Physics 2101 for Section 9&14 Feb. 14th: Ch. 9

Lecture notes:
http://www.phys.lsu.edu/classes/spring2013/phys2101-9-14/

Old Exams:
http://www.phys.lsu.edu/faculty/gonzalez/Teaching/Phys2101/Phys2101OldTests/
Quick Review: the Center of Mass

\[ x_{\text{com}} = \frac{1}{M} \sum_{i=1}^{N} m_i x_i \quad \Rightarrow \quad x_{\text{com}} = \frac{1}{M} \int x \, dm \quad \Rightarrow \quad x_{\text{com}} = \frac{1}{V} \int x \, dV \]

\[ y_{\text{com}} = \frac{1}{M} \sum_{i=1}^{N} m_i y_i \quad \Rightarrow \quad y_{\text{com}} = \frac{1}{M} \int y \, dm \]

\[ z_{\text{com}} = \frac{1}{M} \sum_{i=1}^{N} m_i z_i \quad \Rightarrow \quad z_{\text{com}} = \frac{1}{M} \int z \, dm \]

\[ M = \sum_{i=1}^{N} m_i \quad \Rightarrow \quad \rho = \frac{dm}{dV} = \frac{M}{V} \]

Here “mass density” replaces mass

(1) Center of mass of a symmetric object always lies on an axis of symmetry.

(2) Center of mass of an object does NOT need to be on the object.
Quick Review: Newton’s 2nd Law for a System of Particles

\[ \mathbf{M\hat{a}}_{\text{com}} = \mathbf{F}_{\text{net}} \]

\[
\begin{align*}
F_{\text{net},x} &= Ma_{\text{com},x} \\
F_{\text{net},y} &= Ma_{\text{com},y} \\
F_{\text{net},z} &= Ma_{\text{com},z}
\end{align*}
\]
Quick Review: Linear Momentum

\[ M \mathbf{a}_{\text{com}} = \mathbf{F}_{\text{net}} \]

\[ F_{\text{net},x} = M a_{\text{com},x} \]
\[ F_{\text{net},y} = M a_{\text{com},y} \]
\[ F_{\text{net},z} = M a_{\text{com},z} \]

\[ \mathbf{p} = m \mathbf{v} \quad - \text{Linear Momentum} \]

\[ \mathbf{F}_{\text{net}} = \frac{d \mathbf{p}}{dt} \]

\[ F_{\text{net}} = \frac{d P}{dt} = \frac{d p_1}{dt} + \ldots + \frac{d p_n}{dt} \]

\[ \Delta P = 0 \quad - \text{Conservation of Linear Momentum} \]
Quick Review: Impulse

Impulse

Change in Momentum is equal to Impulse acting on it

\[ d\vec{p}_R = \vec{F}_{L\rightarrow R}(t)dt \]

\[ \int d\vec{p}_R = \int \vec{F}_{L\rightarrow R}(t)dt \]

Definition

\[ \vec{J} \equiv \int_{t_i}^{t_f} \vec{F}_{\text{net}}(t)dt \]

Impulse-momentum theorem

Vector! Must satisfy for each direction!

Impulse

Change in Momentum is equal to Impulse acting on it

\[ \vec{p}_f - \vec{p}_i = \Delta \vec{p} = \vec{J} \]

Collision

Before

During

System

After

What changes momentum of each?
Conservation of Linear Momentum

If

\[ F_{\text{net}} = \frac{d \vec{P}}{dt} = \frac{d \vec{p}_1}{dt} + ... + \frac{d \vec{p}_n}{dt} = 0 \]

\[ \Delta \vec{P} = \vec{P}_f - \vec{P}_i = \left( \vec{P}_{1f} + ... + \vec{P}_{nf} \right) - \left( \vec{P}_{1i} + ... + \vec{P}_{ni} \right) = 0 \]

- Conservation of Linear Momentum

If system is closed and isolated, the total linear momentum \( \vec{P} \) cannot change.

What about energy?
Collisions: Elastic vs. Inelastic

Elastic collision: TOTAL KE is conserved (~ Conservative forces)

AND if system is closed and isolated, the total linear momentum \( P \) cannot change (whether the collision is elastic or inelastic!).

\[
\Delta KE = K_f - K_i = \left( K_{1f} + \ldots + K_{nf} \right) - \left( K_{1i} + \ldots + K_{ni} \right) = 0
\]

\[
\Delta P = P_f - P_i = \left( \rightarrow P_{1f} + \ldots + \rightarrow P_{nf} \right) - \left( \rightarrow P_{1i} + \ldots + \rightarrow P_{ni} \right) = 0
\]

Inelastic collision: KE is not conserved (~ thermal energy)

However, if system is closed and isolated, the total linear momentum \( P \) cannot change (whether the collision is elastic or inelastic!).

\[
\Delta KE = K_f - K_i = \left( K_{1f} + \ldots + K_{nf} \right) - \left( K_{1i} + \ldots + K_{ni} \right) \neq 0
\]

\[
\Delta P = P_f - P_i = \left( \rightarrow P_{1f} + \ldots + \rightarrow P_{nf} \right) - \left( \rightarrow P_{1i} + \ldots + \rightarrow P_{ni} \right) = 0
\]
A 9-kg object is at rest. Suddenly, it explodes and breaks into two pieces. The mass of one piece is 6 kg and the other is a 3-kg piece. Which one of the following statements concerning these two pieces is correct?

