Low energy spectra of $A = 159$ and 161 nuclei

J.G. Hirsch

*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México*
Apartado postal 70-543, 04510 México, D.F., Mexico

C.E. Vargas

*Departamento de Física, Centro de Investigación y Estudios Avanzados del Instituto Politécnico Nacional*
Apartado postal 14-740, 07000 México, D.F., Mexico

J.P. Draayer

*Department of Physics and Astronomy, Louisiana State University*
Baton Rouge, LA 70803-4001, U.S.A.

Recibido el 15 de febrero de 2000; aceptado el 28 de mayo de 2000

The pseudo SU(3) model is used to describe the low-energy spectra of $^{159}$Tb, $^{159}$Dy, $^{160}$Eu, $^{161}$Tb and $^{161}$Tm and B(E2) electromagnetic transition strengths in $^{159}$Tb. The building blocks of the model are the SU(3) proton and neutron states having pseudo spin zero and 1/2, which describe even and odd number of nucleons, respectively. The many-particle states are built as SU(3) coupled states with a well-defined particle number and total angular momentum. The Hamiltonian includes spherical Nilsson single-particle energies, the quadrupole-quadrupole and pairing interactions, as well as two rotor terms which are diagonal in the SU(3) basis. As expected, the use of realistic single-particle energies plays a key role in an appropriate description of these nuclei. The interactions are microscopic, and closely related to those known to be the most relevant for the low-energy spectra.

*Keywords*: Pseudo SU(3) model; rare earth nuclei; odd-mass nuclei; excitation energies; B(E2) values

Se emplea el modelo pseudo SU(3) para describir el espectro de baja energía de los núcleos $^{159}$Tb, $^{159}$Dy, $^{159}$Eu, $^{161}$Tb y $^{161}$Tm, y la intensidad de las transiciones electromagnéticas B(E2) en $^{159}$Tb. El modelo es construido a partir de estados de SU(3) para protones y neutrones con pseudo espín cero y 1/2, que describen el caso de número par e impar de nucleones, respectivamente. Los estados del sistema nuclear se construyen como estados acoplados de SU(3) con número bien definido de partículas y buen momento angular. El Hamiltoniano incluye energías de partícula independiente de Nilsson esféricas, interacciones cuadripolares y de apareamiento, así como dos términos de rotor que son diagonales en la base de SU(3). Como se esperaba, el uso de energías de partícula independiente realistas tiene un papel crucial, en la apropiada descripción de estos núcleos. Las interacciones son microscópicas y están estrechamente vinculadas a aquellas conocidas por su relevancia en el espectro de baja energía.

*Descriptores*: Modelo pseudo SU(3); núcleos de tierras raras; masa impar; energías de excitación; valores B(E2)

PACS: 21.60.Fw; 21.60.Cs; 23.20.Lv; 27.70.+q

1. Introduction

The remarkable advances in computer power together with the use of complex algorithms have allowed systematic shell model studies of nuclei of the $sd$-shell [1] and $pf$-shell up to $A = 56$ [2]. A systematic and proper truncation of the model space is required to obtain a shell-model description of heavy nuclei [3].

In light deformed nuclei the dominance of the quadrupole-quadrupole interaction led to the introduction of the SU(3) shell model [4]. It was found that the ground state wave function of well-deformed light nuclei normally consists of only a few SU(3) irreps [5–8]. The strong spin-orbit interaction renders the usual SU(3) scheme useless in heavy nuclei, but at the same time pseudo-spin emerges as a good symmetry [9–11]. The pseudo SU(3) model capitalizes on the existence of pseudo-spin symmetry.

