M1 strengths (scissors and twist modes) in heavy deformed nuclei (\(^{156-160}\)Gd)

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Recibido el 28 de febrero de 2000; aceptado el 5 de mayo de 2000

The Elliott SU(3) Model, extended for heavy nuclei using pseudo-spin, is used to study low-lying magnetic dipole excitations in deformed nuclei. The Hamiltonian includes single-particle energies as well as quadrupole-quadrupole and pairing interaction terms. Since the quadrupole-quadrupole interaction dominates, a strong SU(3)-dictated truncation of the model space is used. With interaction strengths fixed by systematics, good agreement with experimental results is found for the excitation and M1 transition spectra of the well-deformed even-even \(^{156-160}\)Gd isotopes.

Keywords: SU(3) Model; pseudo-spin; magnetic dipole excitations; \(^{156-160}\)Gd

El modelo SU(3) de Elliott, extendido usando el pseudo-spin para nucleos pesados, es empleado para estudiar excitaciones magnéticas dipolares de baja energía en núcleos deformados. El hamiltoniano incluye energías de partícula independiente e interacciones de apareamiento y cuadrupolo-cuadrupolo. Dado la dominación de la interacción cuadrupolar se emplea un espacio fuertemente truncado basado en la simetría SU(3). Usando intensidades de las interacciones basadas en la sistemática se obtiene un buen acuerdo con los datos experimentales para las energías de excitación y las transiciones M1 de los isotopos deformados par-par \(^{156-160}\)Gd.

Descriptores: Modelo SU(3); pseudo-spin; excitaciones magnéticas dipolares; \(^{156-160}\)Gd

PACS: 21.60.Fw; 23.20.-g; 27.70.+q

1. Introduction

In recent years considerable experimental [1–5] and theoretical [6–9] effort has been focused on the study of low-lying enhanced M1 transition strengths (2–4 single-particle units) found in rare earth and actinide nuclei. A simple geometrical interpretation of this phenomena is that of a collective magnetic dipole excitation resulting from relative rotational oscillations of the proton and neutron distributions against each other, a scissors-like action that led to the christening of this excitation as the "scissors" mode [10]. It is also clear, however, that non-collective features in the nuclear interaction are necessary for a complete interpretation of the experimental results. In particular, the fragmentation of the M1 transition strength among several levels that are closely packed and clustered around a few strong transition peaks [11] cannot be reproduced by a model that includes only collective degrees of freedom. Early on (see Ref. 12 and references therein), models based on more microscopic principles, while still employing the basic concept of the relative rotational motion of the proton and neutron distributions, were proposed for explaining the origin of this collective phenomena. In this article, the pseudo SU(3) shell model is used to describe the non-collective as well as the underlying collective nature of low-lying M1 transitions in strongly deformed nuclei.

The pseudo SU(3) model is a many-particle shell-model theory that takes full advantage of pseudo-spin symmetry [13, 14], which in heavy nuclei is manifested in the near degeneracy of the orbit pairs \([(l-1)_{j=l+1/2}, (l+1)_{j=l-1/2}]\). Since the Lie algebra of the pseudo oscillator is the same as for the normal oscillator, the pseudo SU(3) symmetry can be used to partition the full space into distinct subspaces. Since its introduction in the late sixties [15, 16], the pseudo SU(3) model had been applied to various properties of heavy deformed nuclei [17–19], but these have been limited to schematic nucleon-nucleon interactions because of technical difficulties related to the calculation of SU(3) matrix elements of more general interactions. However, a code is now available [20] that removes these limitations and allows for the introduction of interactions, like pairing which is important for an adequate description of experimental results, into pseudo SU(3) model calculations [21].

The pseudo SU(3) model can be used to give a microscopic shell-model interpretation of the "scissors" mode [22, 23]. As noted above, a geometrical interpretation of this phenomena in terms of a scissors-like relative motion
of the proton and neutron distributions—called the Two Rotor Model (TRM)—parameterizes the motion in terms of an angle $\theta$ between the principal axes of axially symmetric proton and neutron distributions. In shell-model schemes—boson as well as fermion, however, this scissor action is associated with the relative motion of "valence" protons and neutrons only, not the entire mass distribution. That such an interpretation is a correct one can be seen from the fact that only in this case do the energetics ($1^+$ state at 2–3 MeV) match observations. In addition, within the shell-model picture one discovers that "twist" modes are possible for triaxial distributions because rotations by $\phi_\pi$ and $\phi_\nu$ about the $z$-axes of the proton and neutron distributions emerge as additional degrees of freedom. (In addition to Ref. 21, see Refs. 24–27). As for the scissor mode, it is the relative difference in these rotations that gives rise to an M1 excitation. This new mode, which together with the scissor construction determines the overall structure of the M1 transition spectrum, has a very simple interpretation within the framework of the pseudo SU(3) model.

