Exam 2: Physics 2113 Spring 2015 **8:00 PM, Monday, 2 March 2015**

Last Name (print) KEY **First Name (print)**____________________

Clearly circle your section:

MON/WEDS/FRI SECTIONS				TUES/THURS SECTIONS	
M. Gaarde 10:30 AM	J. Dowling 12:30 PM	J. Moreno 2:30 PM	D. Rupnik 3:30 PM	D. Rupnik 12:00 PM	R. Beaird 1:30 PM

Answer all 3 problems & all 3 questions.

Be sure to write your name. Please read the questions carefully.

You may use only scientific or graphing calculators. *In particular you may not use the calculator app on your phone or tablet!*

You may detach and use the formula sheet provided at the back of this test. No other reference materials are allowed.

You may not answer or use cell phones during the exam. Please note that the official departmental policy for exams is as follows:

> "During your test, the only electronic device you may have with you at your seat is a scientific or graphing calculator. You may not have your cell phone, tablet, smartphone, smart watch, PDA, pager, digital camera, computer, or any other device capable of taking pictures or video, sending text messages, or accessing the Internet. This means not just on your person, but close enough to you that you could reach it during the test. Any student found with such a device during a test will be assumed to be violating the LSU Honor Code and will be referred to the Dean of Students for Judicial Affairs."

The simplest remedy is to bring nothing to this test but the calculator, and leave your backpack or purse at home. *If you have brought your cell phone or tablet with you, please leave it at the front of the room under the watchful eye of your instructor.*

Some questions are multiple-choice. You should work these problems starting with the basic equation listed on the formula sheet and write down all the steps. Although the work will not be graded, this will help you make the correct choice and to determine if your thinking is correct.

On problems that are not multiple-choice, be sure to show all of your work, since no credit will be given for an answer without explanation or work. These will be graded in full, and you are expected to show all relevant steps that lead to your answer.

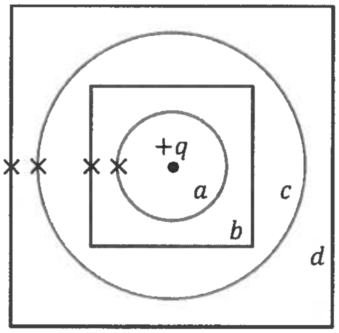
Please use complete sentences where explanations are asked for. For numerical answers that require units you must give the correct units for full credit.

YOU GET 60 min (1 hr) to complete the examination.

1. (Question) [12 points] The figure shows four Gaussian surfaces, two spheres and two cubes, that are all centered on a positive point charge q. Each Gaussian surface also has an "×" symbol marking a specific point on that Gaussian surface.

El. field of a point charge:

$$|\vec{E}| = k\frac{|q|}{r^2}$$



Gauss' law:

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \oint \vec{E}\cdot d\vec{A}$$

$\downarrow$ through a closed surface

(a) [4 points] Rank the Gaussian surfaces according to the net flux through each of them.

$\Phi_a > \Phi_b > \Phi_c > \Phi_d$

$\Phi_a = \Phi_c > \Phi_b = \Phi_d$

$\Phi_d > \Phi_c > \Phi_b > \Phi_a$

Φ: depends on q_{enc}; shape and size of a closed surface does not matter

$\Phi_b = \Phi_d > \Phi_a = \Phi_c$

[✓ $\Phi_a = \Phi_b = \Phi_c = \Phi_d$]

(b) [4 points] Rank the surfaces according to the magnitude of the electric field at the points marked by the "×" symbols.

[✓ $E_a > E_b > E_c > E_d$]

$E_a = E_c > E_b = E_d$

$E_d > E_c > E_b > E_a$

$$|\vec{E}| \propto \frac{1}{r^2}$$

$E_b = E_d > E_a = E_c$

$E_a = E_b = E_c = E_d$

(c) [4 points] On which surface, if any, is the magnitude of the electric field constant everywhere on the surface? $|\vec{E}| = \text{const}$ at $|\vec{r}| = \text{const} \Rightarrow$ concentric sphere

Surface a only.