A fully microscopic description of low-energy bands in even-even nuclei has been developed using the pseudo SU(3) model. The first applications used pseudo SU(3) as a dynamical symmetry, with a single SU(3) irrep describing the whole yrast band up to backbending [12]. The development of a computer code to calculate reduced matrix elements of physical operators between different SU(3) irreps [13] represented a breakthrough in the development of the pseudo SU(3) model. Allowing the mixing of SU(3) irreps through the inclusion of a single-particle term and the pairing interaction in the Hamiltonian, a powerful shell-model theory for a description of normal parity states in heavy deformed nuclei emerged. For example, the low-energy spectra of many Gd and Dy isotopes, their B(E2) and B(M1) transition strengths for both their scissors and twist modes [14] and their fragmentation were successfully described with a realistic Hamiltonian [15].

In the present article we present a refined version of the pseudo SU(3) formalism which uses a realistic Hamiltonian with single-particle energies plus quadrupole-quadrupole and monopole pairing interactions with strengths taken from
4. Summary and conclusions

In this work, we discussed global properties of nuclear structure using random Hamiltonians.

We first considered the problem from a shell model perspective, focussing on a system of identical neutrons in the $sd$ shell. We confirmed the conclusion reached earlier by Johnson and collaborators [11, 12] that nuclei with many nucleons favor $J^P = 0^+$ ground states, even without a dominant pairing component in the force. We demonstrated further that this is not a consequence of time-reversal invariance of the random Hamiltonian. Finally, we showed that systems of identical nucleons interacting via random two-body interactions tend to favor a seniority structure, in accord with conclusions reported recently in [13]. There is little or no evidence for the occurrence of vibrational and rotational bands.

We then considered the same issues in the context of the IBM, a collective model that from the outset emphasizes the role of monopole and quadrupole pairs. Here too we found that $0^+$ ground states predominate, exactly as in the shell model analysis. In contrast, we found that the IBM strongly favors both vibrational and rotational structures, as evident from energy ratios of low-lying states and their corresponding $BE(2)$ ratios. These conclusions emerged from a much wider class of Hamiltonians than is usually thought to be 'realistic'. The inclusion of three-body random interactions did not change these basic conclusions, as long as the number of bosons is sufficiently large. This suggests that the observed vibrational and rotational features represent general and robust properties of the IBM model space, and do not depend significantly on details of the interaction. Since the structure of the model space is completely determined by the degrees of freedom, our results emphasize once again the importance of the selection of the relevant degrees of freedom.

The results that we obtained with random Hamiltonians in the shell model and the IBM are in qualitative agreement with the empirical observation of robust features in the low-lying spectra of medium and heavy even-even nuclei and their tripartite classification into seniority, anharmonic vibrator and rotor regimes [1, 2]. The analysis with random interactions shows that seniority arises as a global property of the shell model space, while vibrational and rotational bands arise as general features of the interacting boson model space. However, the IBM is based on the assumption that low-lying collective excitations in nuclei can be described as a system of interacting monopole and quadrupole bosons, which in turn are associated with generalized pairs of like-nucleons with angular momentum $L = 0$ and $L = 2$. Therefore it would be very important to establish whether vibrational and rotational features can also arise from ensembles of random interactions in the nuclear shell model, if appropriate (minimal) restrictions are imposed on the parameter space.

Acknowledgments

It is a pleasure to thank Rick Casten, Jorge Flores and Francisco Iachello for illuminating discussions. This work was supported in part by DGAPA-UNAM under project IN101997, by CONACyT under projects 32416-E and 32397-E, and by NSF under grant PHY-9970749.

known systematics [16, 17]. The model is applied to five odd-mass rare earth nuclei: \(^{159}\)Eu, \(^{159}\)Tb, \(^{159}\)Dy, \(^{161}\)Tb and \(^{161}\)Tm. We will also discuss the importance of various terms in the Hamiltonian.

In Sec. 2 the pseudo SU(3) classification scheme is presented. The pseudo SU(3) Hamiltonian and its parametrization is discussed in Sec. 3. The energy spectra of \(^{159}\)Tb is analyzed in Sec. 4, together with the effect that some terms in the Hamiltonian have on it. Sec. 5 contains energy spectra of \(^{159}\)Eu, \(^{159}\)Dy, \(^{161}\)Tb and \(^{161}\)Tm, and in Sec. 6 the electromagnetic B(E2) transitions in \(^{159}\)Tb are discussed. Conclusions are drawn in Sec. 7.