2. Shell-model geometry

The starting point for a geometrical interpretation of the scissor mode within the framework of the pseudo SU(3) shell model is the well-known relation of the SU(3) symmetry group to the symmetry group of the triaxial rotor, Rot(3) [29, 30]. As has been shown [23, 31], a similar relation holds for the case of two coupled quantum rotors, one representing protons and the other neutrons. Based on a correspondence between the Casimir operators $C_2$ and $C_3$ of SU(3) and invariants of the rotor group,

$$\text{Tr}[Q^c(\theta, \phi_\pi, \phi_\nu)]^2 = \frac{2}{3} C_2 + 2,$$

$$\text{Tr}[Q^c(\theta, \phi_\pi, \phi_\nu)]^3 = \frac{3}{4} C_3,$$

the irreducible representation (irrep) labels $(\lambda, \mu)$ of the total SU(3) can be mapped onto the collective shape variables ($\beta$ and $\gamma$) of the joint rotor system [30].

$$\beta^2 = \frac{4\pi}{3} \left( \frac{1}{(A r_{\text{rms}})^2} \right) \left[ \lambda^2 + \lambda \mu + \mu^2 + 3 (\lambda + \mu + 1) \right],$$

$$\tan \gamma = \frac{\sqrt{3} (\mu + 1)}{2 \lambda + \mu + 3}.$$  

This correspondence also suggests that one should rewrite the SU(3) Hamiltonian

$$H = c_1 Q \cdot Q + c_2 L^2 + c_3 K_L^2 + c_4 (L_x^2 + L_y^2),$$

$$= H_{\text{rot}} + H_{\text{int}}$$

in terms of a rotational part,

$$H_{\text{rot}} = a L^2 + b K_L^2,$$

and an intrinsic part that describes the proton-neutron interaction,

$$H_{\text{int}} = c (L_x - L_y)^2 - d C_2$$

$$= c l^2 - d C_2,$$

in such a way that $H_{\text{int}}$ can be approximated by a twodimensional, anisotropic oscillator. Explicitly,

$$H_{\text{int}} = c (l_x^2 + l_y^2) + 4d[l_x (\lambda_x + 1) (\lambda_x + 1) + l_y (\lambda_y + 1) (\lambda_y + 1)]$$

$$+ (\mu_x + 1) (\mu_y + 1) \phi^2 + E_0,$$

$$= \hbar \omega_\theta \left( n_\theta + \frac{1}{2} \right) + \hbar \omega_\phi \left( n_\phi + \frac{1}{2} \right) + E_0,$$

where $E_0$ is a constant and the oscillator frequencies $\omega_\theta$ and $\omega_\phi$ are defined by

$$\omega_\theta = \sqrt{12 \ c \ d (\lambda_x + 1) (\lambda_y + 1)},$$

$$\omega_\phi = \sqrt{12 \ c \ d (\mu_x + 1) (\mu_y + 1)}.$$  

Here $\lambda_\theta$ and $\lambda_\phi$, respectively, are the relative angular momenta about an axis perpendicular to and in the symmetry plane of the proton-neutron system.

The structure of the intrinsic Hamiltonian allows for an interpretation of the coupled SU(3) irreps $(\lambda_\pi, \mu_\pi)$ and $(\lambda_\nu, \mu_\nu)$ for protons and neutrons, respectively, in terms of oscillator functions. According to the Littelwood rules [32] for coupling Young diagrams, the allowed product configurations can be expressed in mathematical terms with the help of three quantum numbers $(m, l, k)$ [31]:

$$(\lambda, \mu) = \bigoplus_{m, l, k} (\lambda_x + \lambda_\nu - 2 m + l, \mu_x + \mu_\nu - 2 l + m)^k,$$

where the parameters $l$ and $m$ are defined in a fixed range given by the values of the initial SU(3) representations. In this formulation $(\lambda, \mu)$ turn out to be independent of $k$ which serves to distinguish between multiple occurrences of equivalent $(\lambda, \mu)$ irreps in the tensor product. The number of $k$ values is equal to the maximum outer multiplicity, $\rho_{\text{max}}$ ($\rho = 1, 2, \ldots, \rho_{\text{max}}$). The $l$ and $m$ labels can be identified [23] with excitation quanta of the two dimensional oscillator given by Eq. (6)

$$m = n_\theta, \quad l = n_\phi.$$  

This corresponds to two distinct types of $1^+$ motion, the scissors and twist modes, and their realization in terms of the pseudo SU(3) model.

