% of its maximum quadrupole collectivity. This is shown in Figure 6 where normalized expectation values of Q-Q in calculated ground states are plotted as a function of the strength of the quadrupole-quadrupole interaction. The normalization factor is the maximum
eigenvalue of Q-Q in the (ds)^4, T=0 space, namely, the expectation value of Q-Q in the leading (λ, μ) = (8, 0) irrep of SU(3). For the results to be representative of a pseudo SU(3) application, C = -2.5 and, as above, χ = 0.06 - 0.08. It follows from this that even for a very strong spin-orbit splitting, the yrast state can have as much as 60-70% of its maximum quadrupole collectivity.

Since for rare earth and actinide nuclei the number of particles in the intruder level is typically about 1/3 of the total number of valence particles, and since the intruder level comes from the (n+1)-st oscillator shell, a rough estimate for the ratio of the contribution of particles in the unique parity intruder levels to that of those in the normal parity orbitals can be given: 

\[
\frac{\langle Q-Q \rangle_{\text{unique}}}{\langle Q-Q \rangle_{\text{normal}}} = 0.6 \times \frac{C_2(m(n+1)/3, 0)}{C_2(2mn/3, 0)} \approx 0.6 \times \frac{(n+1)}{2n} = 0.2.
\]

This analysis assumes the (λ, μ) = (m(n+1)/3, 0) and (2mn/3, 0) irreps are representative of those that are

![Diagram](image_url)

Fig. 6 Normalized expectation values of Q-Q in calculated ground states of the (ds)^4 system for Hamiltonian (9), with D = -0.2 and a spin-orbit term with C values as indicated, are plotted as a function of the strength χ of the quadrupole-quadrupole interaction. The normalization factor is the eigenvalue of Q-Q in the leading (λ, μ) = (8, 0) irrep of SU(3). For the results to be representative of a pseudo SU(3) application, C = -2.5 and, as indicated, χ = 0.06 - 0.08. It follows from these results that even for a very strong spin-orbit splitting, the yrast state can achieve as much as 60-70% of its maximum total quadrupole collectivity.
allowed by the exclusion principle for the unique and normal parity spaces, respectively. This suggests that particles in the unique parity orbitals can be expected to contribute to the quadrupole moment of a deformed system roughly in proportion to their number with a strength that is about half the strength with which the normal parity particles contribute. Since the number of particles in the unique parity space is typically not small, they are important in determining quadrupole moments and E2 strengths.

8. Miscellaneous Applications

The pseudo-spin scheme and its pseudo-SU(3) extension have been used in a number of different applications over what is now about a fifteen year period: determining decoupling parameters and magnetic moments for certain rare earth and actinide nuclei, predicting alpha-particle transfer strengths for certain Ni and Zn isotopes, studying backbending in $^{126}$Ba, examining the forking of rotational bands in the $^{68}$Ge nucleus, looking into the role of $Q_x$, $Q_y$, and the strong coupling limit of the pseudo-SU(3) model, explaining systematics of unique parity spin sequences (favored versus unfavored) in deformed odd-A nuclei, as well as determining E2, M1 & M3 transition strengths for certain rare earth and actinide nuclei. In each case the truncation applied was severe, usually down to one or at most five SU(3) irreps in the leading two or three spatial symmetries.

The reason for this diverse but rather restricted set of applications that have been carried out to date is the lack of available resources: reliable codes for calculating $\text{U(N) \supset SU(3)}$ reduced matrix elements, human (students, postdocs, colleagues, etc.), and other (money, computational power, etc.). It now seems clear that the first of these hurdles will be crossed within the year due to the effort of Chairul Bahri, a very talented LSU graduate student from Indonesia. The second remains a problem but now that more structure theorists are looking for group theoretical answers to nuclear structure problems, some relief may be on the horizon. In this regard it is important to acknowledge strong support, interest and encouragement from high-spin nuclear physicists. Some giant steps forward have also been made on the computational side of things through the efforts of Soon Park, a graduate student in computer science at LSU who has chosen to work on problems of relevance to nuclear theory. He has developed two very valuable programs: an on-line space and time optimal numerical database system for large-scale scientific applications and a vectorized and parallelized version of the Lanzcos procedure for matrix diagonalization.

