ABSTRACT

The superiority of the pseudo-spin scheme over the usual single-particle shell-model picture for heavy nuclei is demonstrated first. The general group structure of the many-particle pseudo LS coupled shell model is then presented and the pseudo SU(3) scheme shown to emerge very naturally from this structure. Its validity is confirmed for situations when the quadrupole-quadrupole interaction dominates the residual two-body interaction. Preliminary results on the role of intruder states in contributing to enhanced quadrupole collectivity in strongly deformed nuclei are presented last.

1. INTRODUCTION

The three-dimensional isotropic harmonic oscillator, $H_0$, with eigenvalue $n\hbar\omega$ where $n$ is the shell number, augmented with the one-body $l\cdot s$ and $l^2$ interactions,

$$H = H_0 + C l\cdot s + D l^2$$

is known to be a good approximation for the nuclear single-particle hamiltonian.\(^{1,2}\)

The $l^2$ term ($D < 0$) pushes high angular momentum states down relative to those with lower $l$ values, a feature that occurs automatically when a Woods-Saxon form is used for the central potential, while the phenomenological $l\cdot s$ term ($C < 0$), which couples space and spin degrees of freedom, is required to achieve shell closures at the magic nucleon numbers 2, 8, 20, 50, 82, 126 and 184. Unfortunately, the required value for $C$ is so large that the spin-orbit term completely destroys the underlying SU(3) symmetry of the oscillator for all but light ($A \leq 28$) nuclei, thereby rendering it of little apparent value in attempts at unraveling the structure of heavy ($A \geq 100$) systems.\(^{3}\) Specifically, for heavy nuclei the $j = n+l/2$ orbital of the $n$-th oscillator shell ($n = 0, 1, ...$), which includes levels with $j = l\pm l/2$ with $l = n, n-2, ..., 1$ or 0, is pushed down among the orbitals of the next lower shell. This yields new shells with normal parity $j = l/2, 3/2, ..., n-l/2$ orbitals plus a $j = (n+l)+l/2 = n+3/2$ unique parity intruder level from the $(n+1)$-th shell immediately above.

A straightforward way to gain an immediate appreciation for the simplicity and significance of the pseudo-spin concept\(^{4,5}\) is shown in Figure 1 where eigenvalues of
**Pseudo-Spin Symmetry**

H are plotted as a function of the Nilsson parameter $\mu = 2D/C$. For the special value $\mu = 0.5$, the orbital pairs with $j = l+1/2$ and $j = (l+2)-l/2$ are degenerate for all $l$ values. Furthermore, the splitting of these pairs follows a $\bar{l}(\bar{l}+1)$ rule where $\bar{l}$ is the average $l$ value of the pair, $\bar{l} = [(l+(l+2))/2 = l+1$. This result can be expressed mathematically as a special "normal $\leftrightarrow$ pseudo" unitary transformation.\(^6\)

**FIGURE 1.** Plot of the eigenvalues of the reduced single-particle hamiltonian given by $H/\hbar\omega = n - \kappa(2l+1) + \mu l^2$, where $\mu = 2D/C$ and $\kappa = -C/2\hbar\omega$, for the specific value $\kappa = 0.05$ and $0.0 \leq \mu \leq 1.0$. Notice that the $j = (l+2)-l/2$ and $j = l+1/2$ levels are degenerate for $\mu = 0.5$. Indeed, as shown in the text, the $\mu = 0.5$ spectrum can be duplicated by the simpler hamiltonian $H/\hbar\omega = n - \kappa \mu \bar{l}^2$ where $\hbar\omega = \hbar\omega$ and $\bar{n} = n-1$ with $j = l+s = \bar{l} - \bar{s}$ where $\bar{l} = l+1$ and $\bar{s} = 1/2$. In addition, in each pseudo shell there is the abnormal parity intruder level, shown as dashed, with $j = (n+1)+1/2 = n+3/2$ from the shell above. Since real (heavy) nuclei fall much closer to the $\mu = 0.5$ (pseudo) than the $\mu = 0.0$ (normal) oscillator limit ($-0.6$ for protons and $-0.4$ for neutrons) a many-particle description using basis states of the pseudo oscillator is expected to be superior to one in terms of basis states of the normal oscillator.
The single-particle hamiltonian transforms under this mapping as follows:

