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IDENTICAL BANDS IN DEFORMED NUCLEI

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ABSTRACT

A quantum rotor with spin, where the rotation is about the $L$-axis rather than the $J$-axis, is offered for an interpretation of identical bands and spin alignment phenomena in strongly deformed nuclei. An application of the theory to the superdeformed bands in $^{192}$Hg and $^{194}$Hg is presented.

1. Introduction

A quantum rotor with intrinsic spin $S$ displays one of two different characteristic forms depending upon whether the system rotates about the $L$-axis or the $J$-axis, where $J=L+S$. The most familiar of these two schemes – called the $J$-rotor in this paper – takes the angular momentum of the rotor to be the total angular momentum. The $J$-rotor model has been used since the earliest days of quantum mechanics to interpret diverse rotational phenomena in both physics and chemistry. The other scheme – called the $L$-rotor in this paper – is less well studied because its relevance has not been fully realized. In this case the rotation is about the $L$-axis rather than the $J$-axis. This $L$-rotor picture is an appropriate scheme for strongly deformed rare-earth and actinide nuclei when the pseudo-spin coupling scheme is employed. Within this framework, identical bands are pseudo-orbital angular momentum sequences associated with different projections of the same pseudo-spin.

2. $L$-Rotator Model

Consider a quantum rotor with core angular momentum $L$, intrinsic spin $S$, and total angular momentum $J=L+S$ (Figure 1a). The Hamiltonian for this system is

$$ H = \sum_{\alpha} a_{\alpha} L_{\alpha}^2 + b L \cdot S + c S^2, \quad (1) $$

when a self-interaction term generated by $L \cdot S$ is included and where $a_{\alpha}$ is the rotational inertial parameter about the intrinsic $\alpha$-th axis (equal to $\frac{1}{2} I_{\alpha}$ where $I_{\alpha}$ is the corresponding moment of inertia). For a fixed-spin system the $S^2$ term is a constant. The second term can be replaced by the $J^2$ operator, since $L \cdot S = \frac{1}{2} (J^2 - L^2 - S^2)$. For the case of an axially symmetric rotor ($a_1 = a_2 = a \neq a_3$) this Hamiltonian reduces to

$$ H = a L^2 + (a_3 - a) L_3^2 + \frac{b}{2} (J^2 - L^2 - S^2) + c S^2. \quad (2) $$

The energies of this elementary system are given by the simple result

$$ E(L,S,J) = (a - \frac{b}{2}) L(L+1) + (a_3 - a) K L^2 + \frac{b}{2} J(J+1) + (c - \frac{b}{2}) S(S+1), \quad (3) $$
where $K_L$ is the eigenvalue of $L_z$. The corresponding eigenstates have the form $\{|\gamma K_L S L M\rangle\}$ where $\gamma$ is a running integer index for distinguishing multiple occurrences of a given $K_L$ and $L$ combination and $M$ is the projection of $J$ on the laboratory-fixed $z$-axis. This $LS$-coupled scheme differs from the eigenstates $|\gamma K L S K'M\rangle$ of the $J$-rotor which rather than $K_L$ and $L$ have the projections $K_S$ of $S$ and $K$ of $J$ ($K = K_L + K_S$) on the intrinsic symmetric axis of the system as good quantum numbers (Figure 1b).

The structure of the $L$-rotor is better revealed when a band label $D = J - L$ with values $D = S, S-1, \ldots, -S$ is introduced. The $E(L,S)J$ of (3) can then be rewritten as

$$E(L,S)J = a(L+1) + b LD + (a_1 - a) K_L^2 + \frac{b}{2} D(D+1) + (c - \frac{b}{2}) S(S+1).$$

(4)

