MODERN APPLICATIONS OF SU(3) AND ITS Sp(6,R) EXTENSION IN NUCLEAR STRUCTURE PHYSICS

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Abstract

Recent experimental and theoretical developments are generating renewed interest in the nuclear SU(3) shell model, and this extends to the symplectic model, with its Sp(6,R) symmetry which is a natural multi-hω extension of the SU(3) theory. First and foremost, the discovery and understanding of pseudo-spin has extended the applicability of the theory from light to heavy nuclei. Second, a user-friendly computer code for calculating reduced matrix elements of operators that couple SU(3) representations is now available. And third, since the theory is designed to cope naturally with deformation, microscopic features of deformed systems can be probed; for example, the theory is being employed to study double beta decay and thereby probe the validity of the standard model. Recent developments relating to pseudo-spin and some early results for double beta decay are addressed in this contribution.

1. Introduction

Interest in the SU(3) shell model, which was introduced to nuclear physics by Elliott [1] more than three decades ago, has come and gone with time. The are several reasons for this: In its simplest form it is a elementary algebraic theory that explains the origin of collective rotations within a shell model framework, but its extensions require the mixing of irreducible representations (irreps) of SU(3) and this, in turn, requires rather sophisticated computational algorithms and the last of these, a code for calculating reduced matrix elements, has only recently been published [2]. In addition, the model did not show particularly well when applied in the fp-shell to neutron rich isotopes of $^{40}$Ca nor when it was used to describe heavier fp-shell species with $^{36}$Ni taken as a core and the remaining $\{f_{7/2}, p_{1/2}, p_{1/2}\}$ orbitals treated as members of a pseudo-ds shell [3]. The pseudo-SU(3) scheme was also reported to fail when applied to rare earth nuclei with only identical nucleons taken to be spectroscopically active [4]. While reasons for the latter are now understood, the early results quenched enthusiasm for further developments that were required to make the SU(3) approach competitive with more standard schemes that had already proven their value.

This changed as a result of several converging circumstances: First, the importance of the proton-neutron interaction in generating deformation is now fully appreciated —
a feature that accounts for a poor showing of the theory in neutron-rich \( fp \)-shell applications. Second, when implementing the normal \( \rightarrow \) pseudo transformation it is essential to pick the phase relation between paired orbitals in such a way as to insure that the real quadrupole operator and that of the pseudo-SU(3) algebra are maximally correlated — only the most favorable of the available choices leads to good pseudo-SU(3) symmetry [5]. Third, as pointed out above, the technical problem of calculating reduced matrix elements for operators that couple different irreps of SU(3) has been solved [2]. Fourth, Sp(6,R) which contains SU(3) as a subgroup, has been shown to be the group that encompasses the full range of quadrupole collective behavior in nuclei [6, 7, 8, 9, 10, 11, 12, 13]. And finally, the goodness of pseudo-spin symmetry makes these discoveries applicable to heavy systems via the pseudo-SU(3) and pseudo-Sp(6,R) extensions of the SU(3) and Sp(6,R) models, respectively.

Space prohibits a full discussion of all of these issues, so only two of the most recent developments are considered below. On the theory side this includes recent development in understanding the origin of pseudo-spin symmetry, which is the topic of Section 2. And in the applied arena, some results are given for new developments in the study of double beta decay in Section 3. Strengths of the zero and two neutrino modes test the validity of the standard model particles and their interactions.

2. Importance and Origin of Pseudo-Spin Symmetry

The pseudo-spin concept in nuclear theory refers to a division of the single-particle total angular momentum into pseudo \( j = \frac{1}{2} \) rather than normal \( j = \frac{1}{2} \) orbital and spin parts [14, 15]. That this division is favored in nature can be seen by noting the approximate degeneracy of pairs of single-particle states, \((l-1, l-1), (l+1, l+1)\) with \(l = \frac{1}{2}\), within the major shells of heavy \(A \geq 100\) nuclei, where the normal spin-orbit interaction is strong [16, 17]. By assigning new \(l_{\pm}\) and \(l_{\pm + 1}\) labels to the \((l-1), (l+1)\) states (for example, a \((9/2, 9/2)\) pair is assigned new \((9/2, 9/2)\) “pseudo” labels), an advantage is gained because these paired states can then be interpreted as members of a weakly split pseudo-spin doublet.