\[
\begin{align*}
\tilde{H}_0 + \text{Cl} \cdot \tilde{s} + Dl^2 &= \tilde{H}_0 + (4D-C)\tilde{l} \cdot \tilde{s} + D\tilde{l}^2 + (\hbar\omega+2D-C), \\
\end{align*}
\]

where \( \tilde{H}_0 = \tilde{n} \hbar\omega = (n-1)\hbar\omega, \tilde{l} \cdot \tilde{s} = -(l \cdot s+1) \) and \( \tilde{l}^2 = l^2 + 4l \cdot s + 2 \). Since the \( (\hbar\omega+2D-C) \) term is a constant, the pseudo form, \( \tilde{H} = \tilde{H}_0 + \tilde{C} \tilde{l} \cdot \tilde{s} + \tilde{D} \tilde{l}^2 \), has the same excitation spectrum as the normal one. \( \tilde{H} = H_0 + \text{Cl} \cdot s + Dl^2 \), when \( \hbar\omega = \hbar\omega, \tilde{C} = (4D-C) \) and \( \tilde{D} = D \). This transformation is meaningful because \( C = 4D \) so \( \tilde{C} = 0 \). Specifically, as indicated in the figure, \( \mu_\nu = 0.4 \) and \( \mu_\pi = 0.6 \) (\( \nu \) for neutrons and \( \pi \) for protons). This places heavy nuclei very close to the exact pseudo-spin limit (\( \mu = 0.5 \)) of the theory. In particular, the average value for \( \mu \) is almost exactly 0.5. The familiar single-particle shell-model hamiltonian for heavy nuclei can therefore be replaced by a less familiar but equivalent pseudo form which is inherently simpler because it has a much smaller spin-orbit term.

2. **PSEUDO LS-COUPLED SHELL MODEL**

The pseudo scheme organizes the normal parity \( j = 1/2, 3/2, \ldots, n-1/2 \) levels of the \( n \)-th oscillator shell into a pseudo oscillator shell with \( \tilde{n} = n-1 \). For example, the \((4s_{1/2}, 2d_{3/2}, 2d_{5/2}, 0g_{7/2})\) normal parity levels of the \( n = 4 \) shell are mapped onto the \((3p_{1/2}, 3p_{3/2}, 1f_{5/2}, 1f_{7/2})\) orbitals of a \( \tilde{n} = 3 \) shell. This mapping of single-particle orbitals defines the pseudo coupling scheme. To grasp its full significance, recall that the dynamical symmetry group for the usual many-particle generalization of the single-particle theory, with particles distributed among the lowest available single-particle levels, is \( U(NM) \), the unitary group in \( (N \text{ by } M) \) dimensions, where \( N = (n+1)(n+2)/2 \) is the degeneracy of the \( n \)-th oscillator shell valence space and \( M = 2 \) or \( 4 \) for a spin or spin-isospin formulation of the theory. The \( U(N) \oplus U(M) \) direct product subgroup of this \( U(NM) \) group separates the full \( (N \text{ by } M) \) dimensional space into its spatial and spin \( (M = 2) \) or spin-isospin \( (M = 4) \) degrees of freedom. Irreps of \( U(N) \), which are labeled by a Young pattern \( [\mathbf{f}] = [f_1, f_2, \ldots, f_N] \), specify the space symmetry while irreps \( [\mathbf{f}^S] = [f_1^S, f_2^S, \ldots, f_M^S] \) of \( U(M) \), which must be related to the \( [\mathbf{f}] \) of \( U(N) \) by row-column interchange to insure overall antisymmetry in \( U(NM) \) as required by the exclusion principle, label the spin or spin-isospin symmetry, as appropriate. An important difference between the two one-body interactions is that the \( l \cdot s \) term couples different spatial symmetries whereas \( l^2 \) does not. When the
strength of all terms like $l \cdot s$ that couple different spatial symmetries is small relative to others like $l^2$ that do not, $[\tilde{f}]$ and therefore $[\tilde{f}^C]$ will be good quantum numbers. Whereas this is so for the many-particle extension of $\tilde{H}$, see below, it certainly is not for $H$. This feature represents significant savings because the full model space can then be partitioned into (pseudo) subspaces ([\tilde{f}] and [\tilde{f}^C] of $\tilde{U}(\tilde{N}) \otimes \tilde{U}(\tilde{M})$) of much smaller dimensions than for the normal scheme. In addition, as is known to be the case for the surface delta interaction$^4$) and as is demonstrated below for the quadrupole-quadrupole interaction, if the residual two-body interaction is a pseudo-space scalar operator it reenforces the goodness of the [\tilde{f}] and [\tilde{f}^C] symmetries.