2. The pseudo SU(3) basis

We will describe \(^{159}\)Tb as a first example. It has 15 protons and 12 neutrons in the 50–82 and 82–126 shells, respectively. The number of nucleons in normal (N) and abnormal (A) parity orbitals is determined by filling the Nilsson levels with a pair of particles for \(\beta \sim 0.25\) in order of increasing energy. This gives

\[
\begin{align*}
&n_x^N = 9, \quad n_x^A = 6, \quad n_y^N = 8, \quad n_y^A = 4. \\
&n_z^N = 9, \quad n_z^A = 6, \quad n_\nu^N = 8, \quad n_\nu^A = 4.
\end{align*}
\]

Notice that the deformed Nilsson mean field is only employed to determine the number of nucleons in normal and unique parity orbitals. The use of the pseudo SU(3) basis to describe the normal parity sector implies that active nucleons can occupy all normal parity orbitals, not only those with the lower single-particle energies.

As has been the case for all pseudo SU(3) studies up to now, we will freeze the nucleons in abnormal parity orbital and describe the dynamics using only nucleons in normal parity states. While it has been shown that this is a reasonable, it is nonetheless a strong assumption. This choice is further reflected through the use of effective charges to describe quadrupole electromagnetic transitions which are larger than those usually employed in typical shell-model calculations for light nuclei. A more sophisticated treatment of the problem, with nucleons in intruder orbitals described on the same footing using SU(3) irreps is under development [8].

Many-particle states of \(n_\alpha\) active nucleons in a given normal parity shell \(n_\alpha\), \(\alpha = \nu\) or \(\pi\), can be classified by the following chains of groups:

\[
\begin{align*}
&\{ n_\alpha \} \quad \{ J_\alpha \} \quad \{ \gamma_\alpha \} \quad (\lambda_\alpha, \mu_\alpha) \quad S_\alpha \quad K_\alpha \\
&U(n_\alpha^N) \supset U(n_\alpha^N/2) \times U(2) \supset SU(3) \times SU(2) \supset \\
&J_\alpha \quad S_\alpha \quad K_\alpha \\
&SO(3) \times SU(2) \supset SU_J(2),
\end{align*}
\]

where above each group the quantum numbers that characterize its irreps are given and \(\gamma_\alpha\) and \(K_\alpha\) are multiplicity labels of the indicated reductions.

The most important configurations are those with highest spatial symmetry [12, 8]. This implies that \(S_{\pi, \nu} = 0\) or 1/2, that is, only configurations with pseudo spin zero for even number of nucleons and 1/2 for odd number of nucleons are taken into account.

After decoupling the pseudo spin in Eq. (2), we get \(\{J_\alpha\} = \{2^{+}\}\), \(\{\gamma_\alpha\} = \{2^{+}\}\) with \(S_\alpha = 1/2\) and \(S_\nu = 0\). Table I lists the 15 pseudo SU(3) irreps, with the largest value of the Casimir operator \(C_2\), which were used in this calculation.

What makes the pseudo SU(3) model a powerful theory is that it allows one to envisage a relatively simple and physically motivated basis truncation scheme. Extended shell-model calculations in the pf- and sdg-shell have shown that in a description of deformed nuclei the Hilbert space can be truncated to only those states that are relevant when both the quadrupole-quadrupole force and the single-particle Hamiltonian are taken into account [18]. While pairing is fundamental to obtaining the correct moment of inertia of the rotational bands, it has a relatively small effect on the overall wave functions [19].