The approximate goodness of the pseudo-spin quantum numbers supports a picture of heavy nuclei as many-body systems with weakly broken pseudo-spin symmetry. Going beyond pseudo-spin, to the collection of all pseudo-spin doublets within a major shell, leads to the concept of a pseudo-oscillator shell for describing the normal parity proton and neutron subspaces of heavy nuclei. And this picture, augmented with a quadrupole-quadrupole residual interaction, leads to the many-particle pseudo-SU(3) theory for heavy deformed nuclei that mimics the normal-SU(3) picture that has proven to be so successful for light nuclei [14, 17, 18]. Indeed, the pseudo-SU(3) picture presents us with the first hope of being able to carry out meaningful shell model calculation for heavy deformed nuclei as an implementation of standard shell model techniques leads to matrices that have prohibitively large dimensions.

Even though experimentally well corroborated and successfully used in numerous theoretical applications, only recently has a truly microscopic justification for the pseudo-spin concept been proposed [19]. The usual understanding of pseudo-spin follows from a very simple single-particle oscillator picture where deviations from the oscillator spectrum follow, albeit approximately, a \(2j(j+1) - l(l+1)\) dependence, which transforms into \(l(l+1)
under the normal \( \rightarrow \) pseudo relabeling. Relativistic mean-field estimates were presented in support of such a dependence in the limit of large nucleon numbers [20]. Also, a unitary operator was proposed which acts on the spin and angle variables and accomplishes the normal \( \rightarrow \) pseudo relabeling within a given shell [21]. Later this approach was revisited and resulted in the introduction of another operator that is specifically designed for shell model applications, being unitary only within the normal-parity subspace of the oscillator [22]. The remainder of this section is devoted to a discussion of recent developments relating to the microscopic origin of good pseudo-spin symmetry.

2.1. Microscopic pseudo-spin transformation

To incorporate both the single-particle and many-particle aspects of the pseudo-spin picture, the normal \( \rightarrow \) pseudo transformation should be of the form

\[
U_{\text{total}} = \prod_{i=1}^{A} U(r_i, p_i, \sigma_i)
\]

where \(r_i\) stand for the position, \(p_i\) for the momentum, and \(\sigma_i\) for the Pauli spin matrices of the individual nucleons. The structure of \(U(r, p, \sigma)\) is fixed by the following constraints:

- a) \(\hat{P} = U \hat{P} U^{-1} = P + 21\) — this sets the transformation rule [21];
- b) \([\hat{r}, \hat{p}] = 0\) — rotational invariance; c) \([\hat{r}, \hat{P}] = [\hat{U}, \hat{T}] = 0\) — parity and time-reversal symmetry; d) \(U^{\dagger} = U^{\dagger} = 1\) — unitarity and conservation of observables; and e) \([\hat{p}, \hat{r}] = 0\) — translational invariance.

Once constraints a), b), c) and d) are applied, three distinguishable choices remain:

\[
U = (d \cdot d')^{-1/2} d, \quad d = (\cos \theta r_0 p + i \sin \theta r/r_0) \sigma,
\]

where \(r_0\) is a characteristic length, and due to the option of rescaling \(r_0\), the value of \(\theta\) can be fixed at \(\pm \frac{\pi}{2}\) or \(\frac{\pi}{2}\). The first choice yields the boson annihilation (+\(\frac{1}{2}\)) / creation (−\(\frac{1}{2}\)) operator form that is specifically designed for shell model applications [22]. However, these two operators are unitary only within a subspace of normal parity states while in the unique parity subspace their action is undefined. When global unitarity is required, only two possibilities remain. The case \(\theta = \frac{\pi}{2}\) corresponds to the \(U_r = i \sigma \cdot r/r\) operator [21] that will henceforth be called the \(p\)-helicity. The \(\theta = 0\) choice is the \(p\)-helicity, \(U_p = \sigma \cdot p/p\). This is the only form that is compatible with the constraint of translational invariance and thus consistent with a realistic many-particle theory that is not confined to the shell model approach. Additional arguments in favor of this choice are given below.