For heavy nuclei the valence protons and neutrons occupy different shells so an identical particle ($\tilde{M}=2$) formulation must be applied to each. Specifying the [\tilde{f}] and [\tilde{f}^C] = [\tilde{f}_1^C, \tilde{f}_2^C] labels is then equivalent to specifying the total number of normal parity particles and their pseudo spin, $\tilde{m} = \tilde{f}_1^C + \tilde{f}_2^C$ and $\tilde{S} = (\tilde{f}_1^C - \tilde{f}_2^C)/2$. Each normal parity $\tilde{m}$-particle space, with $\tilde{m} = \tilde{m}_\pi$ for protons and $\tilde{m} = \tilde{m}_\nu$ for neutrons, is partitioned into subspaces with $\tilde{S} = 0, 1, 2, 3, \ldots, \tilde{S}_{\max}$ for $\tilde{m}$ even while for $\tilde{m}$ odd it divides into subspaces with $\tilde{S} = 1/2, 3/2, 5/2, \ldots, \tilde{S}_{\max}$ where $\tilde{S}_{\max}$ is the minimum of $\tilde{m}/2$ and $\tilde{N} - \tilde{m}/2$. Distinct spatial configurations therefore differ by integer pseudo spin values ($\Delta \tilde{S} = 1, 2, \ldots, \text{etc.}$). Also, to each $\tilde{S}$ there is a complementary set of spatial configurations. To the extent the pseudo-spin symmetry is good, one therefore expects to observe sets of states, like rotational sequences, that differ in total angular momenta ($J = \tilde{L} + \tilde{S}$) by integer (even-$A$ compared with even-$A$) or half-integer (odd-$A$ with even-$A$) amounts. This answers in the affirmative a question raised by Stephens et al., in a recent PRL,$^9$) namely, "whether low-lying collective states having alignment 1 would occur in a nucleus with rather good pseudo-spin symmetry." In considering this matter, it is very important to understand that pseudo-spin alignment can be either proton or neutron in origin or a combination of the two because the basis states are coupled products of proton and neutron configurations: $|\Psi\rangle = |(\tilde{\alpha}_\pi \tilde{L}_\pi \tilde{\alpha}_\nu \tilde{L}_\nu)^\pi_x \times (\tilde{S}_\pi \tilde{S}_\nu)^\tilde{S}\rangle$, where $\tilde{\alpha}_\kappa$ labels multiple occurrences of the $\tilde{L}_\kappa$ values and $\kappa = (\pi, \nu)$. The symmetry group of this combined $\pi-\nu$ system is $[\tilde{U}_\pi(\tilde{N}_\pi) \otimes \tilde{U}_\pi(2)] \otimes [\tilde{U}_\nu(\tilde{N}_\nu) \otimes \tilde{U}_\nu(2)]$. This direct product can be reordered as in $|\Psi\rangle$ so the pseudo-space and pseudo-spin associations are made first, $[\tilde{U}_\pi(\tilde{N}_\pi) \otimes \tilde{U}_\nu(\tilde{N}_\nu)] \otimes [\tilde{U}_\pi(2) \otimes \tilde{U}_\nu(2)]$. In this expression, $\tilde{N}_\kappa = [(\tilde{n}_\kappa + 1)(\tilde{n}_\kappa + 2)/2$ is the pseudo-space degeneracy of the $\kappa = (\pi, \nu)$ subshell. This means the observed alignment feature is consistent with good total pseudo-spin symmetry, $\tilde{S} = \tilde{S}_\pi + \tilde{S}_\nu$, provided the $\pi-\nu$ interaction like the $\pi-\pi$ and $\nu-\nu$ terms.
conserves $\bar{S}$. As is demonstrated below, the real quadrupole-quadrupole, $Q^\pi \cdot Q^\nu$, which does connect configurations with different $\bar{S}$ symmetry, but only weakly as compared to the symmetry preserving couplings, is such an interaction.

