Homework 4

L. Lehner, Phys 2221 Th March 11th 2007

March 18, 2008

Abstract

Multiple-variables calculus & Lagrange multipliers.

1. Solve from Boas. page 294, problems 1 & 7.

2. Solve from Boas. page 322, problem 2.

3. Solve from Boas. page 334, problem 2.

4. Solve from Boas. page 337, problem 11.

5. Which of the following forces is conservative? If so, find the potential energy $U(r)$.
   - (a) $\mathbf{F} = (ayz + bx + c, axz + bz, axy + by)$
   - (b) $\mathbf{F} = (-ze^{-x}, e^{-x} + y/z, 0)$

6. The motion of a charged particle in an electromagentic field can be obtained from the Lorentz equation which determines the force a particle with charge q suffers. The component of this force are given by
$$F_i = q(E_i + \epsilon_{ijk} v_j B_k) \tag{1}$$
(with $i = 1, 2, 3$; $\mathbf{E}, \mathbf{B}$ the electric and magnetic fields.)

   - (a) Show that when there is no electric field and when the particle enters the magnetic field in a direction perpendicular to the lines of magnetic flux the trajectory is a circle with radius $r = \frac{mv}{qB}$

   - (b) Choose now the z-axis to lie in the direction of the magnetic field $\mathbf{B}$ and let the plane containing the magnetic field E by the y-z plane. I.e. $\mathbf{B} = (0, 0, B); \mathbf{E} = (0, E_y, E_z)$. Show that the z component of the motion is given by $z(t) = z_0 + v_{z_0} t + \frac{qE_z}{2m} t^2$

   - (c) Write the equations of motions (but no need to solve them) that determine the motion in the x and y directions.

7. Consider a particle of mass m which can move in the region $x > 0$ under the influence of a potential given by $U = c(a/x + x/a)$ with c, a some positive constants.

   - (a) Do a rough sketch of the potential and find its minimum (if any).

   - (b) If the object will move in a small neighborhood of the minimum of the potential, argue it will undergo oscillation and give the frequency of this motion.

8. Two masses $m_1 = 100g$ and $m_2 = 200g$ can slide freely in a horizontal frictionless track and are connected by a spring whose constant is $k = 0.5N/m$. Find the frequency of oscillatory motion for this system.