Thirty years ago, R. D. Ratna Raju, working with his thesis advisor, K. T. Hecht, and with some support from another then thirty-year-old researcher, J. P. Draayer, introduced the pseudo-SU(3) scheme to the nuclear physics community [1]. This scheme was a natural outgrowth of the pseudo-spin concept [2, 3], a fascinating notion in its own right that continues to challenge the nuclear physics community as to whether it is an accidental symmetry found in heavy nuclei or something far more fundamental that has deep physical significance [4, 5].

Over the thirty year time span since that ground-breaking work, and hence the 60th birthday celebration of R. D. Ratna Raju, the pseudo-spin/SU(3) concept has evolved into a full-fledged shell model theory. Key to this has been the development of a code that allows one to calculate reduced matrix elements of one- and two-body operators, plus the matrix elements of various transition operators, within an SU(3) framework [6]. As a consequence, today it is possible to carry out fully microscopic shell-model calculations in the SU(3) basis – be it the normal SU(3) scheme that works well for light nuclei up through the middle of the ds-shell or the pseudo-SU(3) scheme that can be used for heavier nuclei of the rare earth and actinide regions.

Here we look at M1 transitions, the so-called scissors mode, in the odd-mass Gd, Dy and Tm nuclides, which is a natural complement to the predictions for magnetic moments in these nuclei made in the first paper on the pseudo-SU(3) scheme [1]. The scissors mode in nuclei refers to a pictorial image of deformed proton and neutron distributions oscillating against one another [7]. A description of this mode within the framework of the IBM [8] led to its detection in $^{154}$Gd using high-resolution inelastic electron scattering [9]. Systematic studies employing nuclear resonance fluorescence scattering (NRF) measurements [10] followed. The non-observation of these low-energy M1 excitations in inelastic proton scattering (IPS) [11] confirmed its orbital character [12]. Over the past two decades an impressive wealth of information about the scissors mode in even-even nuclei has been obtained and analyzed [13].
Low-energy M1 transitions in odd-mass nuclei were first observed in $^{163}$Dy in 1993 [14]. Unambiguous spin and parity assignments of excited states in these nuclei are difficult to make due to the half-integer character of the angular momentum of the states [15]. Furthermore, the M1 strengths in odd-mass nuclei are highly fragmented. Since the intensities are far smaller than in even-even nuclei, their identification against the background [16], which is complicated by the presence of a small amount of impurities in the target [13], requires much higher experimental resolution [17].

Theoretical descriptions of scissors modes in odd-mass nuclei have been offered within the context of the IBBM [18, 19], the particle-core-coupling model [20] and the QPNM [21]. While the different models agree in relating the presence of the uncoupled nucleon with the observed fragmentation, the detailed description of this mode, with a nearly flat spectrum in some nuclei and well-defined peaks in others is still not understood. Recently, the interplay between the spin and orbital M1 channels was examined [22] in the energy range between 4-10 MeV [23].

In this paper we analyze scissors-like M1 transitions in $^{157}$Gd, $^{163}$Dy and $^{169}$Tm. These nuclei have been studied experimentally by a number of researchers [14, 15, 24]. A fully microscopic description of M1 transitions strengths between 2 and 4 MeV in these rare-earth nuclei was carried out using the pseudo SU(3) shell model. Good qualitative descriptions of the fragmentation of the M1 transition strength is obtained by including, for the first time, states with pseudo-spin 1 (in addition to $\bar{s} = 0$) and 3/2 (in addition to $\bar{s} = 1/2$). For normal parity levels our findings suggest that while orbital couplings are important, in odd-even mass nuclei it is spin-flip type couplings that dominate M1 strengths in the low-energy domain. These spin-flip type transitions were also found to be essential for describing the rapidly changing fragmentation patterns found in neighboring odd-A nuclei. Freezing the unique parity orbitals, which is the usual assumption, prevents the theory from giving a quantitative description of the M1 strength, a result that is not surprising since intruder states have the largest $l$ values and therefore contribute maximally to orbital-type M1 transitions.