2.2. Single-particle Hamiltonian and wavefunctions

If (1) accomplishes the pseudo-spin transformation, then in addition to satisfying general constraints, it should decouple the spin and orbital degrees of freedom in the heavy nuclei. The applicability of the mean field approach allows for a reasonable direct check on this by considering transformed single-particle Hamiltonians and wavefunctions. Corrections for center-of-mass motion are relatively small in heavy nuclei and therefore not expected to worsen such an argument.
For simplicity a spherically symmetric field is considered. In this case the Hamiltonian and its wavefunctions are given by

\[ H = \frac{p^2}{2M} + V(r) + W(r) \cdot \sigma, \]  
\[ \psi_{n,j,m}(r) = i^j R_{s,j}(r)(Y_l \otimes \chi)_{j,m}, \]

where \( n \) is the radial quantum number (number of nodes), \( Y_l \) is a spherical harmonic, and \( \chi \) is a Pauli spinor (\( \chi = \frac{1}{2} \)).

In a coordinate representation the \( p \)-helicity has the following operator form:

\[ U_p = -iK [\hat{I} - \lambda - 1]^{-1} \hat{r} (\sigma \cdot \nabla), \]

where \( K = i^l (\hat{I} + \frac{1}{2} \hat{\lambda} + \frac{1}{2} \hat{\lambda}^2 + \frac{1}{2} \hat{\lambda}^3) \) has the orbital momentum as its eigenvalues, \( \Lambda = \nabla = r \partial / \partial r \) generates shear, and \( \Gamma(x) \) denotes the gamma function. The unitarity of \( K \) follows from the conjugation rules \( \hat{I}^2 = -(\hat{\lambda} + 3) \) and \( \hat{I} = \hat{I} \). Then (3) and (4) transform into

\[ H_p = U_p H U_p^\dagger = \frac{p^2}{2M} + \tilde{V}(r) + \tilde{W}(r) \cdot \sigma, \]
\[ \psi_{n,j,m}^p(r) = U_p \psi_{n,j,m}(r) = i^j \tilde{R}_{s,j}(r)(Y_l \otimes \chi)_{j,m}, \]

where \( \tilde{V}(r) \) and \( \tilde{W}(r) \) are now strongly nonlocal functions given by

\[ \tilde{V}(r) = K [V(r) - 2W(r) - (\hat{I} + 1) v(r)] \hat{K}^l, \]
\[ \tilde{W}(r) = K [v(r) - W(r)] \hat{K}^l. \]

with \( v(r) = (\hat{I} - \lambda - 1)^{-1} r V(r) - (\hat{I} + 2) W(r) [\hat{I} - \lambda - 2]^{-1} \) (primes denoting derivatives) and \( \tilde{R}_{s,j}(r) = -F (\hat{I} + \frac{1}{2} \hat{\lambda} + \frac{1}{2} \hat{\lambda}^2 + \frac{1}{2} \hat{\lambda}^3) \Gamma (\hat{I} - \frac{1}{2} \hat{\lambda}) \Gamma (\hat{I} + \frac{1}{2} \hat{\lambda}) ) R_{s,j}(r). \) Although (6) in its general form does not provide incontrovertible evidence for a reduction in the magnitude of the spin-orbit splitting (see below), the latter is likely to hold at low \( I \) within the nuclear surface region so long as the effective value of \( \Lambda \) operator exceeds \( I - 1 \).

To qualitatively understand the behavior of \( \tilde{R}_{s,j}(r) \), observe that the \( U_p \) transformation involves three consecutive operations: a Fourier transform, a switch from \( l \) to \( I \), and an inverse Fourier transform, which together determine the mapping for the radial function. This mapping generates the following universal behavior:

\[ \tilde{R}_{s,j}(r) \propto \left\{ \begin{array}{ll}
    r^l, & r \to 0, \\
    r^{-(l+3)}, & r \to \infty.
\end{array} \right. \]

The standard \( r^l \) dependence in the interior region follows because deep in the bulk of a heavy nucleus \( \tilde{V}(r) \) is not expected to deviate significantly from the flat behavior of \( V(r) \). The \( r^{-(l+3)} \) asymptotic behavior means a more diffuse surface. This and strong nonlocalities in the surface region come with the \( p \)-helicity transformation.