a) The speed of the 6-kg piece will be one eighth that of the 3-kg piece.

b) The speed of the 3-kg piece will be one fourth that of the 6-kg piece.

c) The speed of the 6-kg piece will be one forth that of the 3-kg piece.

d) The speed of the 3-kg piece will be one half that of the 6-kg piece.

e) The speed of the 6-kg piece will be one half that of the 3-kg piece.
Concept Questions

A sled of mass $m$ is coasting at a constant velocity on the ice covered surface of a lake. Three birds, with a combined mass 0.5$m$, gently land at the same time on the sled. The sled and birds continue sliding along the original direction of motion. How does the kinetic energy of the sled and birds compare with the initial kinetic energy of the sled before the birds landed?

a) The final kinetic energy is one half of the initial kinetic energy.

b) The final kinetic energy is two third of the initial kinetic energy.

c) The final kinetic energy is one quarter of the initial kinetic energy.

d) The final kinetic energy is one ninth of the initial kinetic energy.

e) The final kinetic energy is equal to the initial kinetic energy.
Velocity of COM

In a closed, isolated system the COM velocity \( \vec{V}_{\text{com}} \) of the system is CONSTANT. Why?

\[
\vec{F}_{\text{net}} = \frac{d\vec{P}_{\text{tot}}}{dt} = 0
\]

\[
\vec{P}_{\text{tot}} = M\vec{V}_{\text{com}} = (m_1 + m_2)\vec{V}_{\text{com}}
\]

\[
\vec{P}_{\text{tot}} = \vec{p}_{1i} + \vec{p}_{2i}
\]

\[
\vec{v}_{\text{com}} = \frac{\vec{P}_{\text{tot}}}{(m_1 + m_2)} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{(m_1 + m_2)} = \frac{\vec{p}_{1f} + \vec{p}_{2f}}{(m_1 + m_2)}
\]

Constant !!
9.9 1D Inelastic Collisions

Inelastic collision: KE is not conserved (~ thermal energy)

However, if system is closed and isolated, the total linear momentum $P$ cannot change (whether the collision is elastic or inelastic!)

Special Case: Completely Inelastic Collision (hit-'n-stick)

\[
\vec{P}_{before} = \vec{P}_{after} \quad 1 - D
\]

\[
(m_1v_{1i} + m_2v_{2i}) = (m_1v_{1f} + m_2v_{2f})
\]

Only COLM: Conservation of Linear Momentum

\[
\begin{align*}
(m_1v_{1i} + m_20) &= (m_1V + m_2V) \\
V &= \frac{m_1v_{1i}}{(m_1 + m_2)}
\end{align*}
\]
Pure Inelastic Collision -- Hit-’n-stick

Special case: $v_{2i} = 0$ & $m_2 = 3m_1$

take $v_{1i} = 1 \text{ m/s}$

Lab frame: $v_{1f} = -1/2 \cdot v_{1i}$ & $v_{2f} = \frac{1}{2} \cdot v_{1f}$

$v_{com} = V = \frac{1m_1(1)}{(m_1 + 3m_1)}$

$V = v_{com} = \frac{1}{4} \cdot m/s$
Inelastic Collisions?
Inelastic Collision

**Ballistic pendulum:** A bullet of mass $m$ and initial velocity $v_0$ collides and sticks to a pendulum of mass $M$ supported on a rope of length $L$. How high does the pendulum go before coming to rest?

Conservation of momentum:

$$p_i = p_f \Rightarrow m v_0 = (m + M) v$$

Kinetic energy to potential energy:

$$\frac{1}{2} m v_0^2 = \frac{1}{2} (m + M) v^2 = (m + M) gh$$

$$h = \frac{m v_0^2}{2(g(m + M))}$$
Inelastic Collision

P. # 54: Ch. 9: A bullet of mass \( m \) is moving directly upward at a velocity \( v_0 \). It strikes a block of mass \( M \) initially at rest. It passes through the block but comes out with a velocity \( v_0/4 \). How high does the block rise above its initial position?
9. 10 1D Elastic Collision: 2 particles

\[ \frac{1}{2} m_1 v^2_{1f} + \frac{1}{2} m_2 v^2_{2f} = \frac{1}{2} m_1 v^2_{1i} + \frac{1}{2} m_2 v^2_{2i} \]

\[ m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i} \]

\[ v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \]

\[ v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \]

6 variables and 2 Equations
2 mass & 4 velocity
1D Elastic Collision: 2 particles with $v_{2i}=0$

**Special Cases:**

1. **Equal masses**
   
   If $m_1 = m_2$ then $v_{1f} = 0$ and $v_{2f} = v_{1i}$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

5 variables and 2 Equations

2 mass & 3 velocity
1D Elastic Collision: 2 particles with $v_{2i}=0$

\[
v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}
\]

\[
v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}
\]

