4 Conclusions

High pressure techniques constitute a powerful tool for the stabilization of metastable oxides. Working under moderate CO pressure (200 bar), we have been able to prepare a new series of ferrimagnetic oxides, obtained by replacing Mn^{3+} by Fe^{2+} in the parent R(Cu_{1-x}Mn_{x})O_3 compounds. The crystal and magnetic structures of the new materials, studied from NPD data, displays a ferrimagnetic coupling between Mn^{3+} and Co^{3+} spins; at lower temperatures the magnetic rare-earth moment participates in the magnetic structure, exhibiting an antiferromagnetic coupling with the Mn sublattice.

References


New dynamical symmetries in the symplectic extension of the Interacting Vector Boson Model

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Abstract

A development of the algebraic Interacting Vector Boson Model (IVBM) is achieved by exploring three new subgroup chains that reduce its dynamical symmetry group $Sp(12, R)$ to the angular momentum subgroup $SO(5)$. The corresponding exactly solvable limiting cases are applied for the description of different types of nuclear collective states. The first reduction that we explore is the one through the maximal compact subgroup $SU(6) \supset U(3) \times SU(3)$, which gives successful reproduction of the experimental energies and of their odd-even staggering in the ground and octupole bands of some even-even rare earth and actinide nuclei up to very high spins.

The empirical observation, that sets of states with fixed angular momentum $I$ fall on parabolas with respect to the variable $K$, is confirmed by exploring the reduction chains $Sp(12, R) \supset Sp(6, R) \supset SO(5)$. The theoretical energy distributions show that the procedure gives a clear distinction of typical collective vibrational and rotational spectra.

The third considered limit of the symplectic IVBM contains a natural way the 6-dimensional D6osov potential by means of the reduction $Sp(12, R) \supset Sp(2, R) \times SO(6)$. It leads to a successful description of mixed-mode spectra with the proton and neutron substructures and interactions between them taken into account explicitly.

The symplectic extension of the Interacting Vector Boson Model permits a richer classification of the states than its unitary versions and proves appropriate for the description of diverse nuclear structure phenomena.

1 Introduction

With the advent of the experimental facilities a vast amount of data on the collective spectra of nuclei throughout the nuclear chart is accumulated [1]. It reveals the complicated character of the nuclear motions, yielding different degrees of mixing of the basic rotational and vibrational collective modes. In turn this
requires a corresponding degree of complication of the nuclear structure models, but still they should remain tractable and easy in the interpretation of the observed data.

The algebraic model like IBM [9], based on the fundamental symmetry principles have proven themselves appropriate for this purpose. Even more they have introduced the benchmarks for the understanding of the nuclear structure evolution [8] and have developed the necessary technique for the description of the square matrix shape (phase/shape) transitions from one to another collective mode [4]. Nevertheless along the way some of the beauty and simplicity of the group theoretical approach to the nuclear structure problems is somewhat lost and mainly geometrical considerations in terms of the collective model [5, 4] variables are considered.

On the other hand, in the relatively early stages of the development of the algebraic methods for application in the nuclear structure theory [6, 7, 8] the separation of the collective degrees of freedom from the intrinsic ones, their development and mixing was attempted in the framework of the symmetrical geometry of the nuclear many-body problem. The advantages of exploitation of the symmetrical structure, in addition to the direct geometrical interpretation, are rather transparent, taking into account that they allow for a change in the number of phonons building the collective state which is possible to incorporate the more and more complex nuclear spectra. In the early eighties the number of bosons preserving version of the phenomenological algebraic Interacting Vector Boson Model (IVBM) [9] was successfully applied [10] for the description of the low-lying collective states in various nuclei. In the present work, we are interested in the symmetrical extension of the IVBM, for which the group of dynamical symmetry is the $S(12, R)$. The extension is realized and has its physical interpretation over the basis of its maximal compact subgroup $U(6)$ of $Sp(12, R)$, which gives the rotational limit [10] of the model. This naturally leads to the additional consideration [11] of not only the energies, but of the staggering effects between the states of the ground and octupole bands up to very high spins.

In the last several years the methods of the dynamical $Sp(12, R)$ there are also new chains of subgroup, starting with the normal Sydney subgroup [12]. The first one considered [12] of $Sp(4, R) < Sp(12, R)$ allows the selection of states with fixed angular momentum $L$, given by the $SO(3)$ irreducible representation states for which the physical problem behind this dynamical symmetry outlines the importance of the physics behind the subhead configurations in the development of the nuclear spectra. Further we investigate the reductions $Sp(12, R) \to SO(6) \subset Sp(12, R)$, which allows the location of a 6-dimensional Davidson potential [13], that is known so in applications it reproduces very well the transition behavior in nuclear spectra. In this work, the above mentioned dynamical symmetries are unified in a generalized reduction scheme for the symmetrical extension of the IVBM, which creates a lot of relations between the subgroups from the considered model.