3. The pseudo SU(3) Hamiltonian

The Hamiltonian contains spherical Nilsson single-particle terms for protons and neutrons \((H_{sp, \pi})\), the quadrupole-quadrupole \((Q \cdot Q)\) and pairing \((H_{pair, \pi})\) interactions as well as three 'rotor-like' terms which are diagonal in the SU(3) basis:

\[
H = H_{sp, \pi} + H_{sp, \nu} - \frac{1}{2} \chi \cdot \hat{Q} - G_\pi H_{pair, \pi} - G_\nu H_{pair, \nu} + a K_\pi^2 + b J_\nu^2 + A_{asym} \bar{C}_2.
\]

The term proportional to \(K_\pi^2\) breaks the SO(3) degeneracy of the different K bands [19], the \(J^2\) term represents a small correction to fine tune the moment of inertia, and the last \(\bar{C}_2\) term is introduced to distinguish between SU(3) irreps with \(\lambda\) and \(\mu\) both even from the others with one or both odd [20].

The Nilsson single-particle energies as well as the pairing and quadrupole-quadrupole interaction strengths were taken from systematics [21, 22]; only \(a\) and \(b\) were used for fitting. Parameter values are listed in Table II and are consistent with those used in the description of neighboring even-even nuclei [15].

<table>
<thead>
<tr>
<th>(\lambda, \mu)</th>
<th>(\lambda, \mu)</th>
<th>total (\lambda, \mu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(20,8)</td>
<td>(28,6)</td>
<td>(30,4)</td>
</tr>
<tr>
<td>(31,2)</td>
<td>(32,0)</td>
<td>(26,9)</td>
</tr>
<tr>
<td>(20,8)</td>
<td>(29,6)</td>
<td>(30,4)</td>
</tr>
<tr>
<td>(31,2)</td>
<td>(32,0)</td>
<td>(26,9)</td>
</tr>
<tr>
<td>(20,8)</td>
<td>(29,6)</td>
<td>(30,4)</td>
</tr>
<tr>
<td>(31,2)</td>
<td>(32,0)</td>
<td>(26,9)</td>
</tr>
<tr>
<td>(20,8)</td>
<td>(29,6)</td>
<td>(30,4)</td>
</tr>
<tr>
<td>(31,2)</td>
<td>(32,0)</td>
<td>(26,9)</td>
</tr>
</tbody>
</table>

TABLE II. Parameters used in Hamiltonian \((3)\) for the five nuclei analysed.

<table>
<thead>
<tr>
<th>(\chi)</th>
<th>(G_{\pi})</th>
<th>(G_{\nu})</th>
<th>(a)</th>
<th>(b)</th>
<th>(A_{\text{asym}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{159}\text{Eu})</td>
<td>0.00753</td>
<td>0.132</td>
<td>0.106</td>
<td>-0.0508</td>
<td>0.0009</td>
</tr>
<tr>
<td>(^{159}\text{Tb})</td>
<td>0.00753</td>
<td>0.132</td>
<td>0.106</td>
<td>0.0198</td>
<td>-0.0031</td>
</tr>
<tr>
<td>(^{159}\text{Dy})</td>
<td>0.00753</td>
<td>0.132</td>
<td>0.106</td>
<td>0.0048</td>
<td>0.0006</td>
</tr>
<tr>
<td>(^{161}\text{Tb})</td>
<td>0.00737</td>
<td>0.130</td>
<td>0.105</td>
<td>-0.0040</td>
<td>-0.0040</td>
</tr>
<tr>
<td>(^{161}\text{Tm})</td>
<td>0.00737</td>
<td>0.130</td>
<td>0.105</td>
<td>0.0120</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

4. The spectra of \(^{159}\text{Tb}\): single-particle energies and pairing

Figure 1a shows the yrast and excited bands in \(^{159}\text{Tb}\). Experimental data [23] are plotted on the left hand side of each set, while those obtained using the Hilbert space and the Hamiltonian parameters discussed in the previous sections are shown on the right hand side. The agreement between the experiment and theory is excellent. Of the four bands reported in the literature in \(^{159}\text{Tb}\), three of them (the yrast, \(5/2^+\) and \(3/2^+\) bands) have a difference between the experimental and predicted levels of less than 50 KeV. The \(1/2^+\) is slightly high in energy, and the model predicts an exaggerated staggering.

