Exam 1: Physics 2113 Fall 2014

6:00PM TUE 16 SEP 2014

Name (Last, First): SOLUTIONS

Section #________________

Instructor's name:________________________________

Answer all 3 problems & all 4 questions.

Be sure to write your name. Please read the questions carefully.

You may use only scientific or graphing calculators. *In particular you may not use the calculator app on your phone or tablet!*

You may detach and use the formula sheet provided at the back of this test. No other reference materials are allowed.

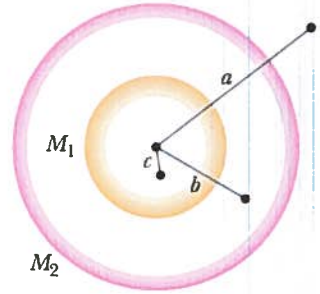
You may not answer or use cell phones during the exam. Please note that the official departmental policy for exams is as follows: "During your test, the only electronic device you may have with you at your seat is a scientific or graphing calculator. You may not have your cell phone, tablet, smartphone, PDA, pager, digital camera, computer, or any other device capable of taking pictures or video, sending text messages, or accessing the Internet. This means not just on your person, but close enough to you that you could reach it during the test. Any student found with such a device during a test will be assumed to be violating the LSU Honor Code and will be referred to the Dean of Students for Judicial Affairs." The simplest remedy is to bring nothing to this test but the calculator, and leave your backpack or purse at home. *If you have brought your cell phone or tablet with you, please leave it at the front of the room under the watchful eye of your instructor.*

Some questions are multiple choice. You should work these problems starting with the basic equation listed on the formula sheet and write down all the steps. Although the work will not be graded, this will help you make the correct choice and to determine if your thinking is correct.

On problems that are not multiple choice, be sure to show all of your work since no credit will be given for an answer without explanation or work. These will be graded in full, and you are expected to show all relevant steps that lead to your answer.
Please use complete sentences where explanations are asked for. For numerical answers that require units you must give the correct units for full credit.

YOU GET 60 min (1 hr)

1. (Question) **[9 points]** Two concentric spherical shells with uniformly distributed masses M_1 and M_2 are situated as shown in the figure. Find the magnitude of the net gravitational force on a particle of mass m, due to the shells, when the particle is located at the following radial distances a, b, and c. Circle the correct answers below.



(i) **[3 points]** $r = a$ (Circle one.)

(a) $F = G(M_1 + M_2)\frac{m}{c^2}$

(b) $F = G(M_1 + M_2)\frac{m}{a^2}$

(c) $F = 0$

(ii) **[3 points]** $r = b$ (Circle one.)

(a) $F = G(M_1 + M_2)\frac{m}{a^2}$

(b) $F = GM_1\frac{m}{b^2}$

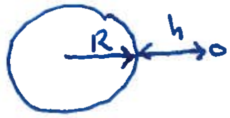
(c) $F = G(M_1 - M_2)\frac{m}{b^2}$

(iii) **[3 points]** $r = c$ (Circle one.)

(a) $F = GM_1M_2\frac{m}{a^2}$

(b) $F = GM_1\frac{m}{c^2}$

(c) $F = 0$



METHOD I: WORK WITH a_g

$= 5.70 \times 10^5\,m$

2. (Problem) **[21 points]** A spherical asteroid has a radius R = 570 km and gravitational acceleration at the surface of $a_g = 3.0\ \text{m/s}^2$.

$a = \frac{GM}{R^2}$ DON'T KNOW M!

(i) **[7 points]** Using conservation of mechanical energy, defined as the sum of the kinetic and potential energies, derive the expression for the escape speed, $v = \sqrt{2a_gR}$. Hint: At the surface, what is a_g in terms of G, M, and R? Show your work!