1.2 Reduction through the compact $U(6)$

The algebraic structure of the model in terms of creation (annihilation) operators $a_{\alpha}(\pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha)$ and the associated momenta $p_{\alpha}(\pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha)$, in a 3-dimensional Hilbert space for the energy spectrum of the nuclear structure, while still retaining the advantages of the use of dynamical symmetries, namely the exact analytic solutions for the energy spectra of nuclei.

2. The symplectic extension of the IVBM dynamical symmetry

The algebraic structure of the IVBM is realized in terms of creation (annihilation) operators $a_{\alpha}(\pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha)$ in a 3-dimensional Hilbert space for the energy spectrum of the nuclear structure, and its associated momenta $p_{\alpha}(\pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha)$, the different chains. These clarify the use of the vector bosons (1) building blocks of the model. The scalar operators:

$$A_{\lambda}(p, n) = \sqrt{\frac{3}{5}} T_{\lambda} + A_{\lambda}(n, p) = \sqrt{\frac{3}{5}} T_{\lambda} + \frac{1}{3} N - 1,$$

are the Weyl generators of the algebra of $U(6)$, where $T_{\lambda}$ and $N$ count the number of particles of each kind. We can use the equivalent set of minimal operators containing in addition to the raising $T_{\lambda}$ and lowering $T_{\lambda}$, components of the pseudospin (see (7)), the Cartan operators $N$ (4) and $N = \frac{1}{2} N$.

The commutation relations between the pair creation and annihilation operators (2) and the number preserving operators (3) are calculated in [8].

$$[a_{\alpha}(\pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha, \pm\alpha), a_{\beta}(\pm\beta, \pm\beta, \pm\beta, \pm\beta, \pm\beta, \pm\beta)] = \delta_{\alpha \beta},$$

and so does the pseudospin algebra $su(2)$. These operators play an important role in the consideration of the nuclear system as composed by two interacting subsystems. Obviously $T_{\lambda}$ commute with the $SO(3)$ generators $\{0, 0, 0, 0, 0, 0\}$, so that the algebra of the two groups is the direct product (9) is mutually complementary. The remaining operators $A_{\lambda}(n, p) (L = 1, 2, M = -L, -L + 1, \ldots, L - 1, L)$ can be treated as vector and angular momentum operators between the states with different number of $p$- and $n$-bosons [10]. At a second glance the above considerations, the rotation limit (10) of the number preserving version of the model is defined by the chains:

$$U(6) \supset SU(3) \supset U(3) \supset SO(3) \supset U(1),$$

where the labels below the subgroups are the quantum numbers (10) corresponding to their irreducible representations. Since the reduction from $U(6)$ to $SO(3)$ is carried out by the mutually complementary group $SU(3)$ and $U(1)$, their quantum numbers are related in the following way:

$$T_{\lambda} = \frac{L}{2} + 1, N = 2L + 1.$$
The multiplicity index $K$ appearing in this reduction is related to the projection of $L$ in the body fixed frame and is used with the parity $(\gamma)$ to label the different bands $(K^\gamma)$ in the energy spectra of the nuclei. We define the parity of the states as $\gamma = (-1)^T$, which allows us to describe both positive and negative bands.

The Hamiltonian, corresponding to the considered limit of CVE, is expressed in terms of the first and second color invariant operators of the different subgroups in the chain (9). As a result of the connections (11) the Casimir operator of $SU(3)$ with eigenvalue $(\lambda^2 + \mu^2 + \lambda \mu + 3\lambda + 3\mu)$ is expressed in terms of the operators $N$ and $T$; so we use the expression:

$$H = aN + bN^2 + cT^2 + \delta_1 \lambda \mu \lambda + \delta_2 T^2.$$  \hspace{1cm} (14)

where $\delta_1$ and $\delta_2$ are the $SO(3)$ second order Casimir operators. $H$ (14) is obviously diagonal in the basis (12) labelled by the quantum numbers of the specific subgroups of the chain (9). Its eigenvalues are the energies of the basis states of the boson representations of $Sp(12, R)$.

$$E((N, T), L, T_0) = aN + bN^2 + cT^2 + \delta_1 \lambda \mu \lambda + \delta_2 T^2.$$  \hspace{1cm} (15)

The subspaces $[N, \ell, 0, 0, 0, 0, [N\ell]],$ where $N = 0, 2, 4, \ldots$, given in Table 1 are of the finite dimension, which simplifies the problem of diagonalization. Therefore the complete spectrum of the system can be calculated through the diagonalization of the Hamiltonian in the subspaces of all the UIR of $SU(3)$, belonging to a given UIR of $Sp(12, R)$, which further clarifies its role of a group of dynamical symmetry.