Surface b only.

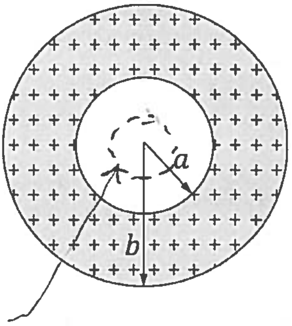
Surface c only.

Surface d only.

[✓ Surfaces a and c only.]

Surfaces b and d only.

None of these.



2. (Problem) [24 points] The figure shows a spherical shell with uniform volume charge density ρ, inner radius a, and outer radius $b = 3a$. Using Gauss's Law as needed, answer the questions below.

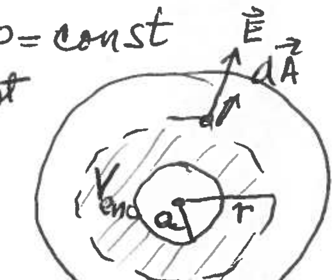
(a) [6 points] What is the magnitude of the electric field at radial distance $r = 0.50\ a$? Why? *Express your final answer in terms of a, b, ρ, and ε_0 as needed.* EXPLAIN YOUR REASONING!

at $r = 0.5a$ $q_{enc} = 0$ for the Gaussian sphere

$\oint \vec{E}\cdot d\vec{A} = |\vec{E}|\oint dA = \frac{q_{enc}}{\varepsilon_0}$, $\oint dA \neq 0 \Rightarrow |\vec{E}| = 0$

(b) [10 points] What is the magnitude of the electric field at radial distance $r = 2a$? *Express your final answer in terms of a, b, ρ, and ε_0 as needed.* SHOW YOUR WORK!

$r = 2a$ ... position within the shell with the charge, $\rho = \text{const}$



$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ $\vec{E} \parallel d\vec{A} \Rightarrow \vec{E}\cdot d\vec{A} = |\vec{E}|dA$, $|\vec{E}| = \text{const}$ at $r = \text{const}$

$\Rightarrow \oint \vec{E}\cdot d\vec{A} = |\vec{E}|4\pi r^2$

$q_{enc} = \rho V_{enc} = \rho\frac{4}{3}\pi(r^3 - a^3)$

$\Rightarrow |\vec{E}| = \frac{\rho}{3\varepsilon_0}\left(r - \frac{a^3}{r^2}\right) = \frac{\rho}{3\varepsilon_0}\left(2a - \frac{a^3}{4a^2}\right) = \frac{\rho a}{3\varepsilon_0}\left(2 - \frac{1}{4}\right)$

$r = 2a$ $|\vec{E}| = \frac{7}{12}\frac{\rho a}{\varepsilon_0} = 0.58\frac{\rho a}{\varepsilon_0}$

(c) [8 points] What is the magnitude of the electric field at radial distance $r = 5a$? *Express your final answer in terms of a, b, ρ, and ε_0 as needed.* SHOW YOUR WORK!

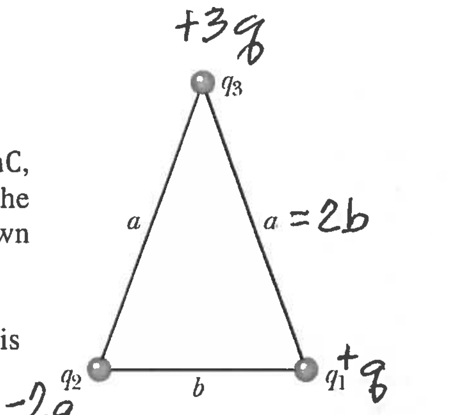
$r = 5a > b$ ... outside of a spherical charge distribution

$\Rightarrow |\vec{E}| =$ electric field of a point charge in the center with $q = \rho V$

