Chapter 4

1. The initial position vector $\vec{r}_o$ satisfies $\vec{r} - \vec{r}_o = \Delta\vec{r}$, which results in

$$\vec{r}_o = \vec{r} - \Delta\vec{r} = (3.0\hat{j} - 4.0\hat{k})\text{m} - (2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k})\text{m} = (-2.0\text{ m})\hat{i} + (6.0\text{ m})\hat{j} + (-10\text{ m})\hat{k}.$$

6. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for dx/dt.

(a) Eq. 4-10 leads to

$$v(t) = \frac{d}{dt}(3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k}) = (3.00\text{ m/s})\hat{i} - (8.00t\text{ m/s})\hat{j}$$

(b) Evaluating this result at t = 2.00 s produces $\vec{v} = (3.00\hat{i} - 16.0\hat{j})$ m/s.

(c) The speed at t = 2.00 s is $v = |\vec{v}| = \sqrt{(3.00\text{ m/s})^2 + (-16.0\text{ m/s})^2} = 16.3\text{ m/s}$.

(d) The angle of $\vec{v}$ at that moment is

$$\tan^{-1}\left(\frac{-16.0\text{ m/s}}{3.00\text{ m/s}}\right) = -79.4° \text{ or } 101°$$

where we choose the first possibility (79.4° measured *clockwise* from the +x direction, or 281° counterclockwise from +x) since the signs of the components imply the vector is in the fourth quadrant.

7. The average velocity is given by Eq. 4-8. The total displacement $\Delta\vec{r}$ is the sum of three displacements, each result of a (constant) velocity during a given time. We use a coordinate system with +x East and +y North.

(a) In unit-vector notation, the first displacement is given by

$$\Delta\vec{r}_1 = \left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{40.0\text{ min}}{60\text{ min/h}}\right)\hat{i} = (40.0\text{ km})\hat{i}.$$

The second displacement has a magnitude of $(60.0\ \frac{\text{km}}{\text{h}})\cdot(\frac{20.0\text{ min}}{60\text{ min/h}}) = 20.0\text{ km}$, and its direction is 40° north of east. Therefore,

$$\Delta\vec{r}_2 = (20.0\text{ km})\cos(40.0°)\hat{i} + (20.0\text{ km})\sin(40.0°)\hat{j} = (15.3\text{ km})\hat{i} + (12.9\text{ km})\hat{j}.$$

And the third displacement is

$$\Delta\vec{r}_3 = -\left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{50.0\ \text{min}}{60\ \text{min/h}}\right)\hat{\text{i}} = (-50.0\ \text{km})\,\hat{\text{i}}.$$

The total displacement is

$$\begin{aligned}\Delta\vec{r} &= \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (40.0\ \text{km})\hat{\text{i}} + (15.3\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}} - (50.0\ \text{km})\,\hat{\text{i}} \\ &= (5.30\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}}.\end{aligned}$$

The time for the trip is (40.0 + 20.0 + 50.0) min = 110 min, which is equivalent to 1.83 h. Eq. 4-8 then yields

$$\vec{v}_{\text{avg}} = \left(\frac{5.30\ \text{km}}{1.83\ \text{h}}\right)\hat{\text{i}} + \left(\frac{12.9\ \text{km}}{1.83\ \text{h}}\right)\hat{\text{j}} = (2.90\ \text{km/h})\,\hat{\text{i}} + (7.01\ \text{km/h})\,\hat{\text{j}}.$$

The magnitude is

$$|\vec{v}_{\text{avg}}| = \sqrt{(2.90\ \text{km/h})^2 + (7.01\ \text{km/h})^2} = 7.59\ \text{km/h}.$$

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{7.01\ \text{km/h}}{2.90\ \text{km/h}}\right) = 67.5^\circ \quad \text{(north of east)},$$

or 22.5° east of due north.