5 variables and 2 Equations
2 mass & 3 velocity

Special Cases:
2) Massive target

If $m_1 \ll m_2$ then $v_{1f} \approx -v_{1i}$ and $v_{2f} \approx \frac{2m_1}{m_2} v_{1i}$
1D Elastic Collision: 2 particles with \( v_{2i}=0 \)

\[
\begin{align*}
v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \\
v_{2f} &= \frac{2m_1}{m_1 + m_2} v_{1i}
\end{align*}
\]

5 variables and 2 Equations
2 mass & 3 velocity

Special Cases:
3) Massive Projectile \[ \text{If } m_1 \gg m_2 \text{ then } v_{1f} \approx v_{1i} \text{ and } v_{2f} \approx 2v_{1i} \]
Problem 10-4

A ball of mass \( m \) is fastened to a cord of length \( L \). The ball is released when cord is horizontal. At bottom of path, the ball elastically strikes block of mass \( M \) initially at rest on frictionless floor.

a) What is the speed of the ball right after the collision?

b) What is the speed of the block right after the collision?
**1-D Collisions**

If system is closed and isolated, the total linear momentum $\vec{P}$ cannot change.

### Inelastic collision: KE is not conserved (~ thermal energy)

$$
\vec{P}_{\text{before}} = \vec{P}_{\text{after}} \quad 1 - D \quad 2 \text{ particles}
$$

$$(m_1 v_{1i} + m_2 v_{2i}) = (m_1 v_{1f} + m_2 v_{2f})$$

### Elastic collision: TOTAL KE is conserved (~ Conservative forces)

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \quad \text{(Eqn. 9-75)}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \quad \text{(Eqn. 9-76)}$$

$$\vec{P}_{\text{before}} = \vec{P}_{\text{after}} \quad 1 - D$$

$$KE_{\text{before}} = KE_{\text{after}} \quad 2 \text{ particles}$$

$$\left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2\right) = \left(\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2\right)$$
Momentum and Kinetic Energy in Collisions

Before

Body 1
\[ m_1 \]
\[ \vec{v}_{1i} \]

Body 2
\[ m_2 \]
\[ \vec{v}_{2i} \]

After

Body 1
\[ m_1 \]
\[ \vec{v}_{1f} \]

Body 2
\[ m_2 \]
\[ \vec{v}_{2f} \]
Examples of Elastic Collisions
1D Elastic Collision with $m_2 = 3m_1$ and $v_{2i} = 0$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{m_1 - 3m_1}{m_1 + 3m_1} v_{1i} = -\frac{1}{2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + 3m_1} v_{1i} = \frac{1}{2} v_{1i}$$
In a closed, isolated system the COM velocity (\( \vec{v}_{\text{com}} \)) of the system is CONSTANT. Why?

\[
\vec{F}_{\text{net}} = \frac{d\vec{P}_{\text{tot}}}{dt} = 0
\]

\[
\vec{P}_{\text{tot}} = M\vec{v}_{\text{com}} = (m_1 + m_2)\vec{v}_{\text{com}}
\]

\[
\vec{P}_{\text{tot}} = \vec{p}_{1i} + \vec{p}_{2i}
\]

\[
\vec{v}_{\text{com}} = \frac{\vec{P}_{\text{tot}}}{(m_1 + m_2)} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{(m_1 + m_2)} = \frac{\vec{p}_{1f} + \vec{p}_{2f}}{(m_1 + m_2)}
\]

**Constant !!**
Summary: 1D Elastic Collisions

Elastic collision: TOTAL KE is conserved (~ Conservative forces)
AND if system is closed and isolated, the total linear momentum $P$
cannot change (whether the collision is elastic or inelastic!).

For example:

$$v_{2i} = 0$$

During Collision: transfer KE and momentum between objects through conservative internal forces

$$\text{KE}_{\text{before}} = \text{KE}_{\text{after}}$$

$$\left( \frac{1}{2} m_1 v_{1i}^2 \right) = \left( \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \right)$$

"COKE"

$$\vec{P}_{\text{before}} = \vec{P}_{\text{after}}$$

$$\left( m_1 v_{1i} \right) = \left( m_1 v_{1f} + m_2 v_{2f} \right)$$

"COLM"
Summary: Velocity & Position of COM
(closed & isolated system)

Special Case: Completely Inelastic Collision (hit-'n-stick)

Before
\[ \vec{v}_{1i} \]
\[ m_1 \]
Projectile
\[ \vec{v}_{2i} = 0 \]
\[ m_2 \]
Target

After
\[ \vec{V} \]
\[ m_1 + m_2 \]

\[ (m_1v_{1i} + m_2v_{2i}) = (m_1V + m_2V) \]
target at rest

\[ V = \frac{m_1v_{1i}}{(m_1 + m_2)} = v_{\text{com}} \]

\[ \hat{x} \]

Position x

Time t

x-t plot

slope in x-t plot gives velocity
Two bodies that form a closed, isolated system undergo an elastic collision in 1-D. Which of the three choices best represents the position-versus-time ($x$-$t$ plot) of those bodies and their center of mass velocity ($v_{com}$)?