Projects currently underway include a study of the role of intruder levels (Jutta Escher), the development of the many-particle pseudo-SU(3) equivalent of the single-particle Nilsson model (Husney Naqvi), a formalism for evaluating electron scattering form factors within the pseudo-SU(3) and pseudo-symplectic model frameworks (Peter Rochford). Another, which will now be described in some detail involves an application of the pseudo-symplectic theory for determining potential energy surfaces and studying giant resonance phenomena. This work is being carried out in collaboration with Peter Rochford, who is a postdoc at LSU,
George Rosensteel, a colleague at Tulane University in New Orleans, and Octavio Castaños and Peter Hess from the University of México. The input of these individuals has been invaluable. Some of the result quoted below are from unpublished work of these researchers. We gratefully acknowledge their part in the pseudo-X programs where \( X = LS, SU(3) \) and \( Sp(3,R) \) symplectic.

9. Giant Resonance Modes

The model Hamiltonian is taken to be the sum of proton and neutron pseudo harmonic oscillator Hamiltonians plus a real quadrupole-quadrupole interaction. And as indicated above, since the current application of the theory is restricted to low-lying states of even-even nuclei, contributions to the dynamics from protons and neutrons in the unique parity levels can and will be suppressed in what follows. Also, to ensure that the mean field of the harmonic oscillator is conserved under perturbation by the quadrupole-quadrupole interaction, \( Q^c - Q^e \) is replaced by \( Q^c - (Q^c \cdot Q^e)_{\text{shell}} \) where \( (Q^c \cdot Q^e)_{\text{shell}} \) is an operator that reproduces single-shell traces of \( Q^c \cdot Q^e \).

\[
H = \hbar \omega \tilde{N} - \frac{1}{2} \lambda (Q^c - (Q^c \cdot Q^e)_{\text{shell}}) + H_r, \tag{10}
\]

\[
\tilde{N} = \tilde{N}_\pi + \tilde{N}_\nu \quad \text{and} \quad Q^c = Q^c_\pi + Q^c_\nu.
\]

The superscript "c" in these equations denotes the real "collective" quadrupole operator, \( Q^c = \sqrt{16\pi/5} r^2 Y_2(\hat{r}) \), as opposed to the symmetrized "algebraic" one of Elliott, \( Q^a = \sqrt{16\pi/5} [b^4 p^2 Y_2(\hat{p}) + r^2 Y_2(\hat{r})]/2 \), where \( b \) is the oscillator length parameter. (This \( Q^a \) is the same as the \( Q \) used in the previous section.) While the matrix elements of \( Q^c \) and \( Q^a \) are identical within an oscillator shell, \( Q^c \) has nonvanishing matrix elements between shells that differ by two quanta whereas \( Q^a \) does not. The major shell separation energy \( \hbar \omega \) is given by the usual empirical rule \( 41A^{-1/3}\text{[Mev]} \). The Hamiltonian includes a residual interaction, \( H_r \), that allows the inertia and band splitting features of the spectrum to be easily reproduced.

The real mass quadrupole operators \( Q^c_\pi \) and \( Q^c_\nu \) can be expressed in terms of the generators of symplectic algebras: \( Q^c_\alpha = Q^c + \sqrt{6}/2[B^c_\alpha + B^c_\alpha] \) where \( \alpha = \pi \) & \( \nu \). The operators \( B^c_{l\max} (B^c_{l\max}) \) with \( l = 0 \) & 2 are the \( 2\hbar \omega \) raising (lowering) operators of the \( Sp(3,R) \) algebras and the \( Q^c_\alpha \) are quadrupole generators of the respective \( SU_{\alpha}(3) \) subalgebras. Through a second quantization formalism these operators \( (O = Q^c_\pi, B^+, B^-) \) can be expanded in terms of pseudo \( SU(3) \) tensors, \( O = xO + \cdots \), where \( O \) has the same tensorial character as \( O \), see (8). The constant \( x \) in this expansion is always greater than unity, ranging from a high of about 1.4 for \( O = Q^c (\tilde{n} = 0) \) to a low of about 1.1 for \( O = B^c_{2\max}(\tilde{n} = 6) \). An average \( x \) value for actinide nuclei is 1.14. In pseudo \( SU(3) \) applications the correction terms were found to yield less than a one percent change in calculated energies and electromagnetic transition rates. Therefore \( O \) was replaced by \( \tilde{O} \) in this study.