8. Our coordinate system has $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north. The first displacement is $\vec{r}_{AB} = (483\ \text{km})\hat{\text{i}}$ and the second is $\vec{r}_{BC} = (-966\ \text{km})\hat{\text{j}}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = (483\ \text{km})\hat{\text{i}} - (966\ \text{km})\hat{\text{j}}$$

which yields $|\vec{r}_{AC}| = \sqrt{(483\ \text{km})^2 + (-966\ \text{km})^2} = 1.08\times10^3\ \text{km}$.

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{-966\ \text{km}}{483\ \text{km}}\right) = -63.4^\circ.$$

We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(c) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives

$$\vec{v}_{\text{avg}} = \frac{(483\text{ km})\hat{\text{i}} - (966\text{ km})\hat{\text{j}}}{2.25\text{ h}} = (215\text{ km/h})\hat{\text{i}} - (429\text{ km/h})\hat{\text{j}}.$$

with a magnitude $|\vec{v}_{\text{avg}}| = \sqrt{(215\text{ km/h})^2 + (-429\text{ km/h})^2} = 480\text{ km/h}.$

(d) The direction of $\vec{v}_{\text{avg}}$ is 26.6° east of south, same as in part (b). In magnitude-angle notation, we would have $\vec{v}_{\text{avg}} = (480\text{ km/h} \angle -63.4°).$

(e) Assuming the *AB* trip was a straight one, and similarly for the *BC* trip, then $|\vec{r}_{AB}|$ is the distance traveled during the *AB* trip, and $|\vec{r}_{BC}|$ is the distance traveled during the *BC* trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483\text{ km} + 966\text{ km}}{2.25\text{ h}} = 644\text{ km/h}.$$

9. The (x,y) coordinates (in meters) of the points are A = (15, –15), B = (30, –45), C = (20, –15), and D = (45, 45). The respective times are t_A = 0, t_B = 300 s, t_C = 600 s, and t_D = 900 s. Average velocity is defined by Eq. 4-8. Each displacement $\Delta\vec{r}$ is understood to originate at point A.

(a) The average velocity having the least magnitude (5.0 m/600 s) is for the displacement ending at point C: $|\vec{v}_{avg}| = 0.0083$ m/s.

(b) The direction of $\vec{v}_{avg}$ is 0° (measured counterclockwise from the +x axis).

(c) The average velocity having the greatest magnitude ($\sqrt{(15\text{ m})^2 + (30\text{ m})^2}\,/\,300\text{ s}$) is for the displacement ending at point B: $|\vec{v}_{avg}| = 0.11\text{ m/s}$.

(d) The direction of $\vec{v}_{avg}$ is 297° (counterclockwise from +x) or –63° (which is equivalent to measuring 63° *clockwise* from the +x axis).

11. We apply Eq. 4-10 and Eq. 4-16.

(a) Taking the derivative of the position vector with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{\text{i}} + 4t^2\,\hat{\text{j}} + t\,\hat{\text{k}}) = 8t\,\hat{\text{j}} + \hat{\text{k}}\,.$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s^2),

$$\vec{a} = \frac{d}{dt}(8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}) = 8\,\hat{\mathrm{j}}\,.$$

13. In parts (b) and (c), we use Eq. 4-10 and Eq. 4-16. For part (d), we find the direction of the velocity computed in part (b), since that represents the asked-for tangent line.

(a) Plugging into the given expression, we obtain

$$\vec{r}\Big|_{t=2.00} = [2.00(8) - 5.00(2)]\hat{\mathrm{i}} + [6.00 - 7.00(16)]\hat{\mathrm{j}} = (6.00\,\hat{\mathrm{i}} - 106\,\hat{\mathrm{j}})\text{ m}$$

(b) Taking the derivative of the given expression produces

$$\vec{v}(t) = (6.00t^2 - 5.00)\,\hat{\mathrm{i}} - 28.0t^3\,\hat{\mathrm{j}}$$

where we have written $v(t)$ to emphasize its dependence on time. This becomes, at t = 2.00 s, $\vec{v}$ = $(19.0\,\hat{\mathrm{i}} - 224\,\hat{\mathrm{j}})$ m/s.

