1. \textit{Sp(4)} classification scheme

The \textit{sp}(4) algebra is realized in terms of creation and annihilation fermion operators with the standard anticommutation relations \( \{c_{j,m,s}, c_{j',m',s'}\} = \delta_{j,j'} \delta_{m,m'} \delta_{s,s'} \), where these operators create (annihilate) a particle of type \( \sigma = \pm 1/2 \) (proton/neutron) in a state of total angular momentum \( j \) (half-integer) with projection \( m \). For \( p \) orbits the dimension of the shell is \( 2\Omega = \Sigma_j (2j + 1) \). In addition to the number operator \( N = N_1 + N_{-1} \) and the isospin projection \( T_0 = (N_1 - N_{-1})/2 \), the generators of \textit{Sp}(4) are \( T_{\pm} = \frac{1}{2\sqrt{\Omega}} \sum_{jm} c_{jm,\pm 1/2} c_{jm,\mp 1/2}^\dagger \), \( A_\mu = \sum_{jm} (-1)^j m c_{jm,\sigma} c_{jm,-\sigma}^\dagger \), \( A_{\pm} = (A_{\mu})^\dagger \). The operators, \( T_0 \) and \( T_{\pm} \), close the \textit{su}(2) subalgebra of the valence isospin, and the generators, \( A_{\pm 0}(A_{0,\pm 1}) \), create (annihilate) a pair of total angular momentum \( J^\pi = 0^+ \) and isospin \( T = 1 \). The basis states are constructed as \( (T = 1) \)-paired fermions \( |n_1, n_0, n_{-1}\rangle = (A_{1}^\dagger)^{n_1} (A_0^\dagger)^{n_0} (A_{-1}^\dagger)^{n_{-1}} |0\rangle \), where \( n_1, n_0, n_{-1} \) are the total number of pairs of each kind, \( pp, pn, nn \), respectively.

The model provides for a classification scheme of nuclei and their \textit{isovector-paired} \( 0^+ \) states (Table 1). The total number of the valence particles, \( N \), enumerates the rows and the valence isospin projection, \( T_0 \), enumerates the columns. The shape of the table is symmetric with respect to the sign of \( T_0 \) and \( N = 2\Omega \). Isotopes (isotones) of an element are situated along the right (left) diagonals.
Table 1. Classification scheme of nuclei with a \(^{40}\text{Ca}\) core (\(\Omega_{J=7/2} = 4\)).

<table>
<thead>
<tr>
<th>(N/T_0)</th>
<th>0</th>
<th>-1</th>
<th>-2</th>
<th>-3</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(^{40}\text{Ca}_{20})</td>
<td>(^{42}\text{Sc}_{21})</td>
<td>(^{44}\text{Ti}_{22})</td>
<td>(^{46}\text{V}_{23})</td>
<td>(^{48}\text{Cr}_{24})</td>
</tr>
<tr>
<td>2</td>
<td>(^{42}\text{Sc}_{21})</td>
<td>(^{44}\text{Ti}_{22})</td>
<td>(^{46}\text{V}_{23})</td>
<td>(^{48}\text{Cr}_{24})</td>
<td>(^{50}\text{Ca}_{25})</td>
</tr>
<tr>
<td>4</td>
<td>(^{44}\text{Ti}_{22})</td>
<td>(^{46}\text{V}_{23})</td>
<td>(^{48}\text{Cr}_{24})</td>
<td>(^{50}\text{Ca}_{25})</td>
<td>(^{52}\text{Ca}_{26})</td>
</tr>
<tr>
<td>6</td>
<td>(^{46}\text{V}_{23})</td>
<td>(^{48}\text{Cr}_{24})</td>
<td>(^{50}\text{Ca}_{25})</td>
<td>(^{52}\text{Ca}_{26})</td>
<td>(^{54}\text{Ca}_{27})</td>
</tr>
</tbody>
</table>