Neglecting all higher order effects, the Hamiltonian (10) can be rewritten as,
\[ H = \hbar \omega N - \frac{1}{2} \hat{\chi} [\hat{Q}^c \cdot \hat{Q}^c - (\hat{Q}^c \cdot \hat{Q}^c)_{\text{shell}}] + a\hat{L}^2 + b\hat{X}_3^c + c\hat{X}_4^c. \]

(11)

In this expression, an average \( \chi \) value has been absorbed into \( \chi \) and the last three terms are an explicit form for \( H_r \). The operator \( \hat{L} \) is the total pseudo angular momentum while the \( \hat{X}_3 \) and \( \hat{X}_4 \) terms are third and fourth order rotational scalars: \( \hat{X}_3 = [\hat{L} \times \hat{Q}^c] \times \hat{Q}^c \) and \( \hat{X}_4 = \left( [\hat{L} \times \hat{Q}^c] \times (\hat{Q}^c \times \hat{L}) \right) \). This special form for the residual interaction means the moment of inertia and band splitting of the low-lying states can be adjusted without otherwise affecting the dynamics. Note that matrix elements of this \( H_r \) vanish in \( 0^+ \) states. An estimate for the strength \( \chi \) of the quadrupole-quadrupole interaction can be made by equating the difference in the expectation value of \( H \) in the \( 0^+ \) state of the \( 0\hbar \omega (\lambda, \mu) \) irrep and the so-called stretched \( 2\hbar \omega (\lambda+2, \mu) \) irrep to the excitation energy, \( 80A^{-1/3} \), of the giant monopole resonance. This estimate for \( \chi \) can be improved upon by using it to calculate eigenfunctions of the hamiltonian, (11), with \( a = b = c = 0 \) in the \( 0^+ \) and \( 2^+ \) spaces and calculating the \( 2^+ \rightarrow 0^+ \) reduced E2 transition probability. If the E2 strength is smaller (bigger) than the experimental result a bigger (smaller) value of \( \chi \) should be chosen. The assumption of a linear relation between E2 values and \( \chi \) suffices in interpolating between results. The residual interaction can be ignored in this process because even though it changes the excitation energy of the \( 2^+ \) state, it has virtually no effect on the calculated \( 2^+ \rightarrow 0^+ \) transition strength. A similar procedure can be used to determine the parameters \( a, b \) and \( c \) of \( H_r \).

Now consider \( ^{238}\text{U} \). It has 10 valence protons in the \( Z = 82 - 126 \) shell and 20 neutrons in the \( N = 126 - 184 \) valence space. The distribution of these particles between normal and unique parity orbitals is made by selecting a reasonable deformation, say \( \beta = 0.25 \), and filling each level of the appropriate Nilsson diagram with a pair of particles. This procedure yields 6 normal parity protons and 12 normal parity neutrons for the most probable occupancies of the pseudo \( n_\pi = 4 \) and \( n_\nu = 5 \) shells. The corresponding occupancies for the unique parity parts are 4 protons and 8 neutrons in the \( j_{13/2} \) and \( j_{15/2} \) orbitals, respectively. The normal parity configurations give rise to the leading pseudo SU(3) irreps (18,0) for protons and (36,0) for neutrons. The strong coupled SU(3) irreps are given by the product of these two irreps, \((18,0) \otimes (36,0) \equiv (\lambda, \mu)\). Of this set of irreps the one with the maximum eigenvalue of \( C_2 \) and hence the largest deformation, which follows because \( \beta^2 \sim \langle Q^a \cdot Q^a \rangle \), is expected to dominate the low-lying structure. This irrep, \((\lambda, \mu) = (54,0)\), is the leading \( 2\hbar \omega \) SU(3) symmetry.