The role played by each term in the Hamiltonian is discussed in detail elsewhere [16]. In this article we wish to emphasize that the role played by the single-particle and pairing terms in Hamiltonian \((3)\).

The whole energy spectra is built up through an interplay of the single-particle and quadrupole-quadrupole terms in the Hamiltonian [18, 8]. These two terms define the relative ordering between the different bands, as well as the main components of the wavefunction. As expected, the use of realistic single-particle energies plays a key role in achieving an appropriate description of odd-mass nuclei.

To make this point clear, in Fig. 1b the theoretical energy spectra calculated with Hamiltonian \((3)\) without single-particle energies are presented on the right of each column of data with the corresponding experimental energies [23] given on the left. It is clear that the ordering is shifted. The ground state is now predicted to have \(J = \frac{1}{2}^+\), the first excited band starts with \(J = \frac{3}{2}^+\). Only the third band with \(J = \frac{5}{2}^+\) remains ordered correctly. The wave functions of all the states in all these three bands are dominated (more than 95%) by the leading SU(3) irrep \((28,8), (10,4)\)\(_{v}(18,4)\)\(_{s}\) [16].

The pairing interaction plays also an essential role despite the strong truncation of the Hilbert space. This can be seen in Fig. 1c, which shows the low-energy spectra of \(^{159}\text{Tb}\) with the same Hamiltonian except that the pairing interaction has been turned off. It clearly exhibits the importance of the pairing interaction in building up the correct moment of inertia: the spectra without pairing is strongly compressed. It can also be seen that pairing affects the other energies in a similar way with an overall effect that resembles the introduction of a multiplicative factor in the Hamiltonian.

5. The energy spectra of \(^{159}\text{Eu}, \ 159\text{Dy}, \ 161\text{Tb} \) and \(^{161}\text{Tm}\)

In Fig. 2a we present the low-lying energy spectra of \(^{159}\text{Eu}\), including the \(K = \frac{5}{2}^+\) and \(\frac{5}{2}^-\) bands built with 7 protons in the normal parity subshell \(\tilde{n} = 3\) and 8 neutrons in \(\tilde{n} = 4\). There is a good agreement between the experimental [23] and theoretical results [17]. The model predicts a second \(\frac{5}{2}^+\) state

**Figure 1.** Energy spectra of \(^{159}\text{Tb}\). 'Exp' represents the experimental results and 'Theo' the calculated ones. (a) The energies obtained with the Hamiltonian parameters listed in Table II; (b) the energies obtained without single-particle energies; and (c) the energies obtained without pairing.
in the $K = \frac{3}{2}$ band which is missing in the experimental spectra, as well as several other states in the excited bands. It is interesting to notice that the ground state in $^{159}$Tb is $\frac{9}{2}^+$ while in $^{159}$Eu it is $\frac{5}{2}^+$. Reproducing this effect is one of the successes of this theory; realistic single-particle energies are required to get this ordering correct.

The low energy spectra of $^{159}$Dy is presented in Fig. 2b. There are three bands, with $K = \frac{3}{2}, \frac{5}{2}$ and $\frac{9}{2}$, respectively. As in the other cases the agreement between theory and experiment is remarkably good. In the $K = \frac{3}{2}$ ground state band the $\frac{11}{2}^+$ state is predicted to have an energy higher than the experimentally observed one. This departure of the experimental ground state band from the rigid rotor behavior may be related with a band crossing. The possibility of describing it by increasing the Hilbert space is under investigation. In the $K = \frac{5}{2}$ band the $\frac{9}{2}^-$ state lies higher than its $\frac{9}{2}^+$ partner which contradicts the experimental results. As in the other cases, the model predicts several excited levels that are as yet undetected [17].