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{\rho\frac{\cancel{4}}{3}(b^3 - a^3)\cancel{\pi}}{\cancel{4}\cancel{\pi}\varepsilon_0(25a^2)} = \frac{\rho}{75\varepsilon_0}\left(\frac{b^3}{a^2} - a\right)$$

using $b = 3a$

$$|\vec{E}| = \frac{\rho}{75\varepsilon_0}\left(\frac{3^3 a^{\cancel{3}}}{\cancel{a^2}} - a\right) = \frac{26}{75}\frac{\rho a}{\varepsilon_0}$$



3. (Problem) [20 points] Three point particles, with charges $q_1 = +10$ mC, $q_2 = -2q_1 = -20$ mC, and $q_3 = 3q_1 = +30$ mC, are positioned at the vertices of an isosceles triangle, as shown in the figure. The lengths shown are $a = 2b = 10$ m and $b = 5.0$ m.

(a) [8 points] What is the potential energy of the three-charge system as it is shown in the figure? SHOW YOUR WORK!

$$\mathcal{U} = k\frac{q_1q_2}{r_{12}} + k\frac{q_1q_3}{r_{13}} + k\frac{q_2q_3}{r_{23}} = k\frac{q^2}{b}\left(-\frac{2}{1}+\frac{3}{2}-\frac{6}{2}\right)$$

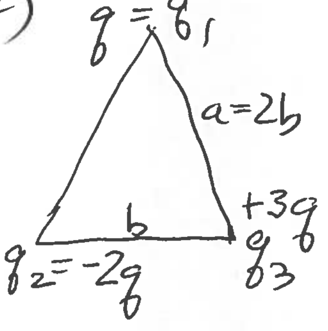
$$\mathcal{U} = \left(8.99\times10^9\,\frac{Nm^2}{C^2}\right)\frac{(10\times10^{-3}C)^2}{5m}\left(-2+1.5-3\right) = \underline{-6.29\times10^5\ J}$$

$q_1 = 10\,mC$
$q_2 = -2q_1$
$q_3 = +3q_1$
$r_{12} = b,\ r_{13} = 2b$
$r_{23} = 2b$

(b) [8 points] Beginning with the charges arranged as shown in the figure, how much work must *you* do to exchange charges q_1 and q_3 with one another? *All charges begin and end at rest.* SHOW YOUR WORK!

$W_{you} = +\Delta\mathcal{U} = \mathcal{U}_f - \mathcal{U}_i$, $\mathcal{U}_i = \mathcal{U}$ in part (a)

• the term $k\frac{q_1q_3}{r_{13}}$ will cancel (same charges and distances in i and f energy)



$$W_{you} = k\frac{q_1q_2}{2b} + k\frac{q_2q_3}{b} - k\frac{q_1q_2}{b} - k\frac{q_2q_3}{2b} =$$

$$= k\frac{q^2}{b}\left(-\frac{2}{2} - \frac{6}{1} + \frac{2}{1} + \frac{6}{2}\right)$$

$$= \left(8.99\times10^9\,\frac{Nm^2}{C^2}\right)\frac{(10\times10^{-3}C)^2}{5m}\left(-7+5\right).$$

$$\underline{W_{you} = -3.60\times10^5\ J}$$

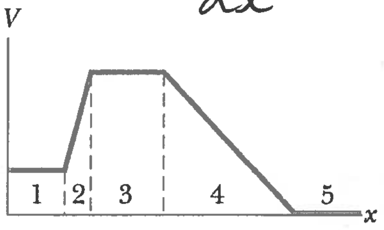
(c) [4 points] Beginning with the charges arranged as shown in the figure, how much work must *you* do to exchange charges q_1 and q_2 with one another? *All charges begin and end at rest.* Note: *you do not need to show your work **for this part only**. Circle the correct answer.*

+53.8 J −30.0 J −26.9 J [✓ 0 J] None of these.