### 2.3. Dirac–Brueckner approach and the helicity transformation

A relativistic extension of the Brueckner theory provides parameter-free microscopic predictions for both infinite and finite nucleon systems [23]. While this approach gives a good description of nuclear matter, the gross features of finite nuclei (especially light species) are reproduced less well, but nonetheless much better than in nonrelativistic theories [24]. For this reason, results of Dirac–Brueckner nuclear matter calculations are used below for examining the \( p \)-helicity transformed two-body nuclear interaction, as well as the mean field, in heavy nuclei.

For a wide range of nuclear densities, including the saturation point, the nucleon-nucleon interaction in the infinite medium is approximated perfectly by a one-boson exchange potential (OBEP) with the boson parameters fitted to the Bonn model and the density-dependent effective nucleon mass \( m^* \) calculated in a self-consistent manner [23]. And to a very good approximation, the density-dependent self-consistent field has the same Lorentz structure as the free Dirac Hamiltonian. Consequently, a single-particle Hamiltonian in the medium commutes with the \( p \)-helicity, and the helicity transformation does not affect the single-particle energies.

However, the two-body interaction changes dramatically. In the representation of plane-wave Dirac spinors for nucleon states, normalized to unity, the \( p \)-helicity operation is equivalent to \( i\gamma^5 \tilde{S} \) when acting on right (ket) states. Here \( \gamma^5 \) is the usual product of Dirac matrices \( \gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 \), and \( \tilde{S} \) is a formal operation for switching the sign of the effective mass \( (m^* + p) = \tilde{f}(m^*, p) \). (Thus, there is no difference between the chiral and the helicity transformations in the \( m^* \to 0 \) limit.) Since the \( \gamma \)-matrices change sign under the \( \gamma^5 \) generated chiral transformation, and the OBEP is bilinear in those matrices, the helicity transformation of the OBEP is reduced to changing the sign of \( m^* \) in the momentum representation. This is easily accomplished in the two-nucleon center-of-mass frame and produces strongly incident-energy dependent and therefore nonlocal interactions. Because only a rough estimate for the potentials is required for this analysis, here these potentials are converted into local approximations by averaging over all allowed values of the relative momentum \( q \) with an appropriate distribution of \( q \) at a fixed momentum transfer \( k \). The localized helicity-transformed OBEP in the momentum space is found to converge rapidly in the shortwave region (\( k > 2|p| \)) to the initial potential, averaged with the hadron distribution. The values of the localized central part of the internucleon potential also coincide at \( k = 0 \) before and after the transformation in accordance with the helicity-invariance of the single-nucleon energy in the infinite medium. The localized estimates for transformed single-particle potentials in coordinate space, Figs. 1 and 2, were calculated in first order perturbation theory with respect to \( \delta V(k) \), the localized difference between the transformed and initial OBEP. Unperturbed potentials were in a Woods-Saxon parametrization [25] with a slight adjustment for the radial dependence which allows for a simple analytic Fourier transform along with a quantitative fit. The estimate was done analytically using a zero-order nuclear density distribution of the same parametrization but with a lesser diffuseness [16] and a Skyrme-type low momentum expansion for \( \delta V(k) \). Due to the strong nonlocality of transformed OBEP, the analytic formulas for single-particle potentials are more complicated than in a conventional scheme with Skyrme forces [26]. Basic complications and approximations of this analysis are: a) \( d \) and \( f \)-waves of the relative motion make an impact that is on the same order of magnitude as the normally included \( s \) and \( p \)-waves; b) \( \delta \tilde{G}(k) \), a difference between the localized
Figure 1: Localized estimates for the neutron central and spin-orbit potentials of 208Pb before and after the helicity transformation (continuous lines and shaded areas, respectively). The two curves that define the borders of the shaded areas were determined by using different reasonable approximations for the relative momentum distribution in a finite nucleus.