2. Discrete derivatives with respect to \(N_+\) and \(N_-\).

The isospin breaking model Hamiltonian\(^1\) includes isovector \((T = 1)\) pairing interaction, diagonal isoscalar \((T = 0)\) force and a symmetry term

\[
H = -\epsilon N - GA_0 A_0 - F(A_{+1} A_{+1} + A_{-1} A_{-1}) - \frac{E}{2\Omega} (T^2 - \frac{3N}{4}) - C N(N-1) - (D - \frac{E}{2\Omega})(T_0^2 - \frac{N}{4}).
\]  

A fit of the eigenvalues of the Hamiltonian (1) to the experimental energies of the lowest isovector-paired \(0^+\) state (the ground state for even-even and some \([N \approx Z]\) odd-odd nuclei and the isobaric analog excited state for the rest odd-odd nuclei) leads to a good theoretical prediction of the relevant energies\(^1\). Different types of discrete derivatives of the energy function\(^2,3\) reveal additional features of the nuclear interaction and provides for a good test of the model.

The first discrete derivative, \(\partial E/\partial N_+\), of the energy of the relevant isovector-paired \(0^+\) states represents the two-proton separation energy, \(S_{2p} = (E(N_+ + 2) - E(N_+))/2\), for even-even nuclei (Figure 1). It increases with increasing of \(N_-\) and deviations from its smooth behavior occur at \(N_+ = N_-\). The second discrete derivative of the energy\(^2,3\), \(\delta V_{pn} = (E(N_+ + 2, N_- + 2) - E(N_+ + 2, N_-) - E(N_+, N_- + 2) + E(N_+, N_-))/4\), represents the residual interaction between the last proton and the last neutron. It has a peak at \(N_+ = N_-\). Both theoretical derivatives agree remarkably with the experiment, especially in reproducing the typical behavior at \(N_+ = N_-\). They provide for a reasonable prediction of the energy difference of proton-rich exotic nuclei and odd-odd nuclei.

3. Discrete derivatives with respect to \(N\) and \(T_0\) and staggering behavior.

A staggering behavior is observed when discrete derivatives with respect to \(N\) and \(T_0\) are considered. They give a relation between even-even and
odd-odd nuclei. The first discrete derivative, $\partial E / \partial N$, of the relevant energies (Figure 2(a)) is the energy difference $(E(N+2) - E(N))/2$ and it accounts for a decrease in the like-particle pairing energy in the odd-odd nuclei due to the blocking effect and an increase in energy due to the $\text{pn}$ pairing, in addition to the change in energy due to the different isospin values (symmetry term).

The second discrete derivative of the energy, $\partial^2 E / \partial N \partial T_0$, is $\delta^{(2)} = (E(N+2,T_0+1) - E(N+2,T_0) - E(N,T_0+1) + E(N,T_0))/2$ (Figure 2(b)). Its positive (negative) value at even-even (odd-odd) nuclei is a consequence of the type of pairing and the different isospin values of the considered states. The staggering amplitude of both derivatives are the same for all $N$ in a $T_0$-multiplet (Figure 3).
4. Conclusions

A dynamical $Sp(4)$ symmetry was used to describe pairing correlations. The energies of the lowest isovector-paired $0^+$ states were fit. It lead to a very good prediction of the relevant energies and an estimation of the interaction strength parameters. The first and second discrete derivatives of the energies were found to agree remarkably with the experiment. The partial discrete derivatives with respect to $N_+$ and $N_-$ give relations only between even-even (odd-odd) nuclei. They do not show a staggering behavior but give some insight into the two-proton (two-neutron) separation energies and the $pn$ correlations. A deviation from their smooth behavior occurs at $N_+ = N_-$. The partial discrete derivatives with respect to $N$ and $T_0$ describe the relative behavior between even-even and odd-odd nuclides and show a prominent staggering behavior. The amplitude of the staggering increases with the difference between protons and neutrons and is the same for all nuclei in one $T_0$-multiplet. In this way, the dependence of the nuclear energy on the isospin projection is more complicated than on the total number of particles.

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References