As noted above, the pseudo SU(3) model [2, 3] capitalize on the existence of pseudo-spin symmetry, which refers to the experimental fact that single-particle orbitals with $j = l - 1/2$ and $j = (l - 2) + 1/2$ in the shell $\eta$ lie very close in energy and can therefore be labeled as pseudo-spin doublets with quantum numbers $\tilde{j} = j$, $\tilde{\eta} = \eta - 1$, and $\tilde{l} = l - 1$. Its origin has been traced back to the relativistic Dirac equation [5]. In the present version of the pseudo-SU(3) model, the intruder level with opposite parity in each major shell is removed from active consideration [25] and pseudo-orbital and pseudo-spin quantum numbers are assigned to the remaining single-particle states. This assumption represents the strongest limitation of the present model.

Many-particle states of $n_o$ active nucleons ($\alpha = p, n$) in a given $(N)$ normal parity shell $\eta'_o$ are classified by the following group chain [26, 27, 28]:

\[
\{1^{\eta}_{o}\} \{f_a\} \gamma_o \left(\lambda_o, \mu_o\right) \tilde{S}_o K_o
\]

\[
U(\Omega'_o) \supset U(\Omega'_o/2) \times U(2) \supset SU(3) \times SU(2) \supset \nonumber
\]

\[
\tilde{L}_o \quad \tilde{J}_o
\]

\[
SO(3) \times SU(2) \supset SU(2).
\]
where above each group the quantum numbers that characterize its irreducible representations (irreps) are given and $\gamma$ and $K_0$ are multiplicity labels of the indicated reductions.

The Hamiltonian used in the calculations includes spherical Nilsson single-particle terms for the protons and neutrons $(H_{sp.n})$, the quadrupole-quadrupole $(Q \cdot Q)$ and pairing $(H_{pair.n})$ interactions, as well as three `rotor-like' terms that are diagonal in the SU(3) basis:

$$H = H_{sp.n} + H_{sp.n} - \frac{1}{2} \chi (Q \cdot \tilde{Q}) - G_\pi H_{pair.n}$$

$$+ G_{\pi} H_{pair.n} + a K_2^2 + b J^2 + A_{asym} \tilde{C}_2.$$  \hspace{1cm} (2)

A detailed analysis of each term of this Hamiltonian and its parametrisation can be found in [28]. The three free parameters $a, b, A_{asym}$ were fixed by the best reproduction of the low-energy spectra: no additional parameters enter into the theory – the calculated M1 transitions reported below were not fit to the data.

A description of the low-energy spectra and B(E2) transition strengths in even-even nuclei [29] and odd-mass heavy deformed nuclei [28, 30] have been carried out using linear combinations of SU(3) coupled proton-neutron irreps with largest $G_\pi$ values and pseudo-spin 0 and 1/2 (for even and odd number of nucleons, respectively), which are mixed by the single-particle terms in the Hamiltonian.

The large number of states that can decay through M1 transitions to the ground state in odd-mass nuclei, led us to enlarge the basis by including states with pseudo-spin 1 and 3/2. These configurations are necessary to describe excited rotational bands and to account for the strong fragmentation of the M1 strengths between 2 and 4 MeV in odd-mass nuclei.

The inclusion of configurations with pseudo-spin 1 and 3/2 in the Hilbert space allows for a description of highly excited rotational bands in odd-mass nuclei. This is illustrated in Ref. [31], where several rotational bands in $^{157}$Gd, $^{160}$Dy and $^{169}$Tm are described, including both excitation energies and intra- and inter-band B(E2) transition strengths, and shown to be in close agreement with the experimental data. In contrast, when the configuration space was restricted to the most spatially symmetric configurations, those with pseudo-spin 0 and 1/2, it was only possible to describe in $^{160}$Dy the first three low-energy bands [30]. The pseudo-spin symmetry is still approximately preserved in the present case, with these three low-energy bands showing only a small amount of pseudo-spin 1 and 3/2 admixing into predominantly pseudo-spin 0 and 1/2 configurations, respectively.

