Formula Sheet for LSU Physics 2113, Final Exam, Spring '15

- **Constants, definitions:**

$g = 9.8\,\frac{\text{m}}{\text{s}^2}$ $R_{Earth} = 6.37\times10^{6}\,\text{m}$ $M_{Earth} = 5.98\times10^{24}\,\text{kg}$

$G = 6.67\times10^{-11}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$ $R_{Moon} = 1.74\times10^{6}\,\text{m}$ Earth-Sun distance = 1.50×10^{11} m

$M_{Sun} = 1.99\times10^{30}\,\text{kg}$ $M_{Moon} = 7.36\times10^{22}\,\text{kg}$ Earth-Moon distance = 3.82×10^{8} m

$\epsilon_o = 8.85\times10^{-12}\,\frac{\text{C}^2}{\text{Nm}^2}$ $k = \frac{1}{4\pi\epsilon_o} = 8.99\times10^{9}\,\frac{\text{Nm}^2}{\text{C}^2}$ $e = 1.60\times10^{-19}\,\text{C}$

$c = 3.00\times10^{8}\,\text{m/s}$ $m_p = 1.67\times10^{-27}\,\text{kg}$ $1\text{ eV} = e(1\text{V}) = 1.60\times10^{-19}\,\text{J}$

dipole moment: $\vec{p} = q\vec{d}$ $m_e = 9.11\times10^{-31}\,\text{kg}$ charge densities: $\lambda = \frac{Q}{L}$, $\sigma = \frac{Q}{A}$, $\rho = \frac{Q}{V}$

Area of a circle: $A = \pi r^2$ Area of a sphere: $A = 4\pi r^2$ $\mu_0 = 4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}$

Area of a cylinder: $A = 2\pi r\ell$ Volume of a cylinder: $V = \pi r^2\ell$ Volume of a sphere: $V = \frac{4}{3}\pi r^3$

- **Kinematics (constant acceleration):**

$v = v_o + at$ $x - x_o = \frac{1}{2}(v_o + v)t$ $x - x_o = v_o t + \frac{1}{2}at^2$ $v^2 = v_o^2 + 2a(x - x_o)$

- **Circular motion:** $F_c = ma_c = \frac{mv^2}{r}$, $T = \frac{2\pi r}{v}$, $v = \omega r$

- **General (work, def. of potential energy, kinetic energy):**

$K = \frac{1}{2}mv^2$ $\vec{F}_{\text{net}} = m\vec{a}$ $E_{\text{mech}} = K + U$

$W = -\Delta U$ (by field) $W_{ext} = \Delta U$ (if objects are initially and finally at rest)

- **Gravity:**

Newton's law: $|\vec{F}| = G\frac{m_1 m_2}{r^2}$ Gravitational acceleration (planet of mass M): $a_g = \frac{GM}{r^2}$

Gravitational Field: $\vec{g} = -G\frac{M}{r^2}\hat{r} = -\frac{dV_g}{dr}$ Gravitational potential: $V_g = -\frac{GM}{r}$

Law of periods: $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$ Potential Energy: $U = -G\frac{m_1 m_2}{r_{12}}$

Potential Energy of a System (more than 2 masses): $U = -\left(G\frac{m_1 m_2}{r_{12}} + G\frac{m_1 m_3}{r_{13}} + G\frac{m_2 m_3}{r_{23}} + \ldots\right)$

Gauss' law for gravity: $\oint_S \vec{g}\cdot d\vec{S} = -4\pi G M_{ins}$

- **Electrostatics:**

Coulomb's law: $|\vec{F}| = k\frac{|q_1||q_2|}{r^2}$ Force on a charge in an electric field: $\vec{F} = q\vec{E}$

Electric field of a point charge: $|\vec{E}| = k\frac{|q|}{r^2}$

Electric field of a dipole on axis, far away from dipole: $\vec{E} = \frac{2k\vec{p}}{z^3}$