The quantity $D$ indicates the (spin) alignment of the band relative to the reference ($D = 0$) band. For $S = 1$, there are three bands with alignments $D = 1, 0$ (reference), and $-1$. When $S = \frac{1}{2}$, only two bands exist with half-integer alignments relative to the reference structure; in nuclear physics, these would be the odd-$A$ neighbor of an even-even parent. In general there are $2S+1$ bands with total angular momentum $J = J_{\text{min}}, J_{\text{min}}+2, \ldots$ when $K_L=0$ and $J = J_{\text{min}}, J_{\text{min}}+1, \ldots$ when $K_L\neq0$. The $J_{\text{min}}$ values are rather complicated functions of $K_L, S, D$, and $x = \text{mod}(L,2)$: $J_{\text{min}} = D + x + 2 [(S-D+3-2x)/4]$ for $K_L=0$ and $J_{\text{min}} = D + K_L + \max(0,[(S-D-2K_L+1)/2])$ for $K_L\neq0$ where $[w]$ is the usual greatest integer function. If the coefficient $b$ of $L^2$ is positive (negative), the $D=S$ band lies highest (lowest) while the $D=S$ band lies lowest (highest) for a given $L$ value. For the special case when $b = 2a$, (3) reduces to

$$E(L,S)J = a J(J+1) + (a_1 - a) K_L^2 + (c - a) S(S+1)$$

(3a)

which gives the energies of a $J$-rotor with the projection governed by $K_L$ rather than $K$. Since the projection quantum numbers are not measurable, when $b=2a$ it is impossible to distinguish the $L$-rotor and the $J$-rotor pictures based solely on excitation energies.

Stretched intraband (interband) electric quadrupole (E2) transitions in deformed nuclei are strongly enhanced (inhibited). All E2 transitions with $\Delta D\neq0$ must therefore be strongly suppressed if the nuclear picture is to describe nuclear physics phenomena. The de-excitation energies within a band (assuming $\Delta L=2$ transitions and a prolate rotor geometry) are given from (4) as

$$\Delta E(LD)(a,b) = E(L+1,S)J+2 - E(L,S)J = 2a(2L+3) + 2bS$$

(5)

or for the special $b=2a$ case from (3a),

$$\Delta E(a) = E(L+2,S)J+2 - E(L,S)J = 2a(2J+3).$$

(5a)

A plot of intraband $\Delta E(LD)(a,b)$ versus $L$ values therefore yields a set of points that lie on lines having identical slopes (4a) but with different intercepts ($6a+2bD$). The E2 selection rules are obtained from the expression

$$B(E2; J \rightarrow J) = \frac{1}{2J'+1} |\langle \gamma' K_L S L J | Q^2 | \gamma K_L' L'S'J' \rangle|^2$$

(6)

where $Q^2$ is the electric quadrupole operator and

$$\langle \gamma K_L S L J | Q^2 | \gamma' K_L' L'S'J' \rangle = \delta_{S'S'} \sqrt{(2J'+1)(2J+1)} W(SJ;L'2J') | \langle \gamma K_L | Q^2 | \gamma' K_L' \rangle |^2 \cdot [S'(J'+J)+1]$$

(7)
The $W$-function in (7) denotes a Racah recoupling coefficient. For intraband $B(E2)$ transitions, this equation reduces to

$$B(E2; J' \to J) = \frac{2S}{4\pi} (2J+1)(2L+1)(2L'+1)$$

$$\times \left| \begin{array}{cc} L & 2 \times L' \times W(SJ'2; L'J) \times Z^2 \beta^2 \times W_{\mu,\nu} \end{array} \right|^2$$

where $Z$ is the atomic number and $\beta^{(0)}$ the usual collective model deformation parameter which in this case may be spin dependent. Representative results are displayed in Figure 2. Note that as the value of $J$ increases, the intraband $B(E2)$ transition strengths for the same $J$ values for the $D=1$ and $D=+1$ bands become equal.

Figure 2. $B(E2)$ strengths for a $K_L=0$ and $S=0$ $L$-rotor and the corresponding $S=1$ series when $b=2a$. Transition strengths between eigenstates with $L < 16 \hbar$ are shown with values less than 0.001 $Z^2 \beta^2$ $W_{\mu,\nu}$ suppressed for clarity.