3. PSEUDO SU(3) COUPLING SCHEME

The importance of the SU(3) model for light nuclei follows from the dominance of the quadrupole-quadrupole interaction, $Q-Q$, over the one-body $l-s$ and $l^2$ terms as well as over all other two-body forms.\textsuperscript{10,11} Even though the spin-orbit interaction is strong, yrast states of nuclei like $^{20}$Ne and $^{24}$Mg are typically 60-80% pure leading SU(3) representations.\textsuperscript{12} This can be understood as follows: First of all, SU(3) is a subgroup of U(N) with Casimir invariant $C_2 = (Q-Q + 3L^2)/4$. Hence Q-Q conserves spatial symmetry. In addition, since the expectation value of Q-Q is proportional to the square of the deformation, Q-Q further subdivides each U(N) irrep [f] into $(\lambda, \mu)$ irreps of SU(3) with the most deformed of these lying lowest and the least deformed highest. The amount and sharpness of the separation, first into irreps of U(N) and then by SU(3), depends on the relative strength of the symmetry preserving and symmetry breaking interactions. And this in turn depends upon whether or not the available space supports strongly deformed configurations. In the ds-shell case, systems like $^{20}$Ne and $^{24}$Mg have leading $(\lambda, \mu)$'s with large deformation so Q-Q overpowers all other interactions and yrast states have relatively good [f] and $(\lambda, \mu)$ quantum labels. In a restricted space like the $d_{5/2}$ subshell of the ds shell, however, the same Hamiltonian will not display the same level of quadrupole collectivity and other interactions like pairing might very well appear to dominate. The success of the SU(3) model in the ds-shell shows that the many-particle dynamics can promote (quadrupole) collectivity over single-particle and other noncollective effects.

We now argue that the pseudo SU(3) scheme,\textsuperscript{13} with $\bar{SU}(3)$ standing in the same relationship to $\bar{U}(\bar{N})$ as SU(3) does to U(N), provides a similar explanation for observed quadrupole collectivity in heavy deformed nuclei. There are, of course, some differences: 1) the valence neutrons and protons occupy different major shells, 2) the normal Q-Q interaction is not the quadratic invariant of $\bar{SU}(3)$, and 3) whereas for the ds shell the coefficient D of $l^2$ is positive, for heavy nuclei it is always negative. To dispense with these differences, first recall that the pseudo-spin scheme is an excellent starting point for a many-particle description of heavy nuclei, whether they are deformed or not. In particular, the $\bar{7} \cdot \bar{S}$ interaction is weak relative to $\bar{7}^2$ so the pseudo-spin symmetry is good. Also, since the surface delta interaction, which is
known to be a reasonably good effective interaction for many applications, is a
pseudo-spin scalar operator, the residual two-body interaction is not expected to
change this picture by inducing additional mixing among pseudo-spin symmetries.4) And even more importantly, although Q·Q is not an invariant of \( \tilde{SU}(3) \), under the
normal-to-pseudo mapping it transforms into its pseudo counterpart plus small
corrections.14)

\[
Q \cdot Q = \kappa \tilde{Q} \cdot \tilde{Q} + \ldots.
\]

Indeed, within the leading pseudo-space symmetry, the sum total of all correction
terms has been shown to induce less than a one percent change in the structure of
calculated yrast states.15) Regarding issues surrounding the fact that protons and
neutron reside in different shells, a very simple calculation has been done which
demonstrates that this makes no difference so long as the protons and neutrons
interact even weakly through their quadrupole fields. Specifically, even if the
separate proton and neutron interactions are dominated by pairing, a small \( Q^p \cdot Q^n \)
interaction between the two suffices to drive the whole system towards the strong
coupled pseudo \( SU(3) \) limit of the theory.16)

To explore the relevance of the pseudo \( SU(3) \) scheme for strongly deformed
nuclei, consider the many-body problem (tildes suppressed) with hamiltonian,

\[
H = H_0 + D \sum_i l_i^2 - \frac{1}{2} \chi Q \cdot Q = H_0 + D \sum_i l_i^2 - \frac{1}{2} \chi (4C_2 - 3L^2).
\]