(c) Differentiating the $\vec{v}(t)$ found above, with respect to t produces $12.0t\,\hat{\mathrm{i}} - 84.0t^2\,\hat{\mathrm{j}}$, which yields $\vec{a} = (24.0\,\hat{\mathrm{i}} - 336\,\hat{\mathrm{j}})\text{ m/s}^2$ at t = 2.00 s.

(d) The angle of $\vec{v}$, measured from $+x$, is either

$$\tan^{-1}\left(\frac{-224\text{ m/s}}{19.0\text{ m/s}}\right) = -85.2° \text{ or } 94.8°$$

where we settle on the first choice (–85.2°, which is equivalent to 275° measured counterclockwise from the $+x$ axis) since the signs of its components imply that it is in the fourth quadrant.

20. The acceleration is constant so that use of Table 2-1 (for both the x and y motions) is permitted. Where units are not shown, SI units are to be understood. Collision between particles A and B requires two things. First, the y motion of B must satisfy (using Eq. 2-15 and noting that θ is measured from the y axis)

$$y = \frac{1}{2}a_y t^2 \quad\Rightarrow\quad 30\text{ m} = \frac{1}{2}\left[(0.40\text{ m/s}^2)\cos\theta\right]t^2.$$

Second, the x motions of A and B must coincide:

$$vt = \frac{1}{2}a_x t^2 \quad\Rightarrow\quad (3.0\text{ m/s})t = \frac{1}{2}\left[(0.40\text{ m/s}^2)\sin\theta\right]t^2.$$

We eliminate a factor of t in the last relationship and formally solve for time:

$$t = \frac{2v}{a_x} = \frac{2(3.0 \text{ m/s})}{(0.40 \text{ m/s}^2) \sin\theta}.$$

This is then plugged into the previous equation to produce

$$30 \text{ m} = \frac{1}{2}\left[(0.40 \text{ m/s}^2)\cos\theta\right]\left(\frac{2(3.0 \text{ m/s})}{(0.40 \text{ m/s}^2)\sin\theta}\right)^2$$

which, with the use of $\sin^2\theta = 1 - \cos^2\theta$, simplifies to

$$30 = \frac{9.0}{0.20}\,\frac{\cos\theta}{1-\cos^2\theta} \;\Rightarrow\; 1-\cos^2\theta = \frac{9.0}{(0.20)(30)}\cos\theta.$$

We use the quadratic formula (choosing the positive root) to solve for $\cos\theta$:

$$\cos\theta = \frac{-1.5 + \sqrt{1.5^2 - 4(1.0)(-1.0)}}{2} = \frac{1}{2}$$

which yields $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$.

21. (a) From Eq. 4-22 (with $\theta_0 = 0$), the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(45.0 \text{ m})}{9.80 \text{ m/s}^2}} = 3.03 \text{ s}.$$

(b) The horizontal distance traveled is given by Eq. 4-21:

$$\Delta x = v_0 t = (250 \text{ m/s})(3.03 \text{ s}) = 758 \text{ m}.$$

(c) And from Eq. 4-23, we find

$$|v_y| = gt = (9.80 \text{ m/s}^2)(3.03 \text{ s}) = 29.7 \text{ m/s}.$$

24. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the initial point (edge of table), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and $y = -1.20$ m indicates the level at which the ball hits the floor, then

$$t=\sqrt{\frac{2(-1.20\text{ m})}{-9.80\text{ m/s}^2}}=0.495\text{s}.$$

(b) The initial (horizontal) velocity of the ball is $\vec{v}=v_0\,\hat{\text{i}}$. Since x = 1.52 m is the horizontal position of its impact point with the floor, we have $x = v_0t$. Thus,

$$v_0=\frac{x}{t}=\frac{1.52\text{ m}}{0.495\text{ s}}=3.07\text{ m/s}.$$

25. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y}=0$ and $v_{0x}=v_0=10\text{ m/s}$.

(a) With the origin at the initial point (where the dart leaves the thrower's hand), the y coordinate of the dart is given by $y=-\frac{1}{2}gt^2$, so that with $y = -PQ$ we have $PQ=\frac{1}{2}(9.8\text{ m/s}^2)(0.19\text{ s})^2=0.18\text{ m}.$

(b) From $x = v_0t$ we obtain x = (10 m/s)(0.19 s) = 1.9 m.

