Pseudo symplectic model and potential energy surfaces

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ABSTRACT: A shell-model scheme for heavy deformed nuclei is proposed. We begin by reviewing results that demonstrate the near equivalence of the quantum rotor and SU(3) pictures. Then we turn to a discussion of the pseudo SU(3) scheme which applies to rare earth and actinide nuclei. With this background established, we go on to show that the symplectic scheme allows one to extend the SU(3) picture, normal and pseudo, to include shell mixing effects generated by the quadrupole-quadrupole interaction. In the final section we show that the symplectic scheme can be used to provide a shell-model interpretation of the potential energy surface concept. Some preliminary results for \(^{238}\)U are presented.

1. INTRODUCTION

There are many barriers to gaining a successful shell-model description of the structure of heavy deformed nuclei: huge shell-model spaces, an unknown nucleon-nucleon interaction, inadequate computer power, etc. Nevertheless, these nuclei remain one of nature's most challenging puzzles and therefore are deserving of the extraordinary effort an understanding of their structure requires. They are quantum systems in which single-particle, many-body and statistical features coexist with a minimum of interference.

In this paper we introduce the pseudo symplectic scheme which is a fully microscopic shell-model theory for heavy deformed nuclei. Our starting point is a review of results that demonstrate the near equivalence of the quantum rotor and SU(3) pictures. We then turn to a discussion of the pseudo SU(3) scheme which is an extension of the Elliott SU(3) Model that applies to rare earth and actinide nuclei. Next we show that the symplectic scheme can be used to extend the SU(3) picture, normal and pseudo, to include shell mixing effects generated by the quadrupole-quadrupole interaction. And this leads us, in the final section, to a shell-model interpretation of the potential energy surface concept.

2. ROTOR-SU(3) EQUIVALENCE

The pioneering work of Elliott (1958) established SU(3) as the underlying symmetry of observed rotational phenomena in light ds-shell nuclei. The success of the model follows from the fact that eigenvalues of the quadrupole-
quadrupole interaction (Q^2, Q^2, "a" for algebraic) go as 4C^2 - 3L^2 where C^2 is the second order invariant of SU(3) and L^2 is the square of the angular momentum operator. Not until recently, however, was the relationship between SU(3) and the rotor fully understood and appreciated. The thesis work of Leschber (1987) shows through numerous examples the close relationship between observables of the triaxial quantum rotor and a simple SU(3) - SO(3) integrality basis interaction. Subsequently, Leschber and Draayer (1987) derived analytic expressions that relate the parameters of these two theories.

A group theoretical reason for the correspondence can be given, namely, the symmetry group of the rotor is a contraction of the SU(3) algebra. However, there are important differences and these teach us a number of things about nuclei that display rotational behavior. For example, SU(3) representations are finite dimensional whereas those of the rotor are infinite. This yields significant differences in predicted E2 transition strengths between high-spin members of a rotational band. The SU(3) results fall off from rotor model values and, indeed, this is in agreement with experiment.

The correspondence we refer to goes well-beyond what can be deduced from the contraction process. Castaños, et al. (1988) have shown that a relationship between shape variables of the rotor and the SU(3) representation labels can be established. Since the SU(3) representation labels are dictated by the filling of shell-model orbitals, this implies a constraint on allowed nuclear shapes. Turning things around, one can use this to study the onset of triaxiality, the probability of finding oblate nuclear species, etc. In a recent paper Wu, et al. (1987) used this type of logic to provide a shell-model interpretation of the binding energy systematics of actinide nuclei.

3. PSEUDO SU(3) SCHEME

It is well-known that for nuclei beyond $^{40}$Ca the spin-orbit interaction pushes the highest j member of a given shell down among orbitals of the next lower shell. This destroys the oscillator structure and with it the underlying SU(3) symmetry. Nonetheless, nuclei of the rare earth and actinide regions display rotational characteristics. How can this be? Though still somewhat of a mystery, a simple answer can be given. The normal parity levels that remain clustered together can be identified as members of an oscillator shell with one less quanta than that of the original set.