$v_{escape} \Rightarrow v = 0$ & $U = 0$ at $r = \infty$

$$K_i + U_i = K_f + U_f \big|_{r=\infty} = 0$$

$$\Rightarrow \frac{1}{2}\cancel{m}v^2 - \frac{G\cancel{m}M}{R} = 0 \Rightarrow v^2 = 2\underbrace{\left(\frac{GM}{R^2}\right)}_{a} \cdot R = 2aR$$

$$\Rightarrow \boxed{v = \sqrt{2aR}}$$ ANS.

UNITS:

$$\sqrt{\frac{m}{s^2}\,m} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$$ ✓

(ii) **[7 points]** Find the numerical value of the escape speed.

$$v = \sqrt{2\,(5.70 \times 10^5\,m)\left(\frac{3.00\,m}{s^2}\right)} = \boxed{1.85 \times 10^3\,\frac{m}{s}}$$ ans.

$R + h = 570\,km + 1000\,km = 1570\,km = 1.57 \times 10^6\,m$

(iii) **[7 points]** With what speed will an object hit the asteroid if it is dropped from $h =$ 1000 km above the surface? $v_i = 0 \Rightarrow K_i = 0$

$$K_i + U_i = K_f + U_f \Rightarrow 0 - \frac{G\cancel{m}M}{(R+h)} = \frac{1}{2}\cancel{m}v^2 - \frac{G\cancel{m}M}{R}$$

$$\Rightarrow v^2 = 2GM\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$= 2\underbrace{\frac{GM}{R^2}}_{a} \cdot R^2\left[\frac{1}{R} - \frac{1}{R+h}\right] = 2a\left[R - \frac{R^2}{R+h}\right]$$

$$\Rightarrow v = \sqrt{2\left(\frac{3.0\,m}{s^2}\right)\left[5.70 \times 10^5\,m - \frac{(5.70 \times 10^5\,m)^2}{(1.57 \times 10^6\,m)}\right]}$$

$$= \sqrt{2.18 \times 10^6\,\frac{m^2}{s^2}} = \boxed{1.48 \times 10^3\,\frac{m}{s}}$$ ANS

METHOD II: SOLVE FOR M IN TERMS OF a_g

$$a_g = \frac{GM}{R^2} \Rightarrow M = \frac{a R^2}{G} = \frac{3.00\,\cancel{m}\,(5.70\times10^{5}\,\cancel{m})^2}{s^2}\;\frac{kg\cdot s^2}{6.67\times10^{-11}\,\cancel{m^3}}$$

2. (Problem) **[21 points]** A spherical asteroid has a radius $R = 570$ km and gravitational acceleration at the surface of $a_g = 3.0\ \text{m/s}^2$. $\rightarrow = 5.70\times10^{5}\,m = R$

$\boxed{M = 1.46\times10^{22}\,kg}$ ANS

(i) **[7 points]** Using conservation of mechanical energy, defined as the sum of the kinetic and potential energies, derive the expression for the escape speed, $v = \sqrt{2a_gR}$. Hint: At the surface, what is a_g in terms of G, M, and R? Show your work!

$K_i + U_i = K_f + U_f$

$K_f = U_f = 0$ at ∞ FOR ESCAPE SPEED

$K_i = \frac{1}{2}mv^2 \qquad U_i = -\frac{GmM}{R}$

$$\Rightarrow \frac{1}{2}\cancel{m}v^2 = \frac{G\cancel{m}M}{R}$$

$$\Rightarrow \boxed{v = \sqrt{\frac{2GM}{R}}}$$

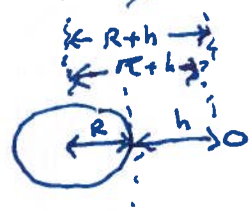
PART. ANS. $= \sqrt{2\underbrace{\left(\frac{GM}{R^2}\right)}_{a_g}R} = \boxed{\sqrt{2a_gR}}$ ANS.

(ii) **[7 points]** Find the numerical value of the escape speed.

$$v = \sqrt{2\;\frac{6.67\times10^{-11}\,m^{\cancel{3}2}}{\cancel{kg}\cdot s^2}\;1.46\times10^{22}\,\cancel{kg}\;\frac{1}{5.70\times10^{5}\,\cancel{m}}}$$

$= \boxed{1.85\times10^{3}\ m/s}$ ANS.