In Fig. 3a we present the low-lying energy spectra of $^{161}$Tb, including the yrast $K = \frac{3}{2}$, one excited band with $K = \frac{1}{2}$ and two excited bands with $K = \frac{9}{2}$. The fine details of the energy spectra are reproduced in the four bands. Notice the large ‘staggering’ effect in the excited $K = \frac{3}{2}$ band, where the pairs $(\frac{5}{2}, \frac{3}{2}), (\frac{7}{2}, \frac{5}{2})$ and $(\frac{9}{2}, \frac{7}{2})$ are inverted relative to the usual ordering.

![Figure 3](image)

**Figure 3.** (a) Energy spectra of $^{161}$Tb and (b) $^{161}$Tm, with the same convention of Fig. 1.

Figure 3b shows the energy spectra of $^{161}$Tm. Four bands are presented: the yrast $K = \frac{3}{2}$ band and the excited bands with $K = \frac{1}{2}, \frac{3}{2}$ and $\frac{9}{2}$. The yrast and first excited bands are very well reproduced. In particular the extreme staggering found in the $K = \frac{3}{2}$ band, where many pairs of states are nearly degenerate, is accurately described. The excited $K = \frac{9}{2}$ is predicted at an energy much larger than observed. The model seems to be unable to reproduce the very small energy (19 keV) of this second excited band head. Having four states under 25 keV, $^{161}$Tm is a nuclei with a very peculiar energy spectra which deserves further consideration.

### 6. B(E2) transition strengths

Up to this point we have centered the discussion on the energetics. But the pseudo SU(3) model is far more powerful than just this, it can also successfully describe the electromagnetic transitions. Theoretical and experimental [23] B(E2) transition strengths between yrast states in $^{159}$Tb are shown in Table III.

The E2 transition operator that was used is given by [12]

$$Q_{\mu} = e_\pi Q_\pi + e_\nu Q_\nu \approx \frac{e_\pi}{\eta_\pi} \bar{Q}_\pi + e_\nu \frac{\eta_\nu}{\eta_\pi} \bar{Q}_\nu, \quad (4)$$

with effective charges $e_\pi = 2.3, e_\nu = 1.3$. These values are very similar to those used in the pseudo SU(3) description of even-even nuclei [12, 15]. They are larger than those used in standard calculations of B(E2) strengths [21] due to the passi-
TABLE III. Theoretical and experimental B(E2) transition strengths for \(^{159}\)Tb.