This mapping defines the pseudo SU(3) scheme (Raju, et al, 1973). It does not mortgage shell-model principles nor is it in any way whatsoever, as the name seems to suggest, a false or fake proposition. The scheme is a renaming and reorganization of the shell-model orbitals. The physics remains unchanged. Of course, the exercise would be meaningless if a description of the physics in terms of the new basis is as complicated as before. And herein lies the real value of the pseudo SU(3) scheme. When the Hamiltonian is expressed in terms of the new labelling scheme, one finds that just as in the ds-shell, the quadrupole-quadrupole interaction dominates. The consequences are therefore known, namely, the Hamiltonian favors representations of SU(3) that correspond to configurations of maximum intrinsic deformation as defined in terms of the new (pseudo) as opposed to the old (normal) oscillator.
To illustrate this point in more detail, in Table 1 we give the SU(3) tensor decomposition of $Q^3$ for the $n = 4$ shell using both the normal and pseudo shell geometries. Since $Q^3$ is a generator of the symmetry group of the normal oscillator it is a pure $(l_0, l_0) (l_0, l_0, l_0) = (1, 1), 2, 0$ tensor in that scheme. However, under a reorganization of the space this changes. In addition to a generator part, other SU(3) tensors appear. As the coefficients indicate, however, the generator part is not only the largest by far but it is also enhanced by the transformation. Indeed, we have found that the nongenerator forms make less than a two percent change in calculated E2 transition rates in applications to rare earth and actinide nuclei (Draayer and Weeks, 1984; Castaños, et al., 1987).

The pseudo SU(3) scheme has been used to study a variety of phenomena:

- Decoupling parameters (Raju, et al., 1973)
- Alpha particle transfer strengths (Braunschweig, et al., 1978)
- Backbending in $^{126}$Ba (Raju, et al., 1979)
- Band crossing in $^{128}$Ba (Draayer, et al., 1981)
- Forking in $^{68}$Ge (Weeks, et al., 1981)
- $Q_\pi$, $Q_\rho$, and strong coupling (Draayer, et al., 1982)
- Unique parity spin sequences (Weeks, et al., 1983)
- Collective E2, M1 and M3 modes (Castaños, et al., 1987)

In each case the calculated results are in good agreement with the known experimental data.

4. SYMPLECTIC EXTENSION

The symplectic model extends the SU(3) scheme by including couplings to other shells generated by the quadrupole operator. In this case the symmetry group is $Sp(3, R)$. It has 21 generators, the 8 of SU(3) plus an operator that counts the total number of oscillator quanta and six $2\hbar\omega$ raising and six $2\hbar\omega$ lowering operators (Rosensteel and Rowe, 1976, 1979; Rowe, 1986):

$\begin{align*}
L_0^3 & \text{ angular momentum} \\
Q_0^5 & \text{ quadrupole (}\hbar\omega) \\
N & \text{ number operator} \\
B^+ & \text{ 6}\, 2\hbar\omega \text{ raising (} L=0 \text{ & } 2 \text{) } \\
B^- & \text{ 6}\, 2\hbar\omega \text{ lowering (} L=0 \text{ & } 2 \text{) }
\end{align*}$

$21$ total operators

The application of these operators to any $\hbar\omega$ SU(3) shell-model representation generates the corresponding $Sp(3, R)$ shell-model representation. Because of the structure of the $B^+$ raising and $B^-$ lowering operators, the symplectic representations have infinite dimensions. Actually, the lowering operators annihilate $\hbar\omega$ states because quanta cannot be removed from the system without violating the Pauli Principle. This means that the $\hbar\omega$ SU(3) representations serve as bandhead labels of symplectic shell-model representations.
The addition of the B⁺ and B⁻ to the SU(3) algebra means that the full effect of the quadrupole-quadrupole interaction (Q①-Q②, "c" for collective) between nucleons in a nucleus can be determined. The Q①s are related to each other as follows: Q ① = Q ③ ± 3 ④( B ③ + B ④). In particular, an effective charge is not required to get correct E₂ strengths, even in cases like 238U which require enhancement factors on the order of 100 single-particle units. It also means deformations can be correctly determined and, as we demonstrate in Figure 1, the theory can be used to probe the microscopic structure of the giant quadrupole resonance.

For applications to heavy deformed nuclei it is important to know that the expansion of the B⁺ and B⁻ operators in terms of their pseudo operator counterparts goes like Q ④, namely, the main contribution is the corresponding pseudo B⁺ and B⁻ operators. This is shown in Table 2. The pseudo symplectic scheme is the pseudo SU(3) scheme enhanced by the addition of the pseudo symplectic raising and lowering operators. As in the pseudo SU(3) case, it is anticipated that differences due to the use of pseudo rather than normal operators will be at most a five percent effect. However, this has not been verified by example.

5. POTENTIAL ENERGY SURFACES

The potential energy surface concept comes from the generalized collective model which describes nuclei in terms of rotational and vibrational degrees of freedom (Gneuss and Greiner, 1971; Hess et al., 1981). The Hamiltonian consists of kinetic and potential parts built out of scalars in the collective coordinates. This Hamiltonian is diagonalized in a five-dimensional harmonic oscillator basis (angular momentum two phonons) with a fixed maximum number of quanta. The parameters of the potential are determined by a least-squares procedure that fits calculated excitation energies and E₂ transition strengths to the corresponding experimental numbers. The potential energy surface is a plot of this potential as a function of the (ς,γ) shape variables.