The last form for \( H \) follows because within a major oscillator shell \( Q \cdot Q = 4C_2 - 3L^2 \).
Here \( C_2 \) is the quadratic Casimir invariant operator of \( SU(3) \) which has eigenvalue
\( \lambda^2 + \lambda \mu + \mu^2 + 3(\lambda + \mu) \) in the \( (\lambda, \mu) \) representation and \( L^2 \) is the square of the total
angular momentum with eigenvalue \( L(L+1) \). Since the \( C_2 \) and \( L^2 \) interactions are
diagonal in an \( SU(3) \) basis they split but do not break the oscillator symmetry. And
because \( \chi \) is always positive the \( SU(3) \) representations with the largest eigenvalue
for \( C_2 \) and hence the greatest intrinsic deformation since \( \beta^2 \sim <Q \cdot Q> = <4C_2> \) in
\( L=0 \) states, lie lowest. Results for \( 0^+ \) states of this hamiltonian, plotted as a function
of \( \chi \) for a typical \( D < 0 \) value for the simple but representative case \( (ds)^4[f]=4 \)
with \( (\lambda \mu) = (8,0), (4,2), (0,4), \) and \( (2,0) \), are given in Figure 2. This pseudo \( ^{20}\text{Ne} \)
case, which differs from the real \( ^{20}\text{Ne} \) nucleus in that \( D < 0 \) instead of being
positive and \( C = 0 \) instead of being nonzero, shows that under conditions very
similar to those for pseudo \( SU(3) \) applications in rare earth and actinide nuclei the
FIGURE 2. Intensity of SU(3) representations in calculated $L^q = 0^+$ states of the model hamiltonian (4) in the space $(ds)^4[f] = [4]$ with $(\lambda, \mu)$'s = (8,0), (4,2), (0,4), and (2,0). The parameter $D$ was set at -0.2 (Mev) while $\chi$ was assigned values between 0.0 and 0.1 (Mev). These parameter values for a pseudo $^{20}$Ne system with no spin-orbit splitting, which differ significantly from the values required for a real $^{20}$Ne application ($C = -2.0$ and $D = +0.2$ for $\mu = -0.2$), simulate an application of the theory to deformed nuclei of the rare earth and actinide regions. A full breakdown of the intensity is given for the $0^+_1$ yrast state while results for the main SU(3) component only are shown for each of the remaining states, $0^+_{2\alpha}, \alpha = 2, 3, \& 4$. Note that SU(3) symmetry breaking decreases sharply as the strength of the deformation inducing quadrupole-quadrupole interaction increases, particularly for the yrast state. A realistic value for $\chi$ is about 0.06 - 0.07. Even at half this value, the yrast state is more than 80% pure $(\lambda, \mu) = (8,0)$. The dashed curve just below the $0^+_1(8,0)$ solid line is the corresponding result when a spin-orbit interaction is included with a strength $C = -0.2$, which is down by a factor of ten from the actual value but again a representative strength for pseudo SU(3) applications.
symmetry breaking decreases sharply as the strength of the deformation inducing quadrupole-quadrupole interaction increases. A $\chi$ of 0.06-0.07 is realistic for both a real and pseudo $^{20}$Ne application of the theory. In the figure the dashed curve is the $0^+_1(8,0)$ intensity when a spin-orbit interaction ($C < 0$) with one-tenth its normal strength is included in H. This value is appropriate for pseudo SU(3) applications. Yrast states of heavy deformed nuclei are therefore expected to be dominated to at least the 80% level by the leading pseudo SU(3) symmetry.

From the results of this simple representative calculation several important conclusions can be drawn. First of all, because $\tilde{C} = 0$ for nuclei of the rare earth and actinide regions, mixing among different pseudo-space symmetries is expected to be small. This means that the normal parity valence space can be truncated down to a reasonable size, for example the $[f] = [4]$ symmetry in the pseudo $^{20}$Ne case just considered, so simple yet realistic calculations can be carried out. Secondly, it is important to again stress that the breaking of the SU(3) symmetry decreases very sharply as the strength of the deformation inducing quadrupole-quadrupole interaction increases. More specifically, the spreading out of the various SU(3) irreps within $[f]$ symmetries by Q-Q reduces the SU(3) symmetry breaking induced by the $l^2$ term. This means, for example, that the normal parity contribution to yrast states of strongly deformed configurations should be dominated by the leading pseudo SU(3) symmetry.