<table>
<thead>
<tr>
<th>(J^+ \rightarrow (J+2)^+)</th>
<th>Theo. ((e^2b^2))</th>
<th>Exp. ((e^2b^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{5}{2}^+ \rightarrow \frac{7}{2}^+)</td>
<td>1.6503</td>
<td>1.4736 ± 0.2047</td>
</tr>
<tr>
<td>(\frac{7}{2}^+ \rightarrow \frac{9}{2}^+)</td>
<td>2.0553</td>
<td>1.8590 ± 0.1023</td>
</tr>
<tr>
<td>(\frac{7}{2}^+ \rightarrow \frac{11}{2}^+)</td>
<td>2.1966</td>
<td>2.2180 ± 0.0537</td>
</tr>
<tr>
<td>(\frac{9}{2}^+ \rightarrow \frac{11}{2}^+)</td>
<td>2.2464</td>
<td>2.3280 ± 0.0645</td>
</tr>
<tr>
<td>(\frac{11}{2}^+ \rightarrow \frac{15}{2}^+)</td>
<td>2.2568</td>
<td>2.1080 ± 0.1433</td>
</tr>
<tr>
<td>(\frac{13}{2}^+ \rightarrow \frac{15}{2}^+)</td>
<td>1.4542</td>
<td>1.9867 ± 0.1316</td>
</tr>
<tr>
<td>(\frac{1}{2}^+ \rightarrow \frac{3}{2}^+)</td>
<td>0.0296</td>
<td>0.0256</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(J^+ \rightarrow (J+1)^+)</th>
<th>Theo. ((e^2b^2))</th>
<th>Exp. ((e^2b^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{5}{2}^+ \rightarrow \frac{7}{2}^+)</td>
<td>2.9988</td>
<td>2.8038 ± 0.1458</td>
</tr>
<tr>
<td>(\frac{7}{2}^+ \rightarrow \frac{9}{2}^+)</td>
<td>1.6914</td>
<td>1.5691 ± 0.3411</td>
</tr>
<tr>
<td>(\frac{7}{2}^+ \rightarrow \frac{9}{2}^+)</td>
<td>1.0471</td>
<td>0.7483 ± 0.0831</td>
</tr>
<tr>
<td>(\frac{9}{2}^+ \rightarrow \frac{11}{2}^+)</td>
<td>0.7084</td>
<td>0.6877 ± 0.0675</td>
</tr>
<tr>
<td>(\frac{11}{2}^+ \rightarrow \frac{13}{2}^+)</td>
<td>0.5201</td>
<td>0.3761 ± 0.0477</td>
</tr>
<tr>
<td>(\frac{13}{2}^+ \rightarrow \frac{15}{2}^+)</td>
<td>0.3726</td>
<td>0.4386 ± 0.0760</td>
</tr>
</tbody>
</table>

The role assigned to nucleons in unique parity orbitals, whose contribution to the quadrupole moments is parameterized in this way.

Calculated B(E2) values are given in units of \(e^2b^2\). Most of the transition strengths are reproduced within the experimental error bars. One exception is the transition \(17/2_1 \rightarrow 13/2_1\) which is underestimated. This could be related with the change in the wavefunction of the first \(15/2\), whose mixing with the second \(17/2\) state seems to be exaggerated in the model, as discussed in [16].

Notice that while the intraband B(E2) transition strengths are on the order of \(e^2b^2\), the interband transitions are much less, typically on the order of \(e^2b^2 \times 10^{-2}\) or fractions thereof [17]. This fact supports the identification of states belonging to bands, and is consistent with the wavefunction analysis. The only measured interband transition is the \(1/2_1 \rightarrow 5/2_1\) and it too is well reproduced by the model.

7. Conclusions

A detailed analysis of the energy spectra of five nuclei \(^{159}\)Tb, \(^{158}\)Eu, \(^{156}\)Dy, \(^{161}\)Tb and \(^{161}\)Tm) has been given. The results demonstrate that the normal parity bands in odd-mass heavy deformed nuclei can be described quantitatively using the pseudo SU(3) model.

The most relevant feature of the present application of the pseudo SU(3) model is a determination of the primary features of the energy spectra of the normal parity rotational bands in \(A = 159\) and \(A = 161\) nuclei using a Hamiltonian with Nilsson single-particle energies and quadrupole-quadrupole and pairing interactions with strengths fixed by systematics—strengths of the primary interactions were not varied to obtain a "best fit" to the date. Two extra rotor-like terms were used to obtain a more precise description of the energies and B(E2) values, but this "fine tuning" did not affect the spectra in a major way and had little influence on the structure of the calculated wavefunctions.

This report exhibits the usefulness of the pseudo SU(3) model as a microscopic shell model theory, one that can be used to describe deformed rare-earth and actinide isotopes by performing a symmetry dictated truncation of the Hilbert space. It opens up the possibility of a more detailed microscopic description of other properties of heavy deformed nuclei, both with even and odd protons and neutrons numbers, like \(g\)-factors, M1 transitions, and beta decays.

Acknowledgments

This work was supported in part by Conacyt (México) and the U.S. National Science Foundation, the latter being through Grants 9500474 and 9603006, as well as an NSF Cooperative Agreement, 9720652, that includes matching from the Louisiana Board of Regents Support Fund.