To get stable minima and ensure against divergent behavior for large values of the deformation it is necessary to use at least a sixth-order polynomial form for the collective model potential: V(ς,γ) = \sum_{pq} V_{pq}(ς)ς^p(γ)γ^q with 2ς + 3γ ≤ 6. In contrast, we find that within the framework of the symplectic model a simple quadrupole-quadrupole interaction suffices to account for the
dynamics. The reason for this difference is a simple but nonetheless profound matter. Specifically, the collective model places no restriction on allowed configurations. This means an attractive quadrupole-quadrupole interaction ($V_{20} = 0$ except for $V_{10} < 0$) favors configurations of maximum deformation, that is, those with the maximum possible number of quanta and all of these quanta aligned along an intrinsic symmetry axis. Higher order terms in the potential, like $8\xi$, cancel this divergence ($V_{20} > 0$).

Since the symplectic model is a fermion based theory, not all configurations are allowed. In particular, the Pauli Principle forces an increase in the system's energy for large values of the deformation. (Drayer, et al., 1989). This can be seen by noting that large deformations come about by exciting valence particles out of the $0\Omega\mu$ space into higher shells. But this adds multiples of $2\Omega\mu$ in energy to the system. Here $\Omega\mu$ is the shell separation energy. So binding energy gains are offset by kinetic energy gains. This is shown in Figure 2. A rise in the total energy (expectation value of the Hamiltonian) is due to the Pauli Principle. Higher order terms that enter into the collective model description are required to correct the fact that there is no direct way in that theory to take account of the Exclusion Principle.

![Figure 1](image)

**FIGURE 1.** Percent analysis of the ground state ($0_1$) and resonant mode ($0_3$) in $^{238\text{U}}$ for two values of $\chi$, the $Q^2$-$Q^2$ coupling strength, $H = H_0 - \frac{1}{2} \chi Q^2-Q^2$. The numbers along the abscissa are in units of $2\Omega\mu$. So for $\chi = 0.002$, the ground state is about 60% in the $0\Omega\mu$ space, $(Q_0,\Omega_0) = (54,0)$, while the resonant state has less than 15% weight in the $2\Omega\mu$ space and reaches out with non-negligible contributions beyond $10\Omega\mu$. The deformation of the resonant mode is about 1.1 times the ground-state deformation.

6. CONCLUSION

We have proposed a scheme called the pseudo symplectic model for studying the structure of heavy deformed nuclei. The underlying symmetry is pseudo SU(3), a scheme that has been shown to give a good account of a number of interesting phenomena in strongly deformed nuclei. The symplectic enhancement allows one to incorporate collective correlations so calculated eigenstates give correct values for E2 transition rates without requiring an effective charge. We showed preliminary results for $^{238\text{U}}$ that indicate very clearly the inadequacy of a $0\Omega\mu$ shell-model theory. These results also suggest that the pseudo symplectic model will prove useful for describing giant resonance phenomena.
We used the symplectic scheme in its simplest form to give a shell-model interpretation of the collective-model-potential energy surface concept.

To conclude, we want to point out what we see as further applications of the theory. First of all, we believe the symplectic scheme holds promise for gaining a deeper understanding of the true nature of rotations: the fraction of the total number of nucleons that participate in the motion, questions concerning transverse versus longitudinal current flows, whether vorticity degrees of freedom play an important role, etc. These are questions that can be probed by experimentalists at facilities like CEBAF. Another topic of interest is superdeformation. We find it difficult to imagine a shell-model scheme contributing much to our understanding of this phenomena if it does not include vertical mixing of the symplectic type.

Indeed, the percentage analysis of the calculated resonant state in $^{238}\text{U}$ suggests the symplectic model may well play an important role in our gaining a deeper understanding of this important physical phenomena. And lastly, we believe the symplectic model, extended to include many bandhead configurations mixed through pairing and other coupling modes offers real hope for gaining a good understanding of the very excellent and detailed data being collected at the various x-sphere facilities, current and future, that we have heard about at this conference.

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FIGURE 2. A schematic diagram that illustrates a shell-model realization of the potential energy surface. All allowed configurations of each major shell fall in a conical strip or a V-shaped band with terminus defined by the representation of SU(3) that gives the maximum deformation in that shell. The potential energy surface is set for \( a = a_0 \) by the representations of the \( \Omega_2 \) shell that lie lowest in energy and for \( a > a_0 \) by the leading SU(3) representations of the higher shells. The rise for \( a > a_0 \) is simply a result of competition between additional binding energy from QC.0C and shell effects.

REFERENCES:

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