### 2.1.2 Description of the ground and octupole bands energies

The most important application of the above limit of the model is the possibility to describe both even and odd parity bands up to very high angular momentum. In order to do this we first have to identify these experimentally observed bands with the sequences of basis states for the even representation of $Sp(12, R)$ given in Table 1. As we deal with the symplectic extension of bosons representations of the number $N\mu$ we are able to consider all even eigenvalues of the number of vector bosons $N\mu$ with the corresponding set of pseudospins $N\lambda$. For the description of the ground band, we choose the $SU(2)$ multiplets $(\lambda, \mu)$ from the different $SO(3)$ irreps given by the numbers of bosons, $N = 0, 4, 8, \ldots$, and pseudospin $T = 0$ in the column labelled by $T_0 = 0$ of Table 1. While for the octupole band the $SU(3)$ multiplets $(\lambda, \mu)$ for $N = 8, 12, \ldots$, and $T = 1$ from the same column $T_0 = 0$ are used. In terms of $(N, T)$ this choice corresponds to $(N = 2n, T = 0)$ for the positive $(K^\gamma = 0^+)$ and $(N = 2n + 2, T = 1)$ for the negative $(K^\gamma = 0^-)$ parity band, respectively.

Further we define the energies of each state with given $L$ as yrast energy with respect to $N$ in the two considered bands. Hence their minimum values are obtained at $N = 2L$ for the ground band, and $N = 2L + 1$ for the octupole band, respectively. Hence the SU(3) multiplets, which reduce to the corresponding values of the angular momenta $L$, correspond for the ground band to $(0, \frac{\ell}{2})$ and for the octupole one to $(\frac{\ell}{2}, \frac{\ell}{2} - 1)$. In the so-defined $SU(3)$ representations for each $N$ the maximal values of $L$ appear for the first time (see Table 1).

According to the correspondence identified above between the basis states $T_0 = 0$ and the experimental data on the ground and octupole bands, the last term in the energy formula (15) vanishes. The phenomenological model parameters $a, b, \delta_1,$ and $\delta_2$ are evaluated by a fit to the experimental data [17]. Their values obtained for some even-parity deformed nuclei belonging to light actinides and rare earth region are given in Table 2. The second column gives the numbers of the experimental states used in the fitting procedure.

The comparison between the experimental spectra and our calculations using the values of the model parameters given in Table 2 for the ground and octupole bands of the nuclei $\text{Ra}^{208}, \text{Y}^{92}, \text{Sm}^{142}$ and $\text{Y}^{148}$ is illustrated in Figure 1.

The values of $\beta$ and $\gamma$ can be determined only from a fit to the positive band energies, while $\eta$ and $\xi$ are estimated from the negative ones, respectively. The values of the parameters (17) determine the behavior of the energies of the two bands and their position with respect to each other. In the actinides the two bands are almost parallel. The shift between them depends on the parameter $\xi$. When they are very close they intersect in the $L$-dependent interaction with a strength $\eta$.

From (16) we can see that eigenstates of the first positive and negative bands consists of rotational $L(L + 1)$ and vibrational $L$ mode. The rotational interaction is with equal strength $\beta$ both of this bands. The obtained values of the parameter $\pi$ are always negative, which means that the negative parity band is less vibrational than the positive one.

The comparison between the experimental spectra and our calculations using the values of the model parameters given in Table 2 for the ground and octupole bands of the nuclei $\text{Ra}^{208}, \text{Y}^{92}, \text{Sm}^{142}$ and $\text{Y}^{148}$ is illustrated in Figure 1.

Figure 2: (Color online) Comparison of the theoretical and experimental staggering functions for the ground and octupole bands of $\text{Ra}^{208}, \text{Y}^{92}, \text{Sm}^{142}$ and $\text{Y}^{148}$.

Odd-even staggering patterns between ground and octupole bands are a typical phenomenon for this type of collectivity. In order to test further our model we applied on the energies the staggering function defined as [18]:

$$S_\mu(L) = \Delta E(L) - \Delta E(L - 1) - \Delta E(L + 1) + \Delta E(L + 2) + \Delta E(L - 2),$$  \hspace{1cm} (16)

where $\Delta E(L) = E(L) - E(L - 1)$. This function is a finite difference of fourth order in respect to $E(L)$ or of fifth order in respect to energy $E(L)$ and is characteristic for the deviation of the rotational behavior from that of the rigid rotor. The calculated and experimental staggering patterns are illustrated in Figure 2. One can see a good agreement with experiment, as well as the reproduction of the "beat" patterns of the staggering behavior. They occur in the region where

### Table 1

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### Table 2

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<th>$\xi$</th>
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<td>Y$^{148}$</td>
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<td>0.0194</td>
<td>-0.0045</td>
<td>0.4590</td>
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The reduction through the non-compact $Sp(4, R)$

2.1. The algebraic realization

- terms of the introduced boson representations of $p(12, R)$ another reduction is realized [19, 6, 12].