Figure 3. Various coupling scenarios for interacting $L$-rotors. a) The $JJ$ scheme in which the rotational angular momentum of the protons couples with its intrinsic spin, and this couples to corresponding neutrons vectors. b) The $LS$ coupling scheme in which the rotational angular momenta of protons and of neutrons are coupled first, followed by coupling with the intrinsic spins. c) The $SU(3)$ strong-coupling scheme in which the coupling of $SU(3)$ quantum number for protons and neutrons is done first and then (as in the $L$-$S$ scheme) this is followed by coupling to the intrinsic spin.
3. Pseudo-spin Realization

An L-rotor picture emerges within the context of a many-body theory whenever space-like and spin-like degrees of freedom decouple. The pseudo-spin coupling scheme for heavy deformed nuclei is an example. In this case, replacing the single-particle orbital angular momentum \( l \) and spin \( s \) operators by their pseudo-orbital \( \vec{l} \) and pseudo-spin \( \vec{s} \) counterparts transforms the one-body orbit-orbit (\( \nu \nu \)) and spin-orbit (\( \nu \omega \)) interactions into their pseudo counterparts \( \nu \nu' \omega \) and \( \nu \nu' \omega' \). As a consequence of the fact that \( \nu \nu' = 0 \), the many-body pseudo-spin \( \vec{s} \) is a good quantum number. Furthermore, this transformation carries the deformation inducing quadrupole-quadrupole interaction \( \langle Q | Q \rangle \), which dominates the residual interaction, into its pseudo (quadrupole-quadrupole) counterpart \( \langle \vec{\Delta} | \vec{\Delta} \rangle \) with (at most) small correction terms. Hence, the L-rotor picture applies for heavy deformed nuclei under the replacement \( L \rightarrow \vec{L} \) and \( S \rightarrow \vec{s} \).

Valence protons and neutrons in heavy nuclei fill different major shells. Therefore, an appropriate model for this case is two interacting L-rotors, one representing the protons and the other the neutrons. (The superposition of these two rotor configurations does not violate the Pauli Exclusion Principle because they refer to different particle types.) The model can assume various forms depending upon whether \( \vec{L}_\pi \) of the protons and \( \vec{L}_\nu \) of neutrons first couple to their own spins \( \vec{J} = (\vec{L}_\pi + \vec{S}_\pi) + (\vec{L}_\nu + \vec{S}_\nu) = \vec{J}_\pi + \vec{J}_\nu \) or if they first couple with each other \( \vec{J} = (\vec{L}_\pi + \vec{L}_\nu) + (\vec{S}_\pi + \vec{S}_\nu) = \vec{L} + \vec{S} \). Since the proton-neutron quadrupole-quadrupole field favors a maximally deformed product configuration, the second scenario is preferred. In the pseudo-spin scheme, this is accomplished through strong coupling of pseudo-SU(3) representations, \( (\vec{\alpha}_\pi \vec{\alpha}_\nu) \times (\lambda_\pi \lambda_\nu) \rightarrow (\lambda \vec{\mu}) \). The pseudo-whips (\( \vec{S}_\pi \) and \( \vec{S}_\nu \)) are coupled to total \( \vec{S} \) in the usual way (see Figure 3).

4. Superdeformed Bands in \( ^{192}\text{Hg} \) and \( ^{194}\text{Hg} \)

Rotational bands in several deformed nuclei have been found to have identical (within \( \pm 2\% \) for normal deformed nuclei and \( \pm 1\% \) for superdeformed nuclei) transition energies. For example, certain superdeformed bands in \( ^{194}\text{Hg} \) appear to be nearly identical to a superdeformed band in \( ^{192}\text{Hg} \). The pseudo-spin scheme can be applied by assigning \( \vec{s} = 0 \) to the superdeformed band in \( ^{192}\text{Hg} \), \( \vec{s} = 0 \) to superdeformed band (1) in \( ^{194}\text{Hg} \), and \( \vec{s} = 1 \) to superdeformed bands (2) and (3) in \( ^{194}\text{Hg} \). In this case the spin alignment \( D \) results from neutrons aligning, and the bands can have distinct deformation parameters: \( \beta(0)(^{192}\text{Hg}) \), \( \beta(0)(^{194}\text{Hg}) \), and \( \beta(1)(^{194}\text{Hg}) \). These deformation parameters can be determined by fitting \( B(E2) \) strengths, or derived microscopically using appropriate pseudo-SU(3) configurations. This letter follows the assignment for final total angular momentum \( J_f \) given in the Reference 13.