The leading irrep $(m, l) = (0, 0)$ of the SU(3) tensor product,

$$(\lambda, \mu) = (\lambda_x + \lambda_\nu, \mu_x + \mu_\nu),$$

is associated with a minimum in the relative angular displacements of the proton and neutron subsystems, generating maximum deformation for the composite system. All other values
for \( m \) and \( l \) provide larger expectation values for the angular variables \( \theta \) and \( \phi \), respectively, which, in turn, are related to internal excitation energies. Assuming prolate shapes for the parent distributions, \( \lambda_\pi > \mu_\nu \), each phonon added to the scissors mode produces a larger increase in energy than a phonon added to the twisting motion. This reflects the softness of the twist degree of freedom relative to the stiffer scissors mode. The configuration

\[
(\lambda, \mu) = (\lambda_\pi + \lambda_\nu - 2, \mu_\pi + \mu_\nu + 1)
\]  

(11)

is the first scissors-like configuration. It is always part of the tensor product decomposition and contains a \( J^m = 1^+ \) state that is the bandhead of a \( K = 1 \) band. Its SU(3) parameters are \((m, l) = (1, 0)\), and therefore corresponds to a single excitation of the \( \theta \) mode. This can be interpreted as the shell-model equivalent of the scissors mode of the TRM. Note, however, that in contrast with the TRM, here the relative angular displacements are associated with the deformed valence nucleon distributions only.

A generalization of this picture is possible for triaxial distributions. In this case, with either \( \mu_\pi \) or \( \mu_\nu \) are non-zero, a second scissors state appears which is given by

\[
(\lambda, \mu) = (\lambda_\pi + \lambda_\nu - 1, \mu_\pi + \mu_\nu - 1)
\]  

(12)

or \((m, l) = (1, 1)\). In terms of an underlying geometrical picture, this structure is a superposition of a \( \phi \) twisting motion on top of the lowest scissors configuration. The energy separation of the \((m, l) = (1, 0)\) and \((1, 1)\) modes is a result of different intrinsic motions.

The most general situation is when \( \mu_\pi \) and \( \mu_\nu \) are both nonzero, that is, when both the proton and neutron distributions are triaxial. Then two more \( 1^+ \) states can be identified, one being the \((m, l) = (0, 1)\) configuration with

\[
(\lambda, \mu) = (\lambda_\pi + \lambda_\nu + 1, \mu_\pi + \mu_\nu - 2).
\]  

(13)

It differs from the others not only through its intrinsic energy but also because it belongs to a \( K = 0 \) band. The fourth state, \((m, l, k) = (1, 1, 2)\) differs from the second, \((m, l, k) = (1, 1, 1)\), only in its value of the outer multiplicity parameter \( \rho \). In summary, a pure pseudo SU(3) picture gives rise to a maximum of four \( 1^+ \) states: scissors, twist, and a doubly degenerate scissors-plus-twist mode.

The experimental results [1], Fig. 1, suggest a much larger number of \( 1^+ \) states with non-zero M1 transition probabilities to the \( 0^+ \) ground state. This can be understood in terms of the fragmentation of the pure symmetry states under the influence of SU(3) breaking residual interactions, which is the topic of the following section.

![Figure 1. The experimental M1 transition strength spectrum of \(^{160}\text{Gd}\) [1]. Note how the transitions are clustered around local centroids and that some of the clusters appear to be more fragmented than others.](image)

3. Shell-model calculations

To investigate the effect of symmetry breaking terms in the interaction on the fragmentation of the geometrical modes [7, 11], a generalization of the Hamiltonian (3) introduced above was used [22],

\[
H_{SU(3)} = -(a_2 + a_{sym})C_2 + a_3 C_3 + bK^2_\pi + cJ^2
\]

\[
+ D_{\pi} \sum_{\lambda_{\pi}} \lambda_{\pi}^2 + D_{\nu} \sum_{\lambda_{\nu}} \lambda_{\nu}^2 - G_{\pi} H_{\pi} - G_{\nu} H_{\nu},
\]

(14)

where the single-particle angular momenta \( \lambda_{\pi} \) and \( \lambda_{\nu} \) are one-body terms and the proton and neutron pairing terms \( H_{\pi} \) and \( H_{\nu} \) are two-body interactions. Also, a parameter \( a_{sym} \) was included to allow an energy shift for SU(3) irreps with either \( \lambda \) or \( \mu \) odd as these belong to different irreps (\( B_\alpha \)), \( \alpha = 1, 2, 3 \), rather than \( A \) of the intrinsic Vierergruppe (\( D_2 \)) symmetry [33].