It is important to understand that the leading pseudo SU(3) symmetry is the most deformed configuration available in the space under consideration. Going beyond normally deformed configurations to superdeformed and perhaps even hyperdeformed structures, therefore implies shifting particles into higher-lying configurations. For example, in the pseudo $^{20}$Ne example one candidate for a superdeformed band is the configuration obtained by lifting two particles out of the $p$ shell into the $fp$ shell. This leads to pseudo SU(3) irreps contained in the product $(8,0) \times (0,2) \times (6,0)$. The leading irrep in this case is $(\lambda,\mu)=(14,2)$. Since in $L=0$ states the square of the deformation is proportional to the expectation value of $C_2$, under the action of the same hamiltonian this arrangement of nucleons will have a deformation that is nearly twice that of the leading $(8,0)$ configuration: $<C_2(14,2)>/<C_2(8,0)> = 3.14$ which yields $\beta(14,2)/\beta(8,0) = 1.77$. Since the hamiltonian does not change, configurations like the one containing the $(14,2)$ are expected to display even less representation mixing than the one containing the $(8,0)$ since for this arrangement of particles the dominance of the Q-Q term in the energy
matrix will be even more pronounced. To summarize, based on the results of this representative study, superdeformed bands are expected to be better pseudo SU(3) nuclei and hence better rotors than normally deformed ones.\textsuperscript{17} This is consistent with recent experimental results.\textsuperscript{9,18}

4. ROLE OF INTRUDER LEVELS

The pseudo scheme removes the highest $j$ state associated with the $n$-th major shell of the oscillator, $j = n+1/2$, from active consideration because it is pushed down among levels of the shell immediately below by the spin-orbit interaction. Furthermore, the $j = n+3/2$ level that penetrates down from the $(n+1)$-st shell into the active model space is normally assumed to play a passive role in the dynamics of low-lying excited states. Specifically, the usual assumption is that for states below the backbending region this unique parity intruder level accepts only paired ($J=0$) states and therefore contributes additional binding energy to the system but nothing to the dynamics. Through and beyond the backbending region, however, alignment sets in and the intruder can no longer be considered to be inactive. We now want to report on preliminary results that challenge this simple assumption. In particular, the results suggest that the coupling of the intruder to its natural partners, even though these may be as much as $\hbar \omega$ away, is strong and leads to sizable contributions to the quadrupole moments and therefore E2 strengths of the many-particle system.

In Figure 3 normalized expectation values of Q-Q in calculated ground states of the $(ds)^4$ system for hamiltonian (4) with $D = -0.2$ and C values as indicated are plotted as a function of the strength $\chi$ of the quadrupole-quadrupole interaction. The normalization factor is the maximum eigenvalue of Q-Q in the $(ds)^4$ space which is just the expectation of Q-Q in the leading $(\lambda,\mu) = (8,0)$ irrep of SU(3). For the results to be representative of a pseudo SU(3) application, $C = -2.5$ and, as above, $\chi = 0.06 - 0.07$. It follows from this that even for a very strong spin-orbit splitting, the yrast state can achieve as much as 60-70% of its maximum total quadrupole collectivity. Since for rare earth and actinide the number of particles in the intruder level is typically about 1/3 of the total number of valence particles, and since the intruder level comes from the $(n+1)$-st oscillator shell, a rough estimate for the ratio of the contribution of particles in the unique parity intruder levels to that of those in the normal parity orbitals can be given: $
abla Q-Q)_{\text{unique}}/\nabla Q-Q)_{\text{normal}} = 0.6 \times C_2(m(n+1)/3,0)/C_2(2mn/3,0) = 0.6 \times [(n+1)/(2n)]^2 = 0.2$. This analysis assumes the $(\lambda,\mu) = (m(n+1)/3,0)$ and $(2mn/3,0)$ irreps are representative of those that are
allowed by the exclusion principle for the unique and normal parity spaces, respectively. This suggests that particles in the unique parity orbitals can be expected to contribute to the quadrupole moment of a deformed system roughly in proportion to their number with a strength that is only slightly less, \( \sim 0.6 \times [(n+1)/n]^2 \), than the strength with which the normal parity particles contribute. Since the number of particles in the unique parity space is not small, they are important in determining quadrupole moments and E2 strengths.