Electric field of an infinite line charge: $|\vec{E}| = \frac{2k\lambda}{r}$

Electric field at the center of uniformly charged arc of angle ϕ: $|\vec{E}| = \frac{\lambda\sin(\phi/2)}{2\pi\epsilon_0 R}$

Torque on a dipole in an $\vec{E}$ field: $\vec{\tau} = \vec{p}\times\vec{E}$, Potential energy of a dipole in $\vec{E}$ field: $U = -\vec{p}\cdot\vec{E}$

- **Electric flux:** $\Phi = \int \vec{E}\cdot d\vec{A}$
- **Gauss' law:** $\epsilon_o \oint \vec{E}\cdot d\vec{A} = q_{enc}$

- **Electric field of an infinite non-conducting plane with a charge density σ:** $E = \frac{\sigma}{2\epsilon_o}$

- **Electric field of infinite conducting plane or close to the surface of a conductor:** $E = \frac{\sigma}{\epsilon_o}$

- **Electric potential, potential energy, and work:**

  $V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$ In a uniform field: $\Delta V = -\vec{E}\cdot\Delta\vec{s} = -Ed\cos\theta$

  $\vec{E} = -\vec{\nabla}V$, $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$

  Potential of a point charge q: $V = k\frac{q}{r}$ Potential of n point charges: $V = \sum_{i=1}^{n} V_i = k\sum_{i=1}^{n}\frac{q_i}{r_i}$

  Electric potential energy: $\Delta U = q\Delta V$ $\Delta U = -W_{\text{field}}$

  Potential energy of two point charges: $U_{12} = W_{\text{ext}} = q_2V_1 = q_1V_2 = k\frac{q_1q_2}{r_{12}}$

- **Capacitance: definition:** $q = CV$

  Capacitor with a dielectric: $C = \kappa C_{air}$ Parallel plate: $C = \varepsilon_\circ\frac{A}{d}$

  Potential Energy in Cap: $U = \frac{q^2}{2C} = \frac{1}{2}qV = \frac{1}{2}CV^2$ Energy density of electric field: $u = \frac{1}{2}\kappa\varepsilon_o|\vec{E}|^2$

  Capacitors in parallel: $C_{eq} = \sum C_i$ Capacitors in series: $\frac{1}{C_{eq}} = \sum\frac{1}{C_i}$

- **Current:** $i = \frac{dq}{dt} = \int\vec{J}\cdot d\vec{A}$, Const. curr. density: $J = \frac{i}{A}$, Charge carrier's drift speed: $\vec{v}_d = \frac{\vec{J}}{ne}$

- **Definition of resistance:** $R = \frac{V}{i}$ Definition of resistivity: $\rho = \frac{|\vec{E}|}{|\vec{J}|}$

- **Resistance in a conducting wire:** $R = \rho\frac{L}{A}$ Temperature dependence: $\rho - \rho_\circ = \rho_\circ\alpha(T - T_\circ)$

- **Power in an electrical device:** $P = iV$ Power dissipated in a resistor: $P = i^2R = \frac{V^2}{R}$

- **Definition of** *emf* : $\mathcal{E} = \frac{dW}{dq}$

- **Resistors in series:** $R_{eq} = \sum R_i$ Resistors in parallel: $\frac{1}{R_{eq}} = \sum\frac{1}{R_i}$

- **RC circuit:** Charging: $q(t) = C\mathcal{E}(1 - e^{-\frac{t}{\tau_c}})$, Time constant $\tau_C = RC$, Discharging: $q(t) = q_oe^{-\frac{t}{\tau_c}}$

- **Magnetic Fields:**

  Magnetic force on a charge q: $\vec{F} = q\vec{v}\times\vec{B}$ Lorentz force: $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

  Circular motion in a magnetic field: $r = \frac{mv}{qB}$ with period: $T = \frac{2\pi m}{qB}$

  Magnetic force on a straight length of wire: $\vec{F} = i\vec{L}\times\vec{B}$

  Magnetic Dipole: $\vec{\mu} = Ni\vec{A}$ Torque: $\vec{\tau} = \vec{\mu}\times\vec{B}$ Potential energy: $U = -\vec{\mu}\cdot\vec{B}$