(iii) **[7 points]** With what speed will an object hit the asteroid if it is dropped from
$h =$ 1000 km above the surface? $R+h = 570\,km + 1000\,km = 1.570\times10^{6}\,m$



$$K_i + P_i = K_f + P_f = 0 - \frac{GM\cancel{m}}{R+h} = \frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{R}$$

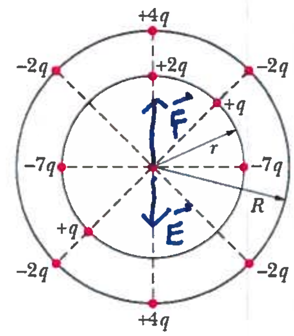
$$\Rightarrow v^2 = 2GM\left[-\frac{1}{R+h} + \frac{1}{R}\right]$$

$$\Rightarrow v = \sqrt{2GM\left[-\frac{1}{R+h} + \frac{1}{R}\right]}$$

$$= \sqrt{2\;\frac{6.67\times10^{-11}\,m^3}{kg\,s^2}\;1.46\times10^{22}\,kg\left[\frac{-1}{1.57\times10^{6}\,m} + \frac{1}{5.70\times10^{5}\,m}\right]}$$

$= \boxed{1.48\times10^{3}\ m/s = 1480\ km/s}$ ANS.

3. (Question) **[9 points]** In the figure, a central particle of charge $-q$ is surrounded by two circular rings of charged particles, of radii r and R, with $R > r$.



(i) **[3 points]** What is the magnitude of the net electrostatic force on the central particle due to the other particles? Circle one. Hint: Consider Symmetry.

(a) $\dfrac{2q}{4\pi\varepsilon_0 r^2}$

(b) $\dfrac{q^2}{4\pi\varepsilon_0 r^2}$

(c) $\dfrac{2q^2}{4\pi\varepsilon_0 r^2}$

(d) $\dfrac{2q^2}{4\pi\varepsilon_0 r}$

(ii) **[3 points]** What is the direction of the net electrostatic *force* on the central particle due to the other particles? Circle *one* and draw the *direction* of the *force* on the diagram and label it "force".

(a) upward ↑

(b) rightward →

(c) leftward ←

(d) downward ↓

(iii) **[3 points]** Now we remove the central particle entirely from the figure. What is the direction of the net electric *field* at the central *point* due to the other particles? Circle *one* and draw the *direction* of the *field* on the diagram and label it "field".

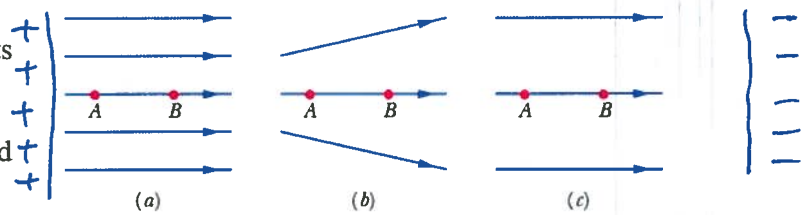
(a) upward ↑

(b) rightward →

(c) leftward ←

(d) downward ↓

4. (Question) **[9 points]** The figure shows three arrangements of electric field lines. In each arrangement, a proton is released from rest at point A and is then accelerated through point B by the electric field. Points A and B have equal separations in the three arrangements.



(i) **[3 points]** As you move from point A to point B, in each arrangement, circle if the electric field strength is increasing, decreasing, or remains the same:

Arrangement (a): increasing decreasing same

Arrangement (b): increasing decreasing same

Arrangement (c): increasing decreasing same

(ii) **[3 points]** Rank the arrangements according to the linear momentum of the proton at point B, greatest first. Circle one.