• $\Delta\mathcal{U} = 0$ for that exchange $\Rightarrow W_{you} = 0$

$E_x = -\frac{dV}{dx}$

4. (Question) [8 points] The figure shown to the right gives the electric potential $V(x)$ along an x-axis. Five regions along the x-axis are labeled in the figure.



$|E_x| = |\text{slope}|$ in V-x plot

(a) [4 points] In which of the regions will the x-component of the electric field E_x have the smallest magnitude? *Circle only one answer.*

Region 1 Region 2 Region 3 Region 4 Region 5

[✓ Regions 1, 3, and 5 tie] $|\vec{E}| = 0$ there Regions 2 and 4 tie None of these.

(b) [4 points] In which region does the x-component of the electric field E_x point in the $+x$ direction? *Circle only one answer.*

• where slope is negative

Region 1 Region 2 Region 3 [✓ Region 4] Region 5

None of these regions. More than one region.

5. (Question) [12 points] Circle **T** if the statement is true and **F** if the statement is false.

A parallel-plate capacitor of capacitance C is charged using a battery of potential V. The battery is **disconnected** and the separation of the plates (d) of the ***isolated*** capacitor is **increased.**

$q = CV$

$C = \varepsilon_0 \frac{A}{d}$

$q = \text{const}$

As a result of the **increase** in the separation distance d...

(a) [3 points] the capacitance of the capacitor **increases** $C \propto \frac{1}{d}$ T [✓ F] actually decreases

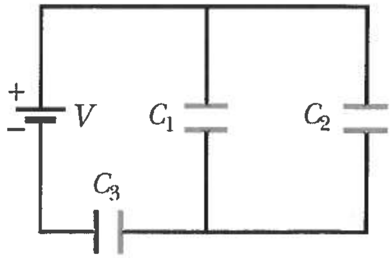
(b) [3 points] the charge on the plates of the capacitor **increases** $q = \text{const}$ T [✓ F] stays constant

(c) [3 points] the potential difference across the capacitor **decreases** $V = \frac{q}{C}$ T [✓ F] increases

(d) [3 points] the electric field between the capacitor plates **decreases** T [✓ F]

$|\vec{E}| = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$ ... stays the same

6. (Problem) [24 points] In the figure to the right, a potential difference $V = 5.00$ V is applied across a capacitor arrangement with capacitances $C_1 = 2.00$ μF, $C_2 = 4.00$ μF, and $C_3 = 1.00$ μF.



(a) [8 points] Find the equivalent capacitance C_{123} of the three-capacitor combination. SHOW YOUR WORK!

C_1 & C_2 ... connected in parallel $\Rightarrow C_{12} = C_1 + C_2 = 6\mu F$

C_{12} & C_3 ... connected in series $\Rightarrow C_{123} = \frac{C_{12}C_3}{C_{12}+C_3} = \frac{(C_1+C_2)C_3}{C_1+C_2+C_3} = \frac{(6)(1)}{7} = \frac{6}{7}\mu F$

$C_{123} = 0.857\mu F = 8.57\times10^{-7}$ F

(b) [8 points] Find the charge on C_3. SHOW YOUR WORK!

$q_3 = q_{12} = q_{123} =$ because C_3 and C_{12} are connected in series

$q_3 = C_{123}V_{123} = C_{123}V = (\frac{6}{7}\times10^{-6}F)(5V) = 4.286\mu C$

(c) [8 points] Find the stored energy U_1 in capacitor C_1. SHOW YOUR WORK!

$V_1 = V - V_3 = V - \frac{q_3}{C_3} = 5V - \frac{30\times\cancel{10^{-6}}\,C}{(7)(1\times\cancel{10^{-6}})F} = \frac{5}{7}V$

$U_1 = \frac{q_1V_1}{2} = \frac{C_1V_1^2}{2} = \frac{(2\times10^{-6})(\frac{5}{7})^2}{2} = 0.510\times10^{-6}J$

$U_1 = 5.1\times10^{-7}J$