The \( L \)-rotor picture predicts three \( \vec{D} \)-bands for an \( \vec{s} = 1 \) configuration; however, some of the bands may not be separately distinguishable. For example, if \( b = 2a \) the transition energies of the \( \vec{D} = 1 \) band match those of the \( \vec{D} = -1 \) band identically. Since the \( B(E2) \) strengths are also equal for large \( J \) values (see Figure 2), the transition strength for states belonging to the \( \vec{D} = \pm 1 \) pair appear to be twice as strong as for the \( \vec{D} = 0 \) band, a result that appears to be in agreement with the experiment.

Equation (5) was fit to each of the four superdeformed Hg bands for \( L \) values in the range \( L = 10 \) to \( 38 \), corresponding to \( E_Y = 255\text{--}735 \text{ keV} \) where the complete experimental results are available. The bands have nearly the same inertial parameter \( a = 4.5 \text{ keV} \). The \( L \)-\( S \) self-interaction strength for \( ^{194}\text{Hg} \) can be deduced from the energy shift of band (2) compared to the reference band (3), and is \( b = 9.0 \text{ keV} \). Since the kinetic \((\hbar^2 \mu / 2I)\) and dynamic \((\hbar^2 / 2I) \vec{L} \cdot \vec{L} \) moments of inertia are not precisely equal and the transition energies not equally spaced (as opposed to the case of rigid rotors), one cannot deduce unambiguously the intercept of the \( \Delta \vec{E} \) versus \( L \) curves. However, if a constant \( k \) replaces the 6a term in (5), this constant \( k \) turns out to be the same as for the same \( \vec{s} \) value (see Figure 4). Within the framework of this model, the integer alignment in the Hg isotopes occurs as a consequence of the \( b = 2a \) condition.

**Figure 4.** Transition energies \( E_Y \) for an \( L \)-rotor system with \( \vec{D} \)-band assignments. The \( L \)-rotor parameters are \( a = 4.5 \text{ keV} \), \( b = 9.0 \text{ keV} \), and \( k = 92 \text{ keV} \) and 74 keV for \( \vec{s} = 0 \) and \( \vec{s} = 1 \), respectively. The top picket-fence structure (heavy lines) for each pair gives the experimental results for superdeformed bands in the specified Hg isotopes while the bottom one (light lines) is the \( L \)-rotor description.
5. Conclusions

An $L$-rotor picture is important when deformed configurations are favored and the coupling between spatial and spin degrees of freedom is weak. The model gives rise to $L(L+1)$ rotational sequences associated with each of the $(2S+1)$ spin orientations identified by band label $D=J-L$. The $L\cdot S$ coupling determines deviations from the structure of the reference band ($D=0$) – which can be significant for low $L$ values but is negligible for high $L$ values – and its strength can be extracted by examining energy shifts in the transition spectra. A value $b=2a$ for the $L\cdot S$ strength leads to integer alignment, and this produces $J(J+1)$ rotational sequences as well.

The appearance of identical superdeformed bands in heavy nuclei has been shown to be consistent with an $L$-rotor picture which emerges naturally within the context of good pseudo-spin symmetry and its pseudo-$SU(3)$ extension. The occurrence of only $\tilde{S}+1$ ($\tilde{S}$) numbers of $\tilde{D}$ bands for integer (half-integer) pseudo-spin nuclei, instead of the expected $2\tilde{S}+1$ distinct bands, occurs for $b=2a$. This band degeneracy doubles the $B(E2)$ transition strengths of $\tilde{D}\neq 0$ bands with respect to those of the reference ($\tilde{D}=0$) band to the same pseudo-spin multiplet. Hence, the measurements of $B(E2)$ rates for heavy deformed nuclei are very crucial to justify this model. Deviations from this simple picture, such as differences in the kinematic and dynamic moments of inertia, are expected to teach us additional physics concerning, for example, the alignment of intruder level particles.

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7. References