The M1 transitions are mediated by the operator

$$T_\pi^\dagger = \frac{3}{\sqrt{4\pi}} \mu_N \left( \gamma_0 L_0^\pi + \gamma_2 S_2^\pi + \gamma_4 L_4^\pi + \gamma_6 S_6^\pi \right)$$

with

$$L_\tau^{(i)} = \sum L_\tau^{(i)}, \quad S_\tau^{(i)} = \sum S_\tau^{(i)}.$$  \hspace{1cm} (3)

$$= \sum I_\pi^{(i)}(i), \quad S_\pi^{(i)} = \sum S_\pi^{(i)}(i).$$  \hspace{1cm} (4)
In Eq. (3) the orbital and 'quenched' (by a factor of 0.7) spin $g$ factors for protons and neutrons are used:

$$
g^*_e = 1, \quad g^*_n = 0, \quad g^*_e = (0.7)5.5857, \quad g^*_n = -(0.7)3.8263. \quad (5)$$

To evaluate the M1 transition operator between eigenstates of the Hamiltonian (2), the pseudo SU(3) tensorial expansion of the T1 operator (3) [27] was employed.

In what follows, the B(M1; $J^f \rightarrow J^i$) transitions in $^{157}$Gd, $^{163}$Dy and $^{169}$Tm are presented. $J^f$ refers to the ground states 3/2$^-$, 5/2$^-$ and 1/2$^-$ in these nuclei. In each figure, insert a) corresponds to the experimental results, while insert b) represents the theoretical values obtained with the T1 operator of Eq. (3). Insert c) shows the values with $g^*_{e,n}$ in Eq. (3) set to zero, i.e. with only the spin part of the T1 operator taken into account, and insert d) shows the results with $g^*_{e,n}$ in Eq. (3) set to zero, i.e. including only the orbital part of T1.

The differences between the M1 transition strength distribution in $^{157}$Gd, $^{163}$Dy and $^{169}$Tm, shown Figs. 1, 2 and 3 respectively (notice the change on the scale), are both striking and well-known [16]. In $^{157}$Gd there are 88 known M1 transitions between 2 and 4 MeV, all smaller than 0.05 $\mu^2$ and distributed in a nearly flat spectrum. In $^{163}$Dy the M1 transition strengths are distributed only among 17 peaks, clustered in three well-defined groups, and most of them have strengths between 0.1 and 0.2 $\mu^2$. $^{169}$Tm has an intermediate degree of fragmentation, with some clustered structures and many transitions of the order of 0.1 $\mu^2$.

Using an enlarged version of pseudo SU(3) shell-model theory described above, we obtained a microscopic description of these M1 transitions and their fragmentation in the three nuclei. The gross features of the M1 strength distributions in each of the nuclei are clearly reproduced, i.e. the different fragmentation patterns. On the other hand, for $^{157}$Gd and $^{163}$Dy the M1 strength distributions are displaced toward higher energies by about 0.75 MeV and the total sums are underestimated. This effect could be related with the absence of spin dependent terms in the Hamiltonian (2). For $^{169}$Tm the distribution in energy of the M1 strengths is correct, but some transition strengths are overestimated by a factor 2 to 3.

The ground state wave functions of the two nuclei with odd number of neutrons, $^{157}$Gd and $^{163}$Dy, have one important difference. In $^{163}$Dy the ground state has only pseudo-spin 0 and 1/2 components, while $^{157}$Gd has a 13% mixing with pseudo-spin 1 and 3/2 components. In the M1 transition matrix elements the presence of these components in the later case gives rise to interference and fragmentation, while its absence in the former nuclei is associated with few large M1 transitions. The odd proton number of $^{169}$Tm allows orbital proton excitations between half-integer components, building up its large M1 summed transition strength.

Having analyzed the similarities and differences between the experimental data and the theoretical predictions, we now discuss the spin and orbital contributions to the M1 transitions. In insert c) of each figure the M1 transition strengths calculated only with the spin operators, i.e. making $g^*_{e,n} = 0$ in Eq. (3), is presented. Insert d) shows the M1 strength when only the orbital part of the operator (3) are included ($g^*_{e,n} = 0$). In all cases the spin coupling is by far the dominant mode, but for $^{169}$Tm the orbital contribution is also large.