$a > b > c$

$a = b > c$

$a = b = c$

(iii) **[3 points]** Instead of releasing a proton at point A, we now release an *electron* from point B. In all three arrangements the electron moves the same way. Which way does it move? Circle one.

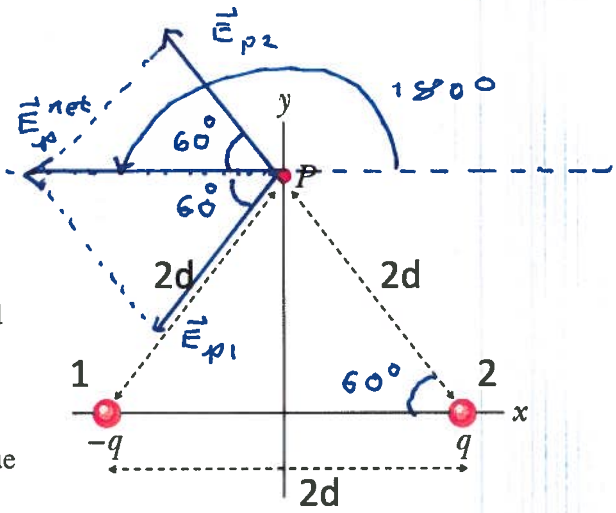
upward ↑

rightward →

leftward ←

downward ↓

5. (Problem) **[20 points]** The figure shows two charged particles on an x axis, $q_1 = -q = -3.20\times10^{-19}\text{C}$ at $x = -3.40\text{m}$ and $q_2 = +q = +3.20\times10^{-19}\text{C}$ at $x = +3.40\text{m}$. The point P forms an equilateral triangle with the two charges with equal sides of length $2d = 6.80$ m and three equal interior angles of 60° each.



(i) **[5 points]** On the figure draw the three electric field vectors, $\vec{\mathbf{E}}_{p1}$ (the field at P due to 1), $\vec{\mathbf{E}}_{p2}$ (the field at P due to 2), and $\vec{\mathbf{E}}_{p,\text{net}}$ (the total net field at P due to both 1 and 2).

(ii) **[5 points]** Compute the magnitudes E_{p1} and E_{p2}. Are they equal? Why or why not? YES! SYMMETRY!

$$E_{p1} = E_{p2} = \frac{kq}{(2d)^2}$$

$$= \frac{8.99\times10^{9}\ \frac{N\cancel{m^2}}{C\cancel{^2}} \cdot 3.20\times10^{-19}\ \cancel{C}}{(6.80\ \cancel{m})^2}$$

$$= \boxed{6.22\times10^{-11}\ N/C} \quad \text{ANS.}$$

(iii) **[5 points]** Compute the magnitude of the total field $E_{p,\text{net}}$. Y-COMPONENTS CANCEL!

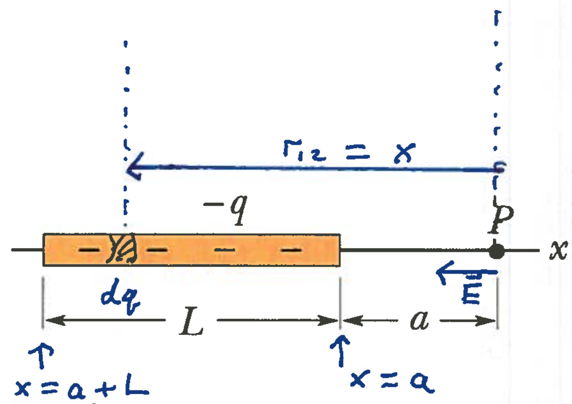
$$E_p^{net} = \underbrace{E_{p1}^{x} + E_{p2}^{x}}_{\text{same}} = 2E_{p1}^{x} = 2E_{p1}\underbrace{\cos 60^\circ}_{1/2}$$

$$= \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot E_{p1} = \boxed{6.22\times10^{-11}\ N/C} \quad \text{ANS.}$$

(iv) **[5 points]** Give the direction of the total field $\vec{\mathbf{E}}_{p,\text{net}}$ in degrees in terms of an angle measured counter-clockwise from the x-direction.

