NEW APPROACH IN THEORY OF CLEBSCH-GORDAN COEFFICIENTS FOR $u(n)$ AND $U_q(u(n))$ *)

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A new method for calculation of Clebsch-Gordan coefficients (CGCs) of the Lie algebra $u(n)$ and its quantum analog $U_q(u(n))$ is developed. The method is based on the projection operator method in combination with the Wigner-Racah calculus for the subalgebra $u(n-1)$ ($U_q(u(n-1))$). The key formulas of the method are couplings of the tensor and projection operators and also a tensor form of the projection operator of $u(n)$ and $U_q(u(n))$. It is shown that the $U_q(u(n))$ CGCs can be presented in terms of the $U_q(u(n-1))$ $q$-9j-symbols.

1 Introduction

It is well known that the Clebsch-Gordan coefficients (CGCs) of the unitary Lie algebra $u(n)$ ($su(n)$) have numerous applications in different fields of the theoretical and mathematical physics. For example, many algebraic models of the nuclear theory such that: the interaction boson model (IBM), the Elliott $su(3)$-model, the $su(4)$-supermultiplet scheme of Wigner, the shall model and so on, demand explicit expressions of the CGCs for $su(6)$, $su(5)$, $su(3)$, $su(4)$ and $su(n)$. Analogously, in different quark models of the hadrons we need the CGCs of $su(3)$, $su(4)$ etc. The theory of the $su(n)$-CGCs is connected with the theory of special functions, the combinatorial analysis, topology, etc.

There are several methods for calculation of CGCs of $su(n)$ ($u(n)$) and other Lie algebras: recurrent method, method of employment of explicit bases of irreducible representations, method of generating invariants, method of tensor operators (where the Wigner-Eckart theorem is used), projection operator method, coherent state method, other combined methods.

We briefly illustrate structure of these methods for the case $su(2)$. Let $\{|j_km_k\rangle\}$ be canonical bases of two irreducible $su(2)$ representations (IRs) $j_k$ ($k = 1, 2$). Then


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