Figure 2: The same as Fig. 1 but in this case for protons.

G-matrix in the transformed space and the physical G-matrix, coincides with δV(κ) in the first order because changes in the short wavelength region are small (see previous paragraph); c) the ratio of proton and neutron densities is taken equal for all r, and Coulomb corrections are not considered; d) M* and kF are fixed at their saturation point values [23].

Although the single-nucleon potentials shown in the figures are only rough local estimates for strongly nonlocal fields, they display several features that are characteristic of pseudo-spin symmetry. First, in accordance with Section 3, the transformation preserves the finite depth of the central potential and increases the surface diffuseness. And because the kinetic energy is conserved by the transformation, this in turn implies that the transformed radial wavefunctions associated with higher energy orbitals (which are most important for heavier nuclei) must be localized at a larger radial distance than for the corresponding conventional functions. Second, a minimum in the spin-orbit potential, which is located in the surface region in the normal representation, gets shifted deeper into the bulk. And from this it follows that the magnitude of the spin-orbit potential in the region where the wavefunctions are localized and which is primarily responsible for the interaction strength, exhibits a dramatic decrease. Also note that the effective pseudo-spin-orbit interaction of the neutrons is more repulsive than one of the protons—in agreement with experiment [21].

To summarize these results, recall that the many-particle p-helicity operator is the simplest form that generates the normal — pseudo relabeling of the spin and orbital momenta while satisfying all other global symmetry requirements. In addition, it appears to transforms wavefunctions in a physically reasonable manner and effectively compensates for the single-particle spin-orbit interaction strength that is observed in the normal (not pseudo) picture. Furthermore, the approximate independence of the single-nucleon spectrum in an infinite medium on the helicity transformation and the consistency of the microscopic estimates for the single-particle nuclear potentials with the Dirac–Brueckner calculations, can be used to relate pseudo-spin symmetry to the boson-exchange nature of nucleon-nucleon interaction. Based on these results and the close relation (coincidence in the chiral symmetry limit) of the helicity and chirality operations, the goodness of pseudo-spin symmetry may be expected to increase with rising densities (or energy per particle) in hadronic systems and actually yield to chiral symmetry in the region of asymptotic freedom.

3. Results for 0ν and 2ν Double Beta Decay

The newest application of the pseudo-SU(3) shell model has been to the evaluation two neutrino (2ν) double beta decay half-lives of heavy deformed potential double beta emitters [27]. Eleven cases have been considered so far and the results are in good agreement with the available experimental data. For example, the radiochemically measured 238U decay [28] raised certain expectations because the experimental setup could not discriminate between the 0ν (zero neutrino) and the 2ν modes and the existing theoretical estimates [29] predicted similar rates for both. The SU(3) shell model results for the 2ν mode are consistent with the experimental result.

The pseudo-SU(3) shell model has also been used to calculated the 0ν matrix elements of six heavy deformed double beta emitters [30]. By using a currently accepted upper limit for the neutrino mass it was also possible to estimated their ββ0p half-lives. In the case of
238U the 0ν half-life is predicted to be at least three orders of magnitude greater than the 2ν one, which gives strong support to the identification of the observed half-life as being 2ν double beta decay. The ββ2ν of 150Nd to the ground state as well as excited states of 150Sm has also been studied [31]. A short review of these calculations is presented below together with a brief discussion of future improvements to the model.

3.1. Theoretically predicted forbidden decays

In all pseudo-SU(3) calculations done to date, only one active shell was allowed for protons and one for neutrons. While this may seem to be a severe truncation, the dynamics of the decays supports such a picture. In particular, note that the Gamow-Teller operator changes a neutron to a proton and can effect a spin-flip but it cannot change the spatial distribution of the system. For the $\beta\beta_{2\nu}$ decay this implies that only one transition is allowed: that which removes a neutron from a normal parity state with maximum angular momentum and creates a proton in the intruder shell ($h_{9/2} \rightarrow h_{11/2}$ in rare earth nuclei and $i_{11/2} \rightarrow i_{13/2}$ in the actinides).