Applying these considerations to \( ^{238}\text{U} \) yields a value of \( 0.00138(\text{MeV}) \) for \( \chi \). With this \( \chi \), the calculated \( 2^+ \rightarrow 0^+ \) transition rate is \( 2.48 \times 10^2 \), which is nearly within the error bars of the adopted experimental number, \( 2.42 \pm 0.04 \). Since the leading pseudo SU(3) irrep for \( ^{238}\text{U} \) is \((54,0)\), it is like \( ^{20}\text{Ne} \) of the ds-shell in that the theory predicts the absence of a \( K^\pi = 2^+ \) (gamma) band. This is consistent with the fact that in \( ^{238}\text{U} \) a second \( 2^+ \) level has not been found below the first excited \( 0^+ \) level. Because \( \mu = 0 \), the \( b \) and \( c \) parameters were set equal to zero. The predicted value for \( a \) is then \( 0.00541 \). A best fit to the experimental energy spectrum and E2 transition strengths for the \( L^\pi = 0^+ - 12^+ \) states was obtained with \( \chi = 0.00135(\text{MeV}) \) and \( a = 0.00465 \). In the calculation a full \( 20\hbar \omega \) symplectic
basis was used, with dimensionalities ranging from 489 for the $L^z = 0^+$ space up to 4069 for $L^z = 12^+$. While the best fit value for $\chi$ is within 5% of the initial estimate, the final value for the parameter "a" is about 15% below the simplest prediction. This larger difference for "a" can be attributed to the fact that $Q^c Q^c$ is more effective in generating rotations than $Q^a Q^a$ suggests.

Rather than showing a near perfect fit to the experimental spectrum, only calculated intensities of the ground (01) and resonant (02) states are given in Figure 7 for the final best-fit $\chi$ value. Recall that for $\chi = 0.0$ the ground and giant monopole states are pure $0h\omega [(\lambda,\mu)=(54,0)]$ and $2h\omega [(\lambda,\mu)=(56,0)]$ excitations, respectively. An important feature is the amount of vertical mixing that is required to reproduce the observed E2 transition strengths. The $(\lambda,\mu)=(54,0)$ 0h\omega configuration makes up 81.0% of the intensity profile of the ground state with slightly over a 15% contribution from the 2h\omega space [11.3% $(\lambda,\mu)=(56,0)$ and 4.0% $(\lambda,\mu)=(52,2)$] and under 5% from the 4h\omega space [1.4% $(\lambda,\mu)=(58,0)$, 1.3% $(\lambda,\mu)=(54,2)$ and 0.4% $(\lambda,\mu)=(50,4)$], etc. The resonant mode, on the other hand, has significant multi-h\omega admixtures extending out to 12h\omega, but in agreement with the most naive picture, the dominant contribution is the $(\lambda,\mu)=(56,0)$ configuration with 45.5% intensity. The $(\lambda,\mu)=(52,2)$ irrep adds an additional 5.1% for a total of 50.6% in the 2h\omega space. The 0h\omega and 4h\omega spaces contribute 16.8% and 23.6%, respectively, to the structure of the resonant mode. At each level, $n = 1, 2, ..., $ the stretched SU(3) irrep, $(\lambda+2n, \mu)$, was found to be the most important. Details for

![Fig. 7](image_url)

Fig. 7 Calculated intensities for the ground state (01 $\rightarrow$ G.S.) and resonant mode (02 $\rightarrow$ R.S.) of the pseudo symplectic Hamiltonian in a full 20h\omega symplectic basis with the best-fit value for the strength of the $Q^c Q^c$ interaction, $\chi = 0.00135$. The abscissa labels the number of 2h\omega excitations. For $\chi=0.0$ the ground state and resonant state are pure SU(3) configurations with $(\lambda,\mu)$ values (54,0) and (56,0), respectively. In agreement with this simple picture, the $(\lambda,\mu)=(54,0)$ 0h\omega irrep makes up 81.0% of the intensity profile of the ground state while the resonant model has 50.6% of its strength lying in the 2h\omega space.
excitation energies and quadrupole transition strengths are given in Table 1. Pseudo SU(3) and collective model results are included for comparison. They show that the pseudo SU(3) B(E2) strengths saturate at $J_t = 6$ while the collective model results show no such trend. The pseudo symplectic B(E2) results, on the other hand, show saturation near to where the effect is observed, $J_t = 12-16$.