180°

6. (Problem) **[24 points]** In the figure a nonconducting charged rod of length L, located along the x-axis as shown, has a charge $-q$ distributed uniformly along its length.



(i) **[6 points]** *Draw* the direction of the electric field vector at the point P *on the figure.*

(ii) **[6 points]** What is the charge density λ? Express the differential charge element dq in terms of λ and dx. *In the figure, draw the differential charge element at some location x on the rod.*

ANS

$$\lambda = \frac{|-q|}{L} = \boxed{\frac{q}{L}} \qquad dq = \boxed{\lambda\, dx = \frac{q}{L}\, dx}$$

ANS

(iii) **[6 points]** Write down an expression for the *vector* differential field $\mathbf{dE}$ at the point P due to a small charged segment dq of length dx for the x your used in part (ii). Express your answer in terms of $k = 1/(4\pi\varepsilon_0)$, q, dx, a, L, and x.

ANS

$$d\vec{E} = -\frac{k\,dq}{x^2}\hat{\imath} = \boxed{-k\lambda\frac{dx}{x^2}\hat{\imath} = -\frac{kq}{L}\frac{dx}{x^2}\hat{\imath}}$$

(iv) **[6 points]** Integrate the expression to obtain the total *vector* electric field at point P in terms of $k = 1/(4\pi\varepsilon_0)$, q, L, and a. INTEGRATE OVER ROD $x: a \to a+L$

$$\vec{E} = \int d\vec{E} = -k\lambda\int_a^{a+L}\frac{dx}{x^2}\hat{\imath}$$

$$\frac{d}{dx}(-x^{-1}) = +x^{-2} = \frac{1}{x^2}$$

$$= +k\lambda\left[\frac{1}{a+L} - \frac{1}{a}\right]\hat{\imath}$$

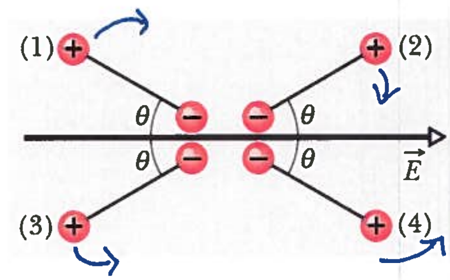
$$\frac{1}{a+L} < \frac{1}{a} \Rightarrow \frac{1}{a+L} - \frac{1}{a} < 0$$

$$= -k\lambda\left[\frac{1}{a} - \frac{1}{a+L}\right]\hat{\imath}$$

$$= \boxed{-\frac{kq}{L}\left[\frac{1}{a} - \frac{1}{a+L}\right]\hat{\imath}}$$

ANS.

7. (Question) **[8 points]** The figure shows four orientations of an electric dipole in an external electric field.



(i) **[2 points]** For each of the four dipoles, draw a curved arrow on the figure from the ⊕ charge indicating if it will rotate clockwise or counterclockwise.

(ii) **[3 points]** Rank the orientations according to the magnitude of the *torque*, greatest first. Circle one.

(a) 1=2=3=4

(b) 1=2>3=4

(c) 1=3>2=4

(iii) **[3 points]** Rank the orientations according to the *potential energy* of the dipole, greatest first. Circle one.

(a) 1=2=3=4

(b) 1=2>3=4

(c) 1=3>2=4

Formula Sheet for LSU Physics 2113, First Exam, Fall '14

- **Constants, definitions:**

$g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$ $\quad R_{Earth} = 6.37 \times 10^6\,\mathrm{m}$ $\quad M_{Earth} = 5.98 \times 10^{24}\,\mathrm{kg}$

$G = 6.67 \times 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}$ $\quad R_{Moon} = 1.74 \times 10^6\,\mathrm{m}$ $\quad$ Earth-Sun distance $= 1.50\times10^{11}\,\mathrm{m}$