$Sp(12, R) \supset Sp(4, R) \otimes SO(3)$. (19)

The infinitesimal operators of the $Sp(4, R)$ algebra are $L_{\mu}$ the L = 0 part

$F^\alpha(\alpha, \beta) = \sum_{\nu=0}^{3} (\alpha_{\nu} + \beta_{\nu}) (\alpha_{\nu} + \beta_{\nu})$ (20)

of the $Sp(12, R)$ generators $F^\alpha(\alpha, \beta), G^\alpha(\alpha, \beta)$ (2) and $A^\alpha(\alpha, \beta) = - \frac{1}{2} \sum_{\nu=0}^{3} (\alpha_{\nu} - \beta_{\nu}) (\alpha_{\nu} - \beta_{\nu})$. (21)

$Sp(12, R)$ is generated by the Weyl generator $A^\alpha(\alpha, \beta)$, (20) and is the same one that corresponds to the pseudoscalar of SU(3) [7] in the direct product with the SU(3)$^3$ in (9). The operator $N$ (4) generates $w(1)$ and plays the role of the first-order invariant of $w(2)$ $\otimes$ $w(2)$ $\otimes$ $w(2)$ (1). The following correspondence between the two considered chains (through $w(6)$ and through $p(12, R)$ of subalgebras of $p(12, R)$ exists (6):

$p(12, R) \supset Sp(4, R) \otimes SO(3)$

$U(0) \supset U(2) \otimes U(1)$. (22)

Each of the $Sp(4, R)$ irrep is contained in the $Sp(12, R)$ boson representations of infinite dimension and consists of countless number of states. A basis for the $Sp(4, R)$ representations is generated by a direct construction of the symmetrically coupled products of the operators $F^\alpha(\alpha, \beta)$ (20) to the lowest weight state $(w_2)$ of angular momentum $L$ that labels the considered $Sp(4, R)$ irrep $[6, 9]$. Each starting $w(2)$ configuration is characterized by a totally symmetric representation $[6, 9]$, formed by $L = N_{\text{max}}$ vector bosons. The process to obtain the rest of the $w(L)$ irrep that are contained in a given $L$ irrep of $Sp(4, R)$ is given in details in [12].

We first decompose the four even numbers $n = 0, 2, 4, 6, \ldots$ where $n/2$ gives the degree of the $F^\alpha(\alpha, \beta)$ that is applied to the $(w_2)$ into a direct sum of $w(L)$ irreps $[6, 9]_{n+2}$ with both $n, n_2$ even and $n = n_1 + n_2$. After symmetrization of the irrep $[6, 9]_{n+2} \rightarrow [6, 9]_{n_1 - n_2, 0} = [6, 9]_{n_1 - n_2}$ we get the decomposition:

$[6, 9]_{n_1 - n_2} \otimes [6, 9]_{n_2} \rightarrow [6, 9]_{n_1} \otimes [6, 9]_{n_2} - 4 \delta_{n_1, n_2}$ (23)

Next all the outer products of the representations $[6, 9]_{n_1}$ with the reduced $[6, 9]_{n_2}$ are calculated and restricted to the two-dimensional Young diagrams of integers. They are transformed into the $w(2)$ representations $[6, 9]$, which correspond to $N = N_{\text{max}} + n, n = 0, 2, 4, 6, \ldots$ and $\sum_{\nu=0}^{3} (\alpha_{\nu} - \beta_{\nu})$ (23) for the even irrep. (24). Table 3 and 4. The columns are defined by the pseudoscalar quantum number $T$ and the rows by the eigenvalues of $N = N_{\text{max}} + n$ for $n = 0, 2, 4, 6, \ldots$ (25).

2.2. Energy distribution of states with fixed $L$

Because of the correspondence (21) and the relation between the SU(2) and SU(2) second order Casimir operators, we use $n$ (11) as the same Hamiltonian (14). Now, even as established above, the bases in both cases are equivalent and as a result the eigenvalues of the Hamiltonian (14) for the states with fixed $L$ are the energies, given by (15). Obviously in (15) the dependence of $N = N_{\text{max}} + n$ for $n = 0, 2, 4, 6, \ldots$ (26) on the number of phonons (vector bosons) is parabolic. The rest of the quantum numbers $T, D_3$ defining the states with fixed $L$ are expressed in terms of $N$ by means of the reduction procedure described above.

This result confirms the empirical investigation (27) of the energy distribution of states with fixed angular momentum. It is rather well described by the simple phenomenological formula $E_0(n) = A n - B n^2$, where $A > 0$ and $B > 0$ are fitting parameters and $n$ is an integer number corresponding to each one of the states with given $L$. A relation $N = 2n$ between the total number of vector bosons $N$ and the introduced in (28) number of ideal monopole bosons $n$ is valid in this application.