To select an appropriate set of SU(3) basis functions, one first determines the proton and neutron occupancies by filling pair-wise from below the single-particle levels of the generalized Nilsson Hamiltonian [34],

\[
h_0 = h_\text{osc} + C1 \cdot s + D1^2
\]

\[
-\mu_\omega \gamma^2 \beta [Y_0^2 + \frac{\sin \gamma}{\sqrt{2}} (Y_2^0 + Y_2^2)],
\]

(15)

for values of \( \beta \) and \( \gamma \) that give the lowest total energy of the combined proton and neutron systems. One then determines the number of valence-space nucleons in the normal and unique parity levels, the latter being intruder states that are pushed down into the valence space from the next higher shell by the strong spin-orbit interaction. An overall simplifying assumption made in most pseudo SU(3) model calculations is that the relevant dynamics can be described by taking into account the nucleons in the normal parity sector only [35]; the nucleons in intruder states (unique parity sector) are assumed to follow in an adiabatic manner the motion of the nucleons in normal parity sector with their effect repre-
Figure 2. Energy and M1 transition spectra for the even-even $^{156-160}\text{Gd}$ isotopes [22]. Experimental excitation energies and E2 transition strengths were used as input in a fitting routine to determine parameters of the Hamiltonian for each system. These were then used to calculate the theoretical spectrum and corresponding M1 transition strengths.

Table I. Deformation and occupation numbers for the Gd isotopes used in this study. These parameters are also used in a determination of the SU(3) basis states.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$n_\alpha^0$</th>
<th>$n_\Delta^0$</th>
<th>$n_\Delta^+$</th>
<th>$n_\alpha^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{156}\text{Gd}$</td>
<td>0.30</td>
<td>$0^\circ \leq \gamma \leq 6^\circ$</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>$^{158}\text{Gd}$</td>
<td>0.31</td>
<td>$0^\circ \leq \gamma \leq 4^\circ$</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$^{160}\text{Gd}$</td>
<td>0.29</td>
<td>$0^\circ \leq \gamma \leq 4^\circ$</td>
<td>8</td>
<td>6</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

sent through a reparameterization of the theory. For the nuclei investigated here, the occupation numbers and the corresponding deformations $\beta$ and $\gamma$ are given in Table I.

In the Ref. 22 all SU(3) basis states with $C_2^{\alpha} \geq C_2^{\alpha_{\text{min}}}$ were selected with $C_2^{\alpha_{\text{min}}}$ set so that all proton ($\alpha = \pi$) and neutron ($\alpha = \nu$) irreps lying below approximately 6 MeV were included in the analysis. Then all possible couplings of these proton and neutron SU(3) irreps, again within 6 MeV of the predicted ground state, were taken to give coupled SU(3) irreps that form basis states of the model space. Also, only states with $J \leq 8$ and $S = 0$ were considered.

The parameters for the Hamiltonian given in Eq. (14) and the effective charges $e_{\pi} = 1 + g_{\text{eff}}$ and $e_{\nu} = g_{\text{eff}}$ used in the $E(2)$ transition operator,

$$T_M^2(E) = A^{1/3} \sum_{\sigma = \pi, \nu} \sum_i e_\sigma r_\sigma^2(i)Y_{2M}(r_\sigma(i)),$$

Table II. Total $B(M1)$ transition strengths $[\mu_{\pi}^2]$ from experiment [1, 28] and calculation [22]. Experimental and theoretical values for $B(E2, 0^+_2 \rightarrow 2^+_1)$ transition in $e^2b^2$ are also given.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\sum B(M1) [\mu_{\pi}^2]$</th>
<th>$B(E2, 0^+_2 \rightarrow 2^+_1) [e^2b^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{156}\text{Gd}$</td>
<td>3.40</td>
<td>2.91</td>
</tr>
<tr>
<td>$^{158}\text{Gd}$</td>
<td>4.32</td>
<td>3.02</td>
</tr>
<tr>
<td>$^{160}\text{Gd}$</td>
<td>4.21</td>
<td>3.29</td>
</tr>
</tbody>
</table>

were determined through a fitting procedure that included as input all known levels with $J \leq 8$ up through 2 MeV in energy and selected $E(2)$ transition strengths. This procedure gave, in general, good agreement between the experimental and theoretical numbers (Fig. 2 and Table II).