In the rare earth and actinide regions this special normal (neutron) → unique (proton) Gamow-Teller transition controls the $\beta\beta_{2\nu}$ decay dynamics. If the occupation of the Nilsson levels is such that the number of protons in the abnormal states does not change for the initial and final state configurations the decay is forbidden. The present version of theory predicts the complete suppression of the $\beta\beta_{2\nu}$ decay for the following five nuclei: $^{144}$Sm, $^{146}$Gd, $^{176}$Yb, $^{232}$Th, and $^{244}$Pu [27]. Configuration mixing will alter this picture somewhat, but because of the alternating parity of the major shells the normal → unique feature will survive. The expectation is that the decays that are predicted to be forbidden in the simplest version of the theory will be strongly suppress in its extension and strong decays may be changed slightly but will remain strong. These and other related matters are currently under investigation.

3.2. The g.s. → g.s. double beta decay mode

In Table 1 results for six $\beta\beta$ emitters are given. In the second and third columns the theoretical [27] and experimental [32] $\beta\beta_{2\nu}$half-lives are cited. The agreement with the available data for $^{150}$Nd and $^{238}$U is reasonable. In the last two columns the theoretical predictions [30] and experimental lower limits [32] of the $\beta\beta_{2\nu}$ half-lives are given. In the calculation of zero neutrino half-lives, the mass of the neutrino was assumed to be one electron volt, $m_\nu \geq 1$eV. In the case of $^{238}$U the predicted 0ν half-life is three orders of magnitude greater than the predicted 2ν half-life, which essentially agrees with the experimental result, confirming that the observed $\beta\beta$ decay of $^{238}$U has to be of the two neutrino mode. In the case of $^{150}$Nd, the pseudo-SU(3) 0ν matrix element reported in the table is about a factor four smaller than the corresponding quasi-particle random phase approximation (QRPA) estimate [29, 33]. This is an especially relevant result. First, it exhibits the stability of the neutrinoless double beta decay matrix elements evaluated in quite different nuclear models, in the case of deformed nuclei. Second, this factor of four, which is small compared with the order of magnitude variations in the 2ν theoretical estimations, is still important in order to extract the parameter $m_\nu$.

As can be seen in the last two columns of Table 1, the $\tau_{1/2}$ predicted for $m_\nu \geq 1$eV are at least three order of magnitude greater than the experimental limits. These results reflect the fact that, at the present stage of the experimental $\beta\beta$ research, the limits $m_\nu \geq 1$eV obtained by the Heidelberg-Moscow collaboration [34] using significative volumes of ultra-pure $^{76}$Ge are the most sensitive. But, if the $\beta\beta_{2\nu}$ decay is observed in $^{76}$Ge, at least a second observation will be essential, and $^{150}$Nd is a likely candidate for this task. In the next few years the limit for $<m_\nu>$ extracted from $\beta\beta_{2\nu}$ experiments is expected to be improved up to 0.1 eV and $^{150}$Nd is one of the selected isotopes [32].

3.3. Double beta decay to excited nuclear states

Table 1: Calculated (thy) and experimental (exp) double beta half-lives (in years) for the two-neutrino and the zero-neutrino modes.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$\tau_{1/2}^{thy}$ [years]</th>
<th>$\tau_{1/2}^{exp}$ [years]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{146}$Nd → $^{146}$Sm</td>
<td>$2.1 \times 10^{31}$</td>
<td>$1.18 \times 10^{28}$</td>
</tr>
<tr>
<td>$^{148}$Nd → $^{148}$Sm</td>
<td>$6.0 \times 10^{20}$</td>
<td>$5.75 \times 10^{24}$</td>
</tr>
<tr>
<td>$^{150}$Nd → $^{150}$Sm</td>
<td>$6.0 \times 10^{18}$</td>
<td>$9.17 \times 10^{48}$</td>
</tr>
<tr>
<td>$^{186}$W → $^{186}$Os</td>
<td>$6.1 \times 10^{24}$</td>
<td>$1.05 \times 10^{34}$</td>
</tr>
<tr>
<td>$^{192}$Os → $^{192}$Pt</td>
<td>$9.0 \times 10^{25}$</td>
<td>$5.13 \times 10^{25}$</td>
</tr>
<tr>
<td>$^{238}$U → $^{238}$Pu</td>
<td>$1.4 \times 10^{21}$</td>
<td>$3.28 \times 10^{26}$</td>
</tr>
</tbody>
</table>