The parity of the states is defined as in [11] as $\pi = (-1)^{20}$. As we consider states with fixed values of $L$ we first evaluate the parameter of inertia $\beta_3$ from in front of the term $L(L+1)$ in (15) by fitting the energies of the ground states band (GSS) with $E^{\text{GSS}} = 0^+, 2^+, 4^+$, ... to their experimental values in each nucleus. Further, the values of $N_{\text{max}}$ corresponding to the experimentally observed $2^{+, 4}$ and the values of the parameters in (15) are evaluated in a multi-step $y$-square fitting procedure making use only of the three sets of states with $J^P = 0^+, 2^+, 4^+$. We choose nuclei with enough of these states to have good statistics in the fit. Most of these states are band heads (all the $0^+$, some of the $2^+$ and $4^+$) of collective bands and so as a result the situation of the whole band depends on them. The set of $N_{\text{max}}$ with minimal value of $x^2$ determines the distribution (the parameters of the Hamiltonian) of the $L^2$ states energies with respect to the number of bosons $N_{\text{max}}$ that build the states. For the distribution of the $0^+$ states we chose both $T = 0$ = 0 and also a possible value of $T = 0$. We determine $T = 0$ and also a possible value of $T = 0$. We determine the $0^+$ states and $2^+$ states (15) are evaluated and fixed. For the $2^+$ we use $T > 0, T = 0$ and so we get the $\alpha_1$ and $\alpha_2$ parameter and finally for the $4^+$ we determine $T > 0$ and also a possible value of $T > 0$. In order to obtain $\alpha_1$ Sets of states with other values of $L = 1, 3, 5, 6,$... and/or with negative parity (T = odd) can be included in the calculation only by determining in a convenient way the values of $T, D_3$ and finding the sequences of $N_{\text{max}}$ corresponding to the observed experimental energies. It is remarkable that once all the parameters of the Hamiltonian are evaluated from the distributions of $0^+$, $2^+$ and $4^+$ states, any other set of states are described without involving additional parameters.

The results of the treatment described above for the collective spectra of two even-even nuclei from the rare earth region are illustrated in Figures 3 and 4. The theoretical distribution of the energies with respect to the attributed values of $N_{\text{max}}$ is the same agreement with the experimental ones can be clearly seen. Additionally, the initial value of $N$ by means of which the lowest state with the considered $J^{+}$ is constructed, the values of $T, D_3$, used, the values of $N_{\text{max}}$ for the Hamiltonian parameters $\beta_3, \alpha_1, \alpha_2$ and $\alpha_1$ with their respective $x^2$ are presented in Table 5. The $x$ in the first column gives the number of the experimentally observed states with the corresponding $J^{+}$.
Table 5.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$I$</th>
<th>$J^*$</th>
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<th>$T$</th>
<th>$\nu^2$</th>
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<tr>
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<tr>
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$H^{178}$

<table>
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<td>2</td>
<td>0.0008</td>
<td>$a_6 = 0.00343$, $b_6 = 0.00006$</td>
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</table>

Type of spectra is the position of the first excited $2^+$ state of the ground state band, which for vibrational nuclei is rather high (over 1 MeV), but for the well-deformed ones lies low around 0.5 MeV [2].

For the nuclei with vibrational spectra $N_{max}$ we apply the procedure described above with values of $T$ that differ significantly ($\Delta T = 4$) for the sets with $L = 0, 2, 4$. This corresponds to rather large changes in the values of the initial $N_{max} = 2T$ which allows one, according to the reduction rules for the basis, to place them on the left side of the symmetrical parabolas. As a result (see Figure 2) the values of $N_{max}$ increase with increasing energy of these states.

For the spectra of $N_{max}$, in addition to the $L = 0^+$, $2^+$, $4^+$ we have included the states with $L = 3^+$, $4^+$, $5^+$.

Figure 3: (Color online) Comparison of the theoretical and experimental energy distributions of the states with $L = 0^+$, $2^+$, $4^+$, $6^+$ of the $^{178}$Hf.

The low-lying states of the ground band, $L = 0^+$, $2^+$, $4^+$, of the typical rotational nuclei [4] $H^{178}$ require small changes in the number of quanta $N_2$ that build the corresponding experimentally observed states. Making use of the latter and the symmetric feature of the second order curves the states with a given $L$ in the rotational spectra are placed on the right-hand-side of the theoretical curves. This corresponds to the second solution $N_2 = \frac{1}{2}L$ of the equation (15) for the ground state with $T = T_2 = 0$, defining the maximum $N_2$ that builds it. This can be used as a restriction on the value of $N_2$ [29]. On a par with $L$, specified for a fixed $T$, the number of bosons that build the states will decrease with increasing energies. Hence, if the number of quanta that is required to build a collective state is taken as a measure of collectivity, the states from a rotational spectra are much more collective than vibrational ones, which is the expected result. In the examples, one can observe the structure of collective bands that are formed by sets of states from the different curves and in particular the ground band, the first two excited $\beta$- and $\gamma$-bands (see Figure 4).