- **Generating Magnetic Fields:** Biot-Savart Law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\vec{r}}{r^3}$

  Magnetic field of a long straight wire: $B = \frac{\mu_0}{4\pi}\frac{2i}{r}$ Magnetic field of a circular arc: $B = \frac{\mu_0}{4\pi}\frac{i}{r}\phi$

  Force between parallel current carrying wires: $F_{ab} = \frac{\mu_0 i_a i_b}{2\pi d}L$

  Ampere's law: $\oint\vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$

  Magnetic field of a solenoid: $B = \mu_0 in$ Magnetic field of a dipole on axis, far away: $\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$

- **Induction:**

  Magnetic Flux: $\Phi = \int \vec{B} \cdot d\vec{A}$

  Faraday's law: $\mathcal{E} = -\frac{d\Phi}{dt}$ $\left(= -N\frac{d\Phi}{dt}\right.$ for a coil with N turns$)$ **Motional emf:** $\mathcal{E} = BLv$

  Induced Electric Field: $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt}$

  Definition of Self-Inductance: $L = \frac{N\Phi}{i}$ **Inductance of a solenoid:** $L = \mu_0 n^2 Al$

  EMF (Voltage) across an inductor: $\mathcal{E} = -L\frac{di}{dt}$

  RL Circuit: Rise of current: $i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{tR}{L}}\right)$, Time constant: $\tau_L = \frac{L}{R}$, **Decay** of current: $i = i_0 e^{-\frac{tR}{L}}$

  Magnetic Energy: $U_B = \frac{1}{2}Li^2$ **Magnetic energy density:** $u_B = \frac{B^2}{2\mu_0}$

- **LC circuits:**

  Electric energy in a capacitor: $U_E = \frac{q^2}{2C} = \frac{CV^2}{2}$ **Magnetic energy in an inductor:** $U_B = \frac{Li^2}{2}$

  LC circuit oscillations: $q = Q\cos(\omega t + \phi)$ $\quad \left(i = \frac{dq}{dt}, \quad q = Cv\right)$ $\quad \omega = \frac{1}{\sqrt{LC}}$ $\quad T = \frac{2\pi}{\omega}$ $\quad f = \frac{1}{T}$

- **Series RLC circuit:** $q(t) = Qe^{-Rt/(2L)}\cos(\omega' t + \phi)$ where $\omega' = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2}$

- **Transformers:**

  Transformation of voltage: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ **Turns ratio:** $\frac{N_p}{N_s}$ **Energy conservation:** $I_p V_p = I_s V_s$

- **Maxwell's Equations:**

  $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ $\quad \oint \vec{B} \cdot d\vec{A} = 0$ $\quad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ $\quad \oint \vec{B} \cdot d\vec{s} = \mu_0\epsilon_0\frac{d\Phi_E}{dt} + \mu_0 i_{enc}$

  Displacement current: $i_d = \epsilon_0 \frac{d\Phi_E}{dt}$

- **Electromagnetic Waves:**

  Wave traveling in +x direction: $E = E_m \sin(kx - \omega t)$ $\quad B = B_m \sin(kx - \omega t)$

  where $\vec{E} \perp \vec{B}$, the direction of travel is $\vec{E} \times \vec{B}$, $E_m/B_m = c$, $f\lambda = c$, $\lambda = 2\pi/k$

  Velocity of light in vacuum $= c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$

  Energy flow: $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ $\quad I = \frac{1}{2c\mu_0}E_m^2$ $\quad E_{rms} = \frac{E_m}{\sqrt{2}}$ $\quad I = \frac{P}{Area}$

  Radiation force and pressure: total absorption: $F_r = \frac{IA}{c}$, $p_r = \frac{I}{c}$ total reflection: $F_r = \frac{2IA}{c}$, $p_r = \frac{2I}{c}$

- **Polarizing Sheets:**

  Unpolarized → polarized: $I = \frac{1}{2}I_0$ **Polarized → polarized:** $I = I_0\cos^2\theta$