The $\beta\beta_{2\nu}$ decay modes of $^{150}$Nd to the ground state (g.s.) and excited states of $^{150}$Sm have also been studied [31]. The $\beta\beta_{2\nu}$ decay to the first excited $0^+$ state was found forbidden and the decay to the second excited $0^+$ state has a half-life four orders of magnitude greater than that to the g.s. The decay to the $2^+$ state is strongly inhibited due to the energy dependence of its matrix elements. These results differ from those previously published [35] where it was suggested that the $\beta\beta_{2\nu}$ decay of $^{150}$Nd to the g.s. and the first excited $0^+$ state of $^{150}$Sm could have similar intensity. The appearance of selection rules which can produce the suppression of the matrix elements governing a $\beta\beta_{2\nu}$ transition is a consequence of the details of the irreps involved. A recent analysis of the case of $^{100}$Mo [36] shows that both matrix elements are very similar and that they are in agreement with the experimental information.

To summarize this section it should suffice to reiterate that the pseudo-SU(3) model, being a fully microscopic (shell model) theory that is symmetry adapted to handling strongly deformed systems, is a natural choice for addressing the double beta decay problem. However, results reported to date rely on rather strong assumptions concerning the many-particle states of parent and daughter systems. Work is underway to enhance the theory so it includes configuration mixing and coupling to other (non-collective) degrees of freedom. In particular, it will be interesting to see what the symplectic extension can add to our understanding of double beta decay. Nonetheless, the nature of the Gamow-Teller
transition suggests that these enhancements will not change the results in a significant way: forbidden decays should become inhibited modes, while strong decays should remain strong. One thing should be clear: nuclear structure can be and is being used to address outstanding questions related to the fundamental nature of particles and their interactions.

4. Conclusions and Future Developments

The status of the SU(3) model and its Sp(6,R) extension have been considered with an eye on "modern applications" of the theory. On the theoretical side there is much more that could be said, but space did not allow for this to be done here. Open questions include the relation of this theory to the simpler mean-field approaches and the importance of deviations from the underlying oscillator assumption. There are also questions relating to vortex degrees of freedom and their affect on the dynamics — an analysis of electron scattering data should help to sort out these matters. The sympletic extension will be critical for this as well since it separates the collective from the non-collective (intrinsic) degrees of freedom in a natural way.

Fortunately, the most difficult of the technical hurdles required for applications of the theory are now resolved [2, 37]. Nonetheless, there are a variety of additional challenges that require further dedicated efforts. One is the coupling of normal-parity orbitals, handled in a pseudo-SU(3) and/or Sp(6,R) framework, with unique-parity intruder states, treated either in a generalized seniority framework or via an extension of its pseudo-SU(3) and/or Sp(6,R) complement [38, 39]. In either case the latter relies on good pseudo-spin symmetry, which is much better understood now as compared to just a few years ago. A particularly interesting feature of the recent work is the prospect of using pseudo-spin to chiral symmetry.

The possibility for new and exciting applications of the theory are driving the revitalization interest in the SU(3) model and its Sp(6,R) extension. In addition to "usual" applications like reproducing excitation spectra and electromagnetic transition rates, these include "novel" results for such things as the double beta decay problem. The list includes: scissors modes; electron scattering form factors; the density, excitation energy, and structure of here-to-for undetected low-lying collective bands; superdeformation; and the whole issue of the structure of nuclei far from stability. These alone are more than sufficient motivation to stimulate further work on the SU(3) and Sp(6,R) models, including the pseudo-extensions of these theories.

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References