26. (a) Using the same coordinate system assumed in Eq. 4-22, we solve for $y = h$:

$$h=y_0+v_0\sin\theta_0 t-\frac{1}{2}gt^2$$

which yields h = 51.8 m for y_0 = 0, v_0 = 42.0 m/s, θ_0 = 60.0° and t = 5.50 s.

(b) The horizontal motion is steady, so $v_x = v_{0x} = v_0\cos\theta_0$, but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

$$v=\sqrt{(v_0\cos\theta_0)^2+(v_0\sin\theta_0-gt)^2}=27.4\text{ m/s}.$$

(c) We use Eq. 4-24 with v_y = 0 and $y = H$:

$$H=\frac{(v_0\sin\theta_0)^2}{2g}=67.5\text{ m}.$$

32. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the point where the ball was hit by the racquet.

(a) We want to know how high the ball is above the court when it is at x = 12.0 m. First, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos \theta_0} = \frac{12.0 \text{ m}}{(23.6 \text{ m/s}) \cos 0°} = 0.508 \text{ s}.$$

At this moment, the ball is at a height (above the court) of

$$y = y_0 + (v_0 \sin \theta_0)\, t - \frac{1}{2} g t^2 = 1.10 \text{m}$$

which implies it does indeed clear the 0.90 m high fence.

(b) At t = 0.508 s, the center of the ball is (1.10 m – 0.90 m) = 0.20 m above the net.

(c) Repeating the computation in part (a) with θ_0 = –5.0° results in t = 0.510 s and $y = 0.040$ m, which clearly indicates that it cannot clear the net.

(d) In the situation discussed in part (c), the distance between the top of the net and the center of the ball at t = 0.510 s is 0.90 m – 0.040 m = 0.86 m.

34. Although we could use Eq. 4-26 to find where it lands, we choose instead to work with Eq. 4-21 and Eq. 4-22 (for the soccer ball) since these will give information about where *and when* and these are also considered more fundamental than Eq. 4-26. With Δy = 0, we have

$$\Delta y = (v_0 \sin \theta_0)\; t - \frac{1}{2} g t^2 \;\Rightarrow\; t = \frac{(19.5 \text{ m/s}) \sin 45.0°}{(9.80 \text{ m/s}^2)/2} = 2.81 \text{ s}.$$

Then Eq. 4-21 yields $\Delta x = (v_0 \cos \theta_0)t$ = 38.7 m. Thus, using Eq. 4-8, the player must have an average velocity of

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{(38.7 \text{ m})\, \hat{\text{i}} - (55 \text{ m})\hat{\text{i}}}{2.81 \text{ s}} = (-5.8 \text{ m/s})\; \hat{\text{i}}$$

which means his average speed (assuming he ran in only one direction) is 5.8 m/s.

35. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at its initial position (where it is launched). At maximum height, we observe $v_y = 0$ and denote $v_x = v$ (which is also equal to v_{0x}). In this notation, we have $v_0 = 5v.$ Next, we observe $v_0 \cos \theta_0 = v_{0x} = v$, so that we arrive at an equation (where $v \neq 0$ cancels) which can be solved for θ_0:

$$(5v)\cos\theta_0 = v \;\Rightarrow\; \theta_0 = \cos^{-1}\left(\frac{1}{5}\right) = 78.5°.$$

37. We designate the given velocity $\vec{v} = (7.6\text{ m/s})\hat{\text{i}} + (6.1\text{ m/s})\hat{\text{j}}$ as $\vec{v}_1 -$ as opposed to the velocity when it reaches the max height $\vec{v}_2$ or the velocity when it returns to the ground $\vec{v}_3 -$ and take $\vec{v}_0$ as the launch velocity, as usual. The origin is at its launch point on the ground.