In the case of $^{163}$Dy, there is an almost null contribution from the orbital part of the transition operator (0.103 $\mu^2$), which in fact interferes destructively with the spin channel (0.543 $\mu^2$) to produce a summed M1 strength of 0.483 $\mu^2$ in the scissors energy region.
The 'angle' between the orbital and spin channels, as defined by Fayache et al. [22] is 110° for $^{163}$Dy. For $^{157}$Gd, this angle has a value of 83° and for $^{169}$Tm it is 96°. From Table I it can be seen that below 2 MeV the spin transitions are clearly dominant. Nevertheless, it should be emphasized that contributions the intruder sector have been neglected.

Fig. 1 Distribution of M1 transitions between 2 and 4 MeV for $^{157}$Gd. Insert a) shows the experimental values [15], insert b) shows the theoretical with the complete T1 operator, insert c) shows the values with $g_{\pi,\nu}^0 = 0$ (only the spin channel) and insert d) with $g_{\pi,\nu}^0 = 0$ (only the orbital channel).
Fig. 2 M1 transitions for $^{163}$Dy. Convention is the same as in Figure 1. Experimental values taken from Ref. [14].

The pseudo SU(3) shell model for odd-mass nuclei has been shown to offer a qualitative microscopic description of the scissors mode and its fragmentation. In order to successfully reproduce the observed fragmentation of the M1 strength, it was necessary to use realistic values for the single particle energies and to enlarge the Hilbert space to include those pseudo SU(3) irreps with the largest $C_2$ values and pseudo-spin 1 and 3/2. This expansion of the model space allowed the T1 operator to connect the ground state with many excited states ($|J_f - J_i| \leq 1$) in the energy range between 2 and 4 MeV. The transitions are dominated by spin couplings, but interference with the orbital mode is very important.
From a historical perspective, the pseudo-SU(3) concept has moved strongly forward since K. T. Hecht first introduced it to the nuclear physics community. However, from this analysis, it is also clear that a fully quantitative treatment of the problem should take into account contributions from the intruder sector—a problem of utmost importance that might be expected to receive greater attention in the future. The present analysis leads us to claim that the theory has been fully tested and has been given an opportunity to realize its full potential. Perhaps at R. D. Rau's 90th birthday we will be able to bring a full report forward that says the theory, as originally envisioned, has been fully developed.
the satisfaction of all stakeholders! In the meantime, detailed studies of M1 transitions in other odd-mass nuclei are under investigation and should offer an opportunity to further apply and test the theory.

The authors thank S. Pittel, A. Frank and P. Van Isacker for constructive comments. This work was supported in part by CONACyT (México) and the US National Science Foundation.

Table 1

<table>
<thead>
<tr>
<th></th>
<th>E &lt; 2 MeV</th>
<th>2 - 4 MeV</th>
<th>4 MeV &lt; E</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{157}\text{Gd})</td>
<td>1.596 ±0.235</td>
<td>0.782</td>
<td>0.613</td>
</tr>
<tr>
<td>Experiment [15]</td>
<td></td>
<td>0.389</td>
<td>0.371</td>
</tr>
<tr>
<td>Theory</td>
<td></td>
<td>0.308</td>
<td>0.355</td>
</tr>
<tr>
<td>Spin only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orbital only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>165\text{Dy}</td>
<td>1.641 ±0.338</td>
<td>0.483</td>
<td>0.303</td>
</tr>
<tr>
<td>Experiment [14]</td>
<td></td>
<td>0.543</td>
<td>0.026</td>
</tr>
<tr>
<td>Theory</td>
<td></td>
<td>0.103</td>
<td>0.012</td>
</tr>
<tr>
<td>Spin only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orbital only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^{165}\text{TM})</td>
<td>1.912 ±0.244</td>
<td>2.833 ±0.812</td>
<td>0.515 ±0.274</td>
</tr>
<tr>
<td>Experiment [24]</td>
<td></td>
<td>3.769</td>
<td>0.435</td>
</tr>
<tr>
<td>Theory</td>
<td></td>
<td>2.332</td>
<td>0.164</td>
</tr>
<tr>
<td>Spin only</td>
<td></td>
<td>1.838</td>
<td>0.321</td>
</tr>
<tr>
<td>Orbital only</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

References