2.3 Reduction through the non-compact Sp(2,R)

2.3.1 Algebraic construction containing the 6-dimensional Davidson potential

The need for a description of nuclei in which rotational-vibrational interactions dominate has led to a search for algebraically solvable potentials and a meaningful set of basis states that make the transition natural of these systems more transparent. An algebraically solvable theory that can describe systems with rotational-vibrational interactions and which has known algebraic solutions when applied to diatomic molecules, is the one containing the Davidson potential [24]. In an algebraic approach for either the nuclear many-body problem or the Bohr-Mottelson collective model, the addition of the Davidson potential to the Bambullovian requires the consideration of a dynamical subgroup chain that contains the direct product $Sp(2,R) \otimes SO(6) \subset Sp(2n,R)$, with $n = 3$ and 5, respectively [27].

Indeed if the local isomorphism of the sp(2,R) = so(1,1) algebra is taken into account its relation to the spectrum generating algebra of the many body nuclear system with the Davidson interaction becomes explicit. This provides motivation for considering this reduction in seeking a description of a more complex nucleus that includes rotational-vibrational interactions. Within the framework of the phenomenological interacting vector boson model (IVBM) [9], the more general case of a 6-dimensional Davidson potential naturally appears. The new reduction chain of its dynamical symmetry group $Sp(2,R)$ through the direct product subgroup $Sp(2,R) \otimes SO(6)$ extends the applicability of the theory to include rotational-vibrational interactions. In order to relate the IVBM to the 6-dimensional Davidson potential, we introduce another reduction of the $Sp(12,R)$ group through its non-compact subgroup [19], [28]:

$$Sp(12,R) \supset Sp(2,R) \otimes SO(6).$$

As can be deduced from the considerations given above, this construction obviously survives the addition of Davidson potential. The infinitesimal generators of the $Sp(2,R)$ algebra

$$F = \sum_{k,m} C_{km}^{01m}u(k)\phi_m^0 = 25^7,$n

$$G = \sum_{k,m} C_{km}^{01m}u(k)\phi_m^0 = -25^7,$n

for $\nu \equiv \sum_{k,m} C_{km}^{01m}(a)\phi_m^0 = -25^7$,

for $\nu \equiv \sum_{k,m} C_{km}^{01m}(a)\phi_m^0 = -25^7 - 1$.

are obtained from the $Sp(12,R)$ generators (2) and (3) by means of contraction with respect to both the spatial $\nu = 0, \pm 1$ and the "pseudo-spin" $\nu = \pm 1/2, \nu = \pm 1/2$ indices. It is straightforward to show that the operators $G^7$, $G^7$, $G^7$, $G^7$ constitute a standard way for the $SU(1,1)$ algebra generators [29]

$$\{G^7, G^7\} = G^7, \{G^7, G^7\} = G^7, \{G^7, G^7\} = G^7, \{G^7, G^7\} = G^7,$n

so the $sp(2,R)$ and the $su(1,1)$ algebras are locally isomorphic with a Casimir operator written as $C_0(SU(1,1)) = G^7 + G^7 + G^7 + G^7$.

By construction, the operators (24) are scalars with respect to 6-dimensional rotations and they commute with the components of the 6-dimensional momentum operators [6].

$$A_{+}^0(\alpha, \beta) = A_{+}^0(\alpha, \beta) = (-1)^{3/2} A_{+}^0(\beta, \alpha),$$

which obey the property:

$$A_{+}^0(\alpha, \beta) = (-1)^n A_{+}^0(\beta, \alpha),$$

and generate the $SO(6)$ algebra. In this way, the direct product of the two groups (25) is realized. The second order invariant for the $SO(6)$ group is

$$\nu^2 = \sum_{L,\mu,\beta} (-1)^{L/2} A_{+}^0(\beta, \alpha),$$

and (26) is related to the second order invariant of the $Sp(2,R)$, as in the direct product (23) the two groups are complementary [19], which means that the irreps of the group $SO(6)$ determines those of $Sp(2,R) \supset SU(1,1)$ and vice versa.

In order to define the basis of the system with (19) as a dynamical symmetry that allows one to include the 6-dimensional Davidson potential, we consider the reduction of the $SO(6)$ algebra to the $SO(3)$ algebra of the angular momentum through the following chain [9], [30].

$$SO(6) \supset SO(3) \otimes SO(3) \supset SO(3),$$

$$\nu = L$$

(27)
which means that the reduction from $SO(6)$ to the rotational group $SO(3)$ is carried out through the component of the elementary group $C_{2n}(SU(3))$ [29]. As a consequence, the irreducible representations $\lambda_1, \lambda_2, \lambda_3$ of $SU(3)$ are determined by $\omega_1, \omega_2, \omega_3$ of $SO(6)$ and by the integer label $\nu$ of the associated irreducible representation of $O(2)$ i.e.,

$$\omega_1 = \left( \frac{1}{2}, 0, 0 \right) \otimes \nu.$$  

And the generators of $SU(3)$ [9] are

$$\frac{X_{\lambda_1}}{N_\lambda} = \frac{1}{2} \left[ X_{\lambda_1}, N_\lambda \right], \quad \frac{X_{\lambda_2}}{N_\lambda} = \frac{1}{2} \left[ X_{\lambda_2}, N_\lambda \right], \quad \frac{X_{\lambda_3}}{N_\lambda} = \frac{1}{2} \left[ X_{\lambda_3}, N_\lambda \right].$$