$M_{Sun} = 1.99 \times 10^{30}\,\mathrm{kg}$ $\quad M_{Moon} = 7.36 \times 10^{22}\,\mathrm{kg}$ $\quad$ Earth-Moon distance $= 3.82\times10^{8}\,\mathrm{m}$

$\epsilon_o = 8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{Nm}^2}$ $\quad k = \frac{1}{4\pi\epsilon_o} = 8.99 \times 10^9\,\frac{\mathrm{Nm}^2}{\mathrm{C}^2}$ $\quad e = 1.60 \times 10^{-19}\,\mathrm{C}$

$c = 3.00 \times 10^8\,\mathrm{m/s}$ $\quad m_p = 1.67 \times 10{-}27\,\mathrm{kg}$ $\quad 1\ \mathrm{eV} = \mathrm{e}(1\mathrm{V}) = 1.60\times10^{-19}\,\mathrm{J}$

dipole moment: $\vec{p} = q\vec{d}$ $\quad m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$ $\quad$ charge densities: $\lambda = \frac{Q}{L}$, $\sigma = \frac{Q}{A}$, $\rho = \frac{Q}{V}$

Area of a circle: $A = \pi r^2$ $\quad$ Area of a sphere: $A = 4\pi r^2$ $\quad$ Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Area of a cylinder: $A = 2\pi r\ell$ $\quad$ Volume of a cylinder: $V = \pi r^2 \ell$

- **Units:**

Joule $= \mathrm{J} = \mathrm{N{\cdot}m}$

- **Kinematics (constant acceleration):**

$v = v_o + at \qquad x - x_o = \frac{1}{2}(v_o + v)t \qquad x - x_o = v_o t + \frac{1}{2}at^2 \qquad v^2 = v_o^2 + 2a(x - x_o)$

- **Circular motion:**

$F_c = ma_c = \frac{mv^2}{r}, \quad T = \frac{2\pi r}{v}, \quad v = \omega r$

- **General (work, def. of potential energy, kinetic energy):**

$K = \frac{1}{2}mv^2 \qquad \vec{F}_{\mathrm{net}} = m\vec{a} \qquad E_{\mathrm{mech}} = K + U$

$W = -\Delta U$ (by field) $\quad W_{ext} = \Delta U = -W$ (if objects are initially and finally at rest)

- **Gravity:**

Newton's law: $|\vec{F}| = G\frac{m_1 m_2}{r^2}$ $\qquad$ Gravitational acceleration (planet of mass M): $a_g = \frac{GM}{r^2}$

Gravitational Field: $\vec{g} = -G\frac{M}{r^2}\hat{r} = -\frac{dV_g}{dr}$ $\qquad$ Gravitational potential: $V_g = -\frac{GM}{r}$

Law of periods: $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$ $\qquad$ Potential Energy: $U = -G\frac{m_1 m_2}{r_{12}}$

Potential Energy of a System (more than 2 masses): $U = -\left(G\frac{m_1 m_2}{r_{12}} + G\frac{m_1 m_3}{r_{13}} + G\frac{m_2 m_3}{r_{23}} + \ldots\right)$

Gauss' law for gravity: $\oint_S \vec{g}\cdot d\vec{S} = -4\pi G M_{ins}$

- **Electrostatics:**

Coulomb's law: $|\vec{F}| = k\frac{|\,q_1\,||\,q_2\,|}{r^2}$ $\qquad$ Force on a charge in an electric field: $\vec{F} = q\vec{E}$

Electric field of a point charge: $|\vec{E}| = k\frac{|\,q\,|}{r^2}$

Electric field of a dipole on axis, far away from dipole: $\vec{E} = \frac{2k\vec{p}}{z^3}$

Electric field of an infinite line charge: $|\vec{E}| = \frac{2k\lambda}{r}$

Torque on a dipole in an electric field: $\vec{\tau} = \vec{p} \times \vec{E}$

Potential energy of a dipole in $\vec{E}$ field: $U = -\vec{p}\cdot\vec{E}$