(a) Different approaches are available, but since it will be useful (for the rest of the problem) to first find the initial y velocity, that is how we will proceed. Using Eq. 2-16, we have

$$v_{1y}^2 = v_{0y}^2 - 2g\Delta y \;\Rightarrow\; (6.1\text{ m/s})^2 = v_{0y}^2 - 2(9.8\text{ m/s}^2)(9.1\text{ m})$$

which yields $v_{0\,y}$ = 14.7 m/s. Knowing that $v_{2\,y}$ must equal 0, we use Eq. 2-16 again but now with $\Delta y = h$ for the maximum height:

$$v_{2\,y}^2 = v_{0\,y}^2 - 2gh \;\Rightarrow\; 0 = (14.7\text{ m/s})^2 - 2(9.8\text{ m/s}^2)h$$

which yields h = 11 m.

(b) Recalling the derivation of Eq. 4-26, but using $v_{0\,y}$ for $v_0 \sin\theta_0$ and v_{0x} for $v_0 \cos\theta_0$, we have

$$0 = v_{0y}t - \frac{1}{2}gt^2, \quad R = v_{0x}t$$

which leads to $R = 2v_{0x}v_{0y}/g$. Noting that $v_{0x} = v_{1x}$ = 7.6 m/s, we plug in values and obtain

$$R = 2(7.6\text{ m/s})(14.7\text{ m/s})/(9.8\text{ m/s}^2) = 23\text{ m}.$$

(c) Since $v_{3x} = v_{1x}$ = 7.6 m/s and $v_{3y} = -v_{0\,y}$ = –14.7 m/s, we have

$$v_3 = \sqrt{v_{3\,x}^2 + v_{3\,y}^2} = \sqrt{(7.6\text{ m/s})^2 + (-14.7\text{ m/s})^2} = 17\text{ m/s}.$$

(d) The angle (measured from horizontal) for $\vec{v}_3$ is one of these possibilities:

$$\tan^{-1}\left(\frac{-14.7\text{ m}}{7.6\text{ m}}\right) = -63° \text{ or } 117°$$

where we settle on the first choice (–63°, which is equivalent to 297°) since the signs of its components imply that it is in the fourth quadrant.

40. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 161 \text{ km/h}$. Converting to SI units, this is v_0 = 44.7 m/s.

(a) With the origin at the initial point (where the ball leaves the pitcher's hand), the *y* coordinate of the ball is given by $y = -\frac{1}{2}gt^2$, and the *x* coordinate is given by $x = v_0t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if *x* = 18.3/2 m, then *t* = (18.3/2 m)/(44.7 m/s) = 0.205 s.

(b) And the time to travel the next 18.3/2 m must also be 0.205 s. It can be useful to write the horizontal equation as $\Delta x = v_0 \Delta t$ in order that this result can be seen more clearly.

(c) From $y = -\frac{1}{2}gt^2$, we see that the ball has reached the height of $|-\frac{1}{2}(9.80 \text{ m/s}^2)(0.205 \text{ s})^2| = 0.205 \text{ m}$ at the moment the ball is halfway to the batter.

(d) The ball's height when it reaches the batter is $-\frac{1}{2}(9.80 \text{ m/s}^2)(0.409 \text{ s})^2 = -0.820 \text{m}$, which, when subtracted from the previous result, implies it has fallen another 0.615 m. Since the value of *y* is not simply proportional to *t*, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial *y*-velocity for the first half of the motion is not the same as the "initial" *y*-velocity for the second half of the motion.

41. Following the hint, we have the time-reversed problem with the ball thrown from the ground, towards the right, at 60° measured counterclockwise from a rightward axis. We see in this time-reversed situation that it is convenient to use the familiar coordinate system with +*x* as *rightward* and with positive angles measured counterclockwise.

(a) The *x*-equation (with x_0 = 0 and *x* = 25.0 m) leads to

$$25.0 \text{ m} = (v_0 \cos 60.0°)(1.50 \text{ s}),$$

so that v_0 = 33.3 m/s. And with y_0 = 0, and *y* = *h* > 0 at *t* = 1.50 s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60.0°$. This leads to *h* = 32.3 m.