Using the relation (31) of the Casimir operators, for their respective eigenvalues in terms of the Elliott's notation $\langle \lambda, \nu \rangle$, one obtains:

$$\omega_1 = \sum_{\nu=0}^{\infty} \left( \frac{1}{2}, 0, 0 \right) \otimes \nu.$$  

Finally, the convenience of this reduction can be further enhanced through the usage of the standard rules for the reduction of the $SU(3)$ to $SU(3)$ chain (31). Hence the basis, labelled by the quantum numbers classified by the group-subgroup chain (31), can be written as

$$\rho \lambda, \nu \mid \lambda, \nu.$$  

The states of the excited bands are not yet states. The proper placement of the bands in the spectrum strongly depends on their band-head configurations and in particular, on the number of bosons, $N = N_{\text{bos}}$. From these, we derive $1$'s for the band-head band and then the bands are developed along the "diagonal" of Table 6 by changing simultaneously $\omega$ and $\nu$ depending on the respective values of $L$ and defining the corresponding values of $\lambda$ (33). For the $\beta$-bands with $K = L$, where $L$ enumerates the $0^+$ excited states in the order of increasing energy, we choose $\omega = L$, $\nu = L$, that leads to a sequence of $\omega(3)$ multiplets $(\lambda = L, L = 0)$ for the states in the bands with $\Delta L = 2$. Obviously in these bands $\Delta L = 2$ and the dependence of the band-head position on the number of bosons $N = N_{\text{bos}}$ that build the initial state $L = 0$ is made explicit. For the excited state $L^+ = L^+ (\nu = 0, L = 2, \nu = 2)$ the bands-head configurations are again obtained by the evaluation of the respective $\nu = N_{\text{bos}}$ and $L$. Since in these bands $\Delta L = 1$ we take from each $N$ two values $L$ and $L^+ = L$ with the initial $L$ even. Then the bands are developed again along the diagonal, but there is a difference in the values of $\nu$ and $\omega$ since prescribed by the reduction rules $\omega = \nu$. In the given examples we consider the low lying $\gamma$-bands as belonging to $\omega$-decoupled $\omega(3)$ multiplets $\omega = L + k, L = 2$ (33).

$$H = \omega + \nu^2 + \omega_1 \omega + \omega_2 \omega + \omega_3 L^2.$$  

The five free parameters, are determined by fitting the theoretical predictions for the energy of the ground and few excited bands in the experimental data, using a $\chi^2$-procedure. The parameters that were obtained, the number of experimental states, $x$, and $N_{\text{bos}}$ are all given in Table 7 for different configurations. We have chosen the nuclei in the presented examples for their long spin sequences in particular in their ground bands, from where most of the parameters are obtained. The first one $^{150}$Nd is a well deformed nucleus from the rare earth region with experimental ratios $R = 4.92$ of the ground band energies $R = 3.89$. One can see in Figure 6, the good agreement between the theoretical and experimental bands for the ground, first two $\beta$ and $\gamma$ bands is more complicated:

$$E_{\text{exc}}(N_{\text{exc}}, L) = E_{\text{exc}}(N_{\text{exc}}, L) + 2b N_{\text{exc}} + 2d N_{\text{exc}} + al + b L + (L + 1) + \frac{\omega_1}{2} + \omega_2 (L + 1).$$  

$$E_{\text{exc}}(N_{\text{exc}}, L) = E_{\text{exc}}(N_{\text{exc}}, L) + 2b N_{\text{exc}} + 2d N_{\text{exc}} + al + b L + (L + 1) + \frac{\omega_1}{2} + \omega_2 (L + 1).$$

Where the so introduced parameters $al, b, al, b, b$ and $c$ are different linear combinations of the interaction strengths of the Hamiltonian (35), which are evaluated [33], taking into account the above assignments for the values of quantum numbers $\omega$ and $\nu$ in the angular momentum $L$ in each band. The other important band characteristics that have to be evaluated in each case, except for the yrast ground band are $N_{\text{bos}}$, that fixes the position of the band-heads and also the value of $a$ that shifts the $K^+ = L^+$ bands in respect to the ground and $\beta, \gamma$ bands.

The variety of possible choices for the correspondence of the excited bands to sequences of states in the symplectic space and the mixing of the rotational and vibrational degrees of freedom (37) allows us to reproduce correctly the behavior of these bands without one another, which can change a lot even in neighboring nuclei (33).