**FIGURE 3.** Normalized expectation values of Q-Q in calculated ground states of the \((ds)^4\) system for hamiltonian (4), with \( D = -0.2 \) and a spin-orbit term with \( C \) values as indicated, are plotted as a function of the strength \( \chi \) of the quadrupole-quadrupole interaction. The normalization factor is the maximum eigenvalue of Q-Q in the \((ds)^4\) space which is just the expectation of Q-Q in the leading \((\lambda, \mu) = (8,0)\) irrep of SU(3). For the results to be representative of a pseudo SU(3) application, \( C = -2.5 \) and, as indicated, \( \chi = 0.06 - 0.07 \). It follows from these results that even for a very strong spin-orbit splitting, the yrast state can achieve as much as 60-70% of its maximum total quadrupole collectivity.
5. CONCLUSION

A key ingredient in this development is the empirical result $C = 4D$ or that the Nilsson parameter $\mu = 2D/C = 0.5$ for heavy nuclei. Actual estimates for $\mu$ are $(0.60 & 0.65)$ for protons with $(50 < Z < 82 & Z > 82)$ and $(0.42 & 0.33)$ for neutrons with $(82 < N < 126 & N > 126)$, respectively. These numbers are close enough to the 0.5 value for exact pseudo-spin symmetry that even with a relatively weak quadrupole-quadrupole interaction the pseudo SU(3) limit of the theory applies. In contrast with this, for the real $ds$-shell the value for the parameter $\mu = -0.2$ (note the sign) and, as is well-known, the $(s_1/2, d_3/2)$ levels do not form a near degenerate pair nor is the $d_5/2$ orbital far removed from them. This analysis underscores the need for a deeper understanding of the $C = 4D$ result. In particular, is the neutron-proton difference $(\mu_n = 0.4$ versus $\mu_\pi = 0.6)$ only a Coulomb effect and even if it is, what is the physics behind the average $\mu = 0.5$ result? It would seem that any theory of nuclear structure, be it a nucleon-meson or quark-gluon formulation, should explain the microscopic origin of the $C = 4D$ result as this is a general feature that applies for all $A \geq 100$ nuclei and appears to be a central issue to understanding many of the regularities found in heavy nuclei.

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$$H = H_0 + \hbar s + \hbar^2$$

is known to be a good approximation for the nuclear single-particle Hamiltonian.\(^{12}\)

The $\hbar^2$ term ($D < 0$) pushes high angular momentum states down relative to those with lower $l$ values, a feature that occurs automatically when a Woods-Saxon form is used for the central potential, while the phenomenological $\hbar s$ term ($C < 0$), which couples space and spin degrees of freedom, is required to achieve shell closures at the magic nucleon numbers 2, 8, 20, 50, 82, 126 and 184. Unfortunately, the required value for $C$ is so large that the spin-orbit term completely destroys the underlying SU(3) symmetry of the oscillator for all but light ($A \leq 28$) nuclei, thereby rendering $\hbar s$ of little apparent value in attempts at unraveling the structure of heavy ($A \geq 100$) systems.\(^{13}\) Specifically, for heavy nuclei the $j = n+1/2$ orbital of the $n$-th oscillator shell ($n = 0, 1, \ldots$), which includes levels with $j = \hbar (n+1/2)$ with $l = n, n-2, \ldots, 1$ or 0, is pushed down among the orbitals of the next lower shell. This yields new shells with normal parity $j = 1/2, 3/2, \ldots, n-1/2$ orbitals plus $j = \hbar (n-1)+1/2 = n+1/2$ unique parity intermediate level from the $(n+1)$-th shell immediately above.

A straightforward way to gain an immediate appreciation for the simplicity and significance of the pseudo-spin concept\(^{4,5}\) is shown in Figure 1, where eigenvalues of