(b) We have

$$v_x = v_{0x} = (33.3 \text{ m/s})\cos 60.0° = 16.7 \text{ m/s}$$
$$v_y = v_{0y} - gt = (33.3 \text{ m/s})\sin 60.0° - (9.80 \text{ m/s}^2)(1.50 \text{ s}) = 14.2 \text{ m/s}.$$

The magnitude of $\vec{v}$ is given by

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(16.7 \text{ m/s})^2 + (14.2 \text{ m/s})^2} = 21.9 \text{ m/s}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{14.2 \text{ m/s}}{16.7 \text{ m/s}}\right) = 40.4°.$$

(d) We interpret this result ("undoing" the time reversal) as an initial velocity (from the edge of the building) of magnitude 21.9 m/s with angle (down from leftward) of 40.4°.

47. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. The *Hint* given in the problem is important, since it provides us with enough information to find v_0 directly from Eq. 4-26.

(a) We want to know how high the ball is from the ground when it is at x = 97.5 m, which requires knowing the initial velocity. Using the range information and θ_0 = 45°, we use Eq. 4-26 to solve for v_0:

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.8 \text{ m/s}^2)(107 \text{ m})}{1}} = 32.4 \text{ m/s}.$$

Thus, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{97.5 \text{ m}}{(32.4 \text{ m/s})\cos 45°} = 4.26 \text{ s}.$$

At this moment, the ball is at a height (above the ground) of

$$y = y_0 + (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = 9.88 \text{ m}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At t = 4.26 s, the center of the ball is 9.88 m – 7.32 m = 2.56 m above the fence.

48. Using the fact that $v_y = 0$ when the player is at the maximum height $y_{\max}$, the amount of time it takes to reach $y_{\max}$ can be solved by using Eq. 4-23:

$$0 = v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \quad t_{\max} = \frac{v_0 \sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max} = (v_0 \sin\theta_0)\ t_{\max} - \frac{1}{2} g t_{\max}^2 = v_0 \sin\theta_0 \left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta_0}{g}\right)^2 = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

To find the time when the player is at $y = y_{\max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2} y_{\max} = \frac{v_0^2 \sin^2\theta_0}{4g} = (v_0 \sin\theta_0)\ t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t_{\pm} = \frac{(2 \pm \sqrt{2}) v_0 \sin\theta_0}{2g}.$$

With $t = t_-$ (for ascending), the amount of time the player spends at a height $y \geq y_{\max}/2$ is

$$\Delta t = t_{\max} - t_- = \frac{v_0 \sin\theta_0}{g} - \frac{(2-\sqrt{2}) v_0 \sin\theta_0}{2g} = \frac{v_0 \sin\theta_0}{\sqrt{2} g} = \frac{t_{\max}}{\sqrt{2}} \quad \Rightarrow \quad \frac{\Delta t}{t_{\max}} = \frac{1}{\sqrt{2}} = 0.707\,.$$

Therefore, the player spends about 70.7% of the time in the upper half of the jump. Note that the ratio $\Delta t / t_{\max}$ is independent of v_0 and θ_0, even though Δt and $t_{\max}$ depend on these quantities.

51. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. We use *x* and *y* to denote the coordinates of ball at the goalpost, and try to find the kicking angle(s) θ_0 so that *y* = 3.44 m when *x* = 50 m. Writing the kinematic equations for projectile motion:

$$x = v_0 \cos\theta_0, \quad y = v_0 t \sin\ \theta_0 - \tfrac{1}{2} g t^2,$$

we see the first equation gives $t = x/v_0 \cos\ \theta_0$, and when this is substituted into the second the result is

$$y = x \tan\theta_0 - \frac{g x^2}{2 v_0^2 \cos^2\ \theta_0}.$$

One may solve this by trial and error: systematically trying values of θ_0 until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution: Using the trigonometric identity $1 / \cos^2\ \theta_0 = 1 + \tan^2\ \theta_0$, we obtain

$$\frac{1}{2} \frac{g x^2}{v_0^2} \tan^2\ \theta_0 - x \tan\theta_0 + y + \frac{1}{2} \frac{g x^2}{v_0^2} = 0$$

which is a second-order equation for $\tan\ \theta_0$. To simplify writing the solution, we denote

$$c = \tfrac{1}{2} g x^2 / v_0^2 = \tfrac{1}{2}\left(9.80 \text{ m/s}^2\right)\left(50 \text{ m}\right)^2 / \left(25 \text{ m/s}\right)^2 = 19.6\text{m}.$$