Table 1. Experimental and calculated excitation energies $[E_i \text{ (Mev)}]$ and electric quadrupole transition strengths $(B(E2) \text{ (e}^2b^2))$ in $^{238}\text{U}$. B(E2) values are quoted for the pseudo SU(3) and collective model (CM) theories in addition to those for the pseudo symplectic scheme $[\text{Sp}(3,R)]$. The pseudo SU(3) results were renormalized to the adopted experimental $B(E2: 2_1 \rightarrow 0_1)$ number, $2.42e^2b^2$.

<table>
<thead>
<tr>
<th>$E_i \text{ (Mev)}$</th>
<th>$B(E2) \text{ (e}^2b^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp</td>
<td>$\text{Sp}(3,R)$</td>
</tr>
<tr>
<td>0.0449</td>
<td>0.0435</td>
</tr>
<tr>
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<td>0.7982</td>
</tr>
<tr>
<td>1.0765</td>
<td>1.1320</td>
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</table>

10. Conclusion

A key feature in the development of the pseudo-spin scheme and the SU(3) and Sp(3,R) derivative models is the empirical result that the Nilsson parameter $\mu = 2D/C = 0.5$ for heavy nuclei. Actual estimates for $\mu$ are $(0.60 \& 0.65)$ for protons with $(50 < Z < 82 \& Z > 82)$ and $(0.42 \& 0.33)$ for neutrons with $(82 < N < 126 \& N > 126)$, respectively. These numbers are close enough to the exact pseudo-spin limit, $\mu = 0.5$, that even with a relatively weak quadrupole-quadrupole interaction the pseudo SU(3) and Sp(3,R) limits of the theory apply. In contrast with this, for the real ds-shell the value for the parameter $\mu = -0.2$ (note the sign) and, as is well-known, the $(s_{1/2}, d_{3/2})$ levels do not form a near degenerate pair nor is the $d_{5/2}$ orbital far removed from them. The analysis underscores the need for a deeper understanding of the $C = 4D$ result. In particular, is the neutron-proton difference $(\mu_N = 0.4 \text{ versus } \mu_p = 0.6)$ more that a Coulomb effect and is there fundamental physics behind the average $\mu = 0.5$ result for the ratio $2D/C$ of the orbit-orbit to spin-orbit self interaction strengths? Work on this is in progress.
Acknowledgements

Discussions with colleagues O. Castaños and P. O. Hess of the University of México, P. Rochford, a postdoc, and students C. Bahri, H. Naqvi, S. Park and J. Escher, from Louisiana State University are gratefully acknowledged. The nuclear theory effort at Louisiana State University is supported in part by grants from the U. S. National Science Foundation, INT-88-01337 and PHY-89-22550.

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Introduction

A single-particle wavefunction consisting of a three-dimensional isoropic harmonic oscillator is associated with the usual one-body spin-orbit (SO) and orbit-orbit (OO) coupling,

\[ \phi = Pq + CJ_s + D, \]

is known to be a useful starting point for nuclear structure studies.\(^1\)\(^2\) The \(P\) term, with \(P < 0\), presents high angular momentum states down relative to those with lower \(l\) values, a feature that occurs naturally when a more realistic interaction like a Woods-Saxon potential is used for the central potential, while the phenomenological \(D\) term with \(C\) = 0 couples multiple space and spin degrees of freedom, is required to achieve shell closure in the single number 2, 8, 20, 50, 82, 126 and 184.

Unfortunately, the obtained value for \(C\) in (1) is so large that the spin-orbit interaction actually destroys the underlying SU(3) symmetry of the oscillator for all but the lightest \(A < 100\) nuclei, and in so doing renders it of little apparent value in attempts at understanding the structure of heavier \((A > 100)\) systems. Specifically, for heavy nuclei the \(P\) term is the orbital of the \(\alpha\)-th oscillator shell, which includes