Recent calculations were done using a refined version of the pseudo SU(3) formalism. The realistic Hamiltonian that was used in these calculations took single-particle energies and the quadrupole-quadrupole and monopole pairing strengths from systematics,

$$H = H_\pi^{sp} + H_\nu^{sp} - \frac{1}{2} \chi Q \cdot Q - G_\pi H_P^{sp} - G_\nu H_P^{sp} + aK_2^{\pi} + bJ^2 + a_{\text{sym}} C_2 + a_3 C_3. \quad (17)$$

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Table III. $B$(M1) transition strengths $[\mu_{B}^{2}]$ in the pure symmetry limit of the pseudo SU(3) model. The strong coupled pseudo SU(3) irrep $(\lambda, \mu)_{gs}$ for the ground state is given with its proton and neutron sub-irreps and the irreps associated with the $1^{+}$ states, $(\lambda', \mu')_{1^{+}}$. In addition, each transition is labeled as a scissors ($s$) or twist ($t$) or combination mode.

| Nucleus $(\lambda, \mu)_{gs}$ $(\lambda, \mu)_{gs}$ $(\lambda', \mu')_{1^{+}}$ $B$(M1) mode | 156−158Gd (10,4) (18,0) (28,4) (26,5) 1.91 $s$ (27,3) 1.61 $s + t$ | 160Gd (10,4) (18,4) (28,8) (26,9) 1.77 $s$ (27,7) 1.82 $s + t$ (27,7) 0.083 $t + s$ (29,6) 0.56 $t$ |

The other parameters of the Hamiltonian (17), namely, $a$, $b$, $a_{s}$, and $a_{sym}$, were determined by fitting the low-energy spectra to the experimental numbers. This refined model was applied to 156Gd. As with the earlier calculations, the low-energy spectra was described well. In addition, the distribution of $B$(M1) transition strengths was determined to be closer to the experimental values than in the previous calculation (Fig. 3).

In all cases the M1 transition operator [19]

$$T_{M1}^{1} = \sqrt{3/4\mu_{N}} \sum_{\tilde{\sigma}} \left( g_{a}^{\tilde{\sigma}} T_{\tilde{\sigma}}^{a} + g_{c}^{\tilde{\sigma}} S_{\tilde{\sigma}}^{c} \right) \tag{18}$$

with orbit $g$-factors

$$g_{a}^{\tilde{\sigma}} = 1, \quad g_{c}^{\tilde{\sigma}} = 0 \tag{19}$$

(i.e., with no effective $g$-factors) was used to determine transition strengths between the $0^{+}$ ground state and $1^{+}$ states. In the pure SU(3) limit of the theory, there are at most four allowed M1 transitions. These, together with their classification as the scissors mode, twist mode, or a combination are listed with the corresponding SU(3) irreps in Table III.

Because of the SU(3) symmetry breaking terms $I_{sp}^{2}$ ($I_{sp}^{2,s}$) and $H_{p}^{2,s}$, this simple picture gives way to a more complex transition spectrum which is in much better agreement with experimental results. One finds a number of transitions that are close to the observed ones. Also, the centroid of the experimental and theoretical M1 transition strength distribution are usually found to lie at approximately the same energy, so good overall agreement with experiment is obtained. The total M1 strength, which for the full Hamiltonian is a bit lower then for its pure SU(3) limit due to destructive interference associated with the mixing (see Tables II and III), shows reasonable agreement with the experimental results, underestimating them slightly for the Gd isotopes. A possible reason for this discrepancy is missing spin 1 admixtures in the wavefunctions which are known to add M1 strength to the system [36].

4. Conclusion

The pseudo SU(3) model offers a microscopic shell-model interpretation of the "scissors" mode. In addition, it reveals an additional "twist" degree of freedom that corresponds to allowed relative angular motion of the proton and/or neutron sub-distributions when they are triaxial. Each M1 modes is associated with a well-defined SU(3) irrep. In the pure SU(3) symmetry limit of the theory there are up to four non-zero M1 transitions from excited $1^{+}$ states to the ground state. The summed strength is in good agreement with experiment. By adding SU(3) symmetry breaking one-body and two-body pairing interactions to the Hamiltonian, it is possible to describe the experimentally observed fragmentation of the M1 strength.

Acknowledgments

Supported by the U.S. National Science Foundation through a U.S.-Mexico Cooperative Research grant (INT-9500474), a regular grant (PHY-9970769), and a Cooperative Agreement (EPS-9720652) that includes matching from the Louisiana Board of Regents Support Fund; and by Conacyt (Mexico).

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