Figure 5: (Color online) Comparison of the theoretical and experimental energies for ground and excited bands of $^{150}$Nd.
bands in its spectrum. The values of $N_{\text{IBM}}$ that determine the start of the considered excited bands are rather high and equal in the first and $\gamma$ bands, which are almost degenerated. Although this is the dynamical symmetry for the $\gamma$-soft model the well deformed ones are well described as well for a convenient choice of the parameters, given in Table 7.

Additionally, in Table 7 we give the results for the $^{148}$Nd, which is a $\gamma$-soft model with $R_1 = 2.49$, that corresponds to the $SO(6)$-limit of the IBM (see Figure 6). The shifting of the band head configurations of these $\gamma$ bands is obtained by means of the different values of $N_{\text{IBM}}$ evaluated for them.

Figure 6: (Color online) Comparison of the theoretical and experimental energies for ground and excited bands of $^{148}$Nd.

The most interesting example is the last nucleus $^{152}$Sm (see Figure 7), which is transitional between the $\gamma$-soft ($R_1 = 2.5$) and rotational nuclei with $R_1 = 3.33$ and is a model at the critical point of phase transition [84] with so-called $X(3)$ symmetry [4]. As shown in Figure 7, the experimental data is reproduced remarkably well, especially for the $\beta$ and $\gamma$ bands, that are well distinguished. This is reflected by the values of $N_{\text{IBM}}$ which vary quite a lot, but less than the parameters $R_4$ of the Hamiltonian.

Figure 7: (Color online) The same as on Fig. 1 for $^{152}$Sm.

From the presented results, it could be seen that the model is appropriate for the description of a rather broad range of nuclei, and most importantly nuclei that display different degree of mixing of the rotational and vibrational degrees of freedom.

References

Symplectic Symmetry in the $fp$ Shell

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Abstract

The theory of spectral distributions is employed to provide important information about symmetries embedded in the GXPF1 and Strasbourg effective realistic interactions for many-nucleon states in the $fp$ shell. This in turn reveals the ability of the interactions to account for many-particle effects such as the formation of correlated nucleon pairs and enhanced quadrupole collective excitations. Furthermore, the realistic interactions are in general found to correlate strongly with a pairing+quadrupole model interaction, especially for the highest possible isospin group of states where the model interaction can be used to provide a reasonable description of the corresponding energy spectra.


1 Introduction

It has been recognized for a long time that ground states of even-even nuclei reflect strongly on the nature of the nuclear interaction, especially its propensity to form correlated, angular momentum $J = 0$ pairs [1]. The recent resurgence of interest in the nature of pairing correlations in atomic nuclei is linked to the search for a reliable microscopic theory for describing the structure of medium mass nuclei around the $N = Z$ line where protons and neutrons occupy the same major shell and their mutual interactions strongly influence the structure and decay modes of their partners. These nuclei also have a significant impact in astrophysical studies [2, 3] and have been extensively explored with radioactive beams.

The $Sp(4)$ dynamical symmetry of like-particle and proton-neutron pairing correlations [4, 5, 6] between nucleons occupying the same major shell has been found to provide a reasonable microscopic description of the pairing-governed isobaric analog $0^+$ states in light and medium mass nuclei [7, 8]. In this regard, it is important to recall that $SU(5)$ [6] (with a Lie algebra that is isomorphic to $sp(4)$) has been shown to play a significant role in the structure of $fp$-shell $N = Z$ nuclei. Indeed, a model based on this symmetry group can be used to track the results of an isospin-invariant pairing plus quadrupole shell-model theory [6]. In addition, based on the theory of spectral distributions [10, 11], the comparison with realistic interactions can determine the extent to which the significantly simpler $Sp(4)$ model Hamiltonian can readily be used to obtain an approximate, yet very good description of low-lying nuclear structure and in turn, one can apply the model to larger model spaces that are otherwise prohibitive in size. Furthermore, we introduce a possible $Sp(4)$ symmetry breaking by an additional quadrupole-quadrupole interaction and examine the capacity of the extended model interaction to mimic realistic interactions. This, in turn, provides a further step towards gaining a better understanding of the underlying foundation of the microscopic interactions.

2 Symplectic $sp(4)$ Pairing Model Interaction

Isovector pairing plays a crucial role in understanding the microscopic structure of light and medium mass nuclei as well as nuclei far off the valley of stability. Indeed, recently we showed that the symplectic $Sp(4)$ dynamical symmetry is fundamental to the nuclear interaction that governs fully-paired isobaric analog $0^+$ states of light and medium mass even-A nuclei with valence protons and neutrons occupying the same shell [8].

The fermion realization of the symplectic $Sp(4)$ symmetry is a natural choice for microscopic descriptions of nuclear systems. In the nuclear shell-model a basic set of wave functions for a system with $n$ (valence) nucleons is constructed by taking a Slater determinant (which is fully antisymmetrized in accordance with the Pauli principle) of a single-particle wave functions represented in second quantization by the fermion operators $c_{i\alpha}^{\dagger}$, where these operators annihilate (create) a particle of type $\alpha = \uparrow \downarrow / (\text{pro-}