Then the second-order equation becomes $c \tan^2 \theta_0 - x \tan \theta_0 + y + c = 0$. Using the quadratic formula, we obtain its solution(s).

$$\tan \theta_0 = \frac{x \pm \sqrt{x^2 - 4(y+c)c}}{2c} = \frac{50 \text{ m} \pm \sqrt{(50 \text{ m})^2 - 4(3.44 \text{ m} + 19.6 \text{ m})(19.6 \text{ m})}}{2(19.6 \text{ m})}.$$

The two solutions are given by $\tan \theta_0 = 1.95$ and $\tan \theta_0 = 0.605$. The corresponding (first-quadrant) angles are $\theta_0 = 63°$ and $\theta_0 = 31°$. Thus,

(a) The smallest elevation angle is $\theta_0 = 31°$, and

(b) The greatest elevation angle is $\theta_0 = 63°$.

If kicked at any angle between these two, the ball will travel above the cross bar on the goalposts.

58. The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10 \text{ m/s})^2}{25 \text{ m}} = 4.0 \text{ m/s}^2.$$

60. We apply Eq. 4-35 to solve for speed v and Eq. 4-34 to find acceleration a.

(a) Since the radius of Earth is 6.37×10^6 m, the radius of the satellite orbit is

$$r = (6.37 \times 10^6 + 640 \times 10^3) \text{ m} = 7.01 \times 10^6 \text{ m}.$$

Therefore, the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi(7.01 \times 10^6 \text{ m})}{(98.0 \text{ min})(60 \text{ s/min})} = 7.49 \times 10^3 \text{ m/s}.$$

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(7.49 \times 10^3 \text{ m/s})^2}{7.01 \times 10^6 \text{ m}} = 8.00 \text{ m/s}^2.$$

62. (a) The circumference is $c = 2\pi r = 2\pi(0.15 \text{ m}) = 0.94$ m.

(b) With $T = (60 \text{ s})/1200 = 0.050$ s, the speed is $v = c/T = (0.94 \text{ m})/(0.050 \text{ s}) = 19$ m/s. This is equivalent to using Eq. 4-35.

(c) The magnitude of the acceleration is $a = v^2/r = (19 \text{ m/s})^2/(0.15 \text{ m}) = 2.4 \times 10^3 \text{ m/s}^2$.

(d) The period of revolution is $(1200 \text{ rev/min})^{-1} = 8.3 \times 10^{-4}$ min which becomes, in SI units, $T = 0.050$ s = 50 ms.

67. To calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed. Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0 t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). It hits the ground at $x = 10$ m and $y = -2.0$ m. Formally solving the second equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10 \text{ m})\sqrt{-\frac{9.8 \text{ m/s}^2}{2(-2.0 \text{ m})}} = 15.7 \text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(15.7 \text{ m/s})^2}{1.5 \text{ m}} = 160 \text{ m/s}^2.$$

66. When traveling in circular motion with constant speed, the instantaneous acceleration vector necessarily points towards the center. Thus, the center is “straight up” from the cited point.

(a) Since the center is “straight up” from (4.00 m, 4.00 m), the x coordinate of the center is 4.00 m.

(b) To find out “how far up” we need to know the radius. Using Eq. 4-34 we find

$$r = \frac{v^2}{a} = \frac{(5.00 \text{ m/s})^2}{12.5 \text{ m/s}^2} = 2.00 \text{ m}.$$

Thus, the y coordinate of the center is 2.00 m + 4.00 m = 6.00 m. Thus, the center may be written as $(x, y) = (4.00 \text{ m}, 6.00 \text{ m})$.