PHYS 2102 Formula Sheet   Spring 2007.

- **Constants, definitions:**

  $\epsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2
  $k = \dfrac{1}{4\pi\epsilon_o} = 8.99 \times 10^9$ Nm^2/C^2
  $e = 1.60 \times 10^{-19}$ C
  1 eV = e(1V) = 1.60 $\times 10^{-19}$ J
  dipole moment: $p = qd$
  charge densities: $\lambda = \dfrac{Q}{L}$, $\sigma = \dfrac{Q}{A}$, $\rho = \dfrac{Q}{V}$
  $m_e = 9.11 \times 10^{-31}$kg
  $m_p = 1.67 \times 10^{-27}$kg
  Area of a circle: $A = \pi r^2$
  Area of a sphere: $A = 4\pi r^2$
  Volume of a sphere: $V = \frac{4}{3}\pi r^3$

- **Kinematics (constant acceleration) :**

  $v = v_0 + at$
  $x - x_0 = \frac{1}{2}(v_0 + v)t$
  $x - x_0 = vt - \frac{1}{2}at^2$
  $x - x_0 = v_0 t + \frac{1}{2}at^2$
  $v^2 = v_0^2 + 2a(x - x_0)$

- **Coulomb's law:** $F = k\dfrac{|\, q_1 \,||\, q_2 \,|}{r^2}$

- **Electric field measured with a test charge:** $\vec{E} = \dfrac{\vec{F}}{q_o}$

- **Force on a charge in an electric field:** $\vec{F} = q\vec{E}$

- **Electric field of a point charge:** $E = \dfrac{F}{q_o} = k\dfrac{|\, q \,|}{r^2}$

- **Electric field of an infinite non-conducting plane with a charge density** σ**:** $E = \dfrac{\sigma}{2\epsilon_o}$

- **Electric field of an infinite line charge:** $E = \dfrac{2k\lambda}{r}$

- **Electric field of a dipole on axis, far away from dipole:** $E = \dfrac{2kp}{z^3}$

- **Torque on a dipole in an electric field:** $\vec{\tau} = \vec{p} \times \vec{E}$

- **Potential energy of a dipole in electric field:** $U = -\vec{p} \cdot \vec{E}$

- **Electric flux:** $\Phi = \int \vec{E} \cdot d\vec{A}$

- **Gauss' law:** $\epsilon_o \oint \vec{E} \cdot d\vec{A} = q_{enc}$

- **Electric field close to the surface of a conductor:** $E = \dfrac{\sigma}{\epsilon_o}$

- **Work, potential energy, and electric potential:**

  $\Delta U = U_f - U_i = -W$
  $\vec{E} = -\vec{\nabla} V$, $E_x = -\dfrac{\partial V}{\partial x}$, $E_y = -\dfrac{\partial V}{\partial y}$, $E_z = -\dfrac{\partial V}{\partial z}$
  $U = -W_\infty$
  potential of a point charge q**:** $V = k\dfrac{q}{r}$
  $V = -\dfrac{W_\infty}{q}$
  potential of n **point charges:** $V = \sum_{i=1}^{n} V_i = k\sum_{i=1}^{n} \dfrac{q_i}{r_i}$
  $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$

  potential energy of two point charges: $U_{12} = W_{ext} = q_2 V_1 = q_1 V_2 = k\dfrac{q_1 q_2}{r_{12}}$

- Kinetic Energy of a particle: $K = \frac{1}{2}mv^2$

- **Capacitance** definition: $q = CV$

  Capacitor with a dielectric: $C = \kappa C_{air}$ farad $= \frac{\text{coulomb}}{\text{volt}}$

  Parallel plate: $C = \varepsilon_\circ \frac{A}{d}$ $\frac{A}{d} = \frac{\text{Area}}{\text{separation}}$

  Spherical: $C = 4\pi\varepsilon_\circ \frac{ab}{b-a}$ a = inner radius; b = outer radius

  Isolated sphere: $C = 4\pi\varepsilon_\circ R$ Cylindrical Cap.: $C = 2\pi\varepsilon_\circ \frac{L}{ln(b/a)}$ L = length

  For a dielectric, substitute $\kappa\varepsilon_\circ$ for $\varepsilon_\circ$ in the above formulae. κ = dielectric constant

  Permittivity: $\varepsilon = \kappa\varepsilon_\circ$

  Potential Energy in Cap: $U = \frac{q^2}{2C} = \frac{1}{2}qV = \frac{1}{2}CV^2$ Energy density of electric field: $u = \frac{1}{2}\varepsilon E^2$

  Capacitors in parallel: $C_{eq} = \sum C_i$ Capacitors in series: $\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$

- **Current:** $i = \frac{dq}{dt}$ In a conductor: $i = nqAv_d$ **Current density:** $j = \frac{i}{A}$

- **Drift speed of the charge carriers:** $\vec{v}_d = \frac{\vec{J}}{ne}$

- **Definition of resistance:** $R = \frac{V}{i}$ **Definition of resistivity:** $\rho = \frac{1}{\sigma} = \frac{E}{J}$ σ **= conductivity**

- **Resistance in a conducting wire:** $R = \rho\frac{L}{A}$

- **Power in an electrical device:** $P = iV$ **Power in a resistor:** $P = i^2R = \frac{V^2}{R}$

- **Definition of** emf**:** $\mathcal{E} = \frac{dW}{dq}$ **Rate of change of chemical to electrical energy:** $P_{emf} = i\mathcal{E}$

- **Resistors in series:** $R_{eq} = \sum R_i$ **Resistors in parallel:** $\frac{1}{R_{eq}} = \sum \frac{1}{R_i}$

- **Loop rule in DC circuits:** the sum of changes in potential across any closed loop of a circuit must be zero.

- **Junction rule in DC circuits:** the sum of currents entering any junction must be equal to the sum of currents leaving that junction.

- **Charging a capacitor in a series RC circuit:** $q(t) = C\mathcal{E}(1 - e^{-\frac{t}{\tau}})$, time constant $\tau = RC$
  Discharging: $q(t) = q_0 e^{-\frac{t}{\tau}}$, time constant $\tau = RC$

- **Magnetic Fields**

  Magnetic force on a charge q: $\vec{F} = q\vec{v} \times \vec{B}$ Lorentz force: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

  Hall voltage: $V = v_d B d = \frac{i}{nle}B$ d = width $\perp$ to field and i, l = thickness $\parallel$ to field and $\perp$ to i

  Circular motion in a magnetic field: $qv_\perp B = \frac{mv_\perp^2}{r}$

  so period: $T = \frac{1}{f} = \frac{2\pi m}{qB} = \frac{2\pi}{\omega}$, where f = frequency, ω = angular frequency

  Magnetic force on a length of wire: $\vec{F} = i\vec{L} \times \vec{B}$

  Definition of a Magnetic Dipole: $\vec{\mu} = Ni\vec{A}$

  Torque on a Magnetic Dipole: $\vec{\tau} = \vec{\mu} \times \vec{B}$

  Energy of a Magnetic Dipole: $U = -\vec{\mu} \cdot \vec{B}$

- **Generating Magnetic Fields**

  $\mu_0 = 4\pi \times 10^{-7} \dfrac{\mathrm{T \cdot m}}{\mathrm{A}}$

  Biot-Savart Law: $d\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{i d\vec{s} \times \vec{r}}{r^3}$

  Magnetic field of a long straight wire: $B = \dfrac{\mu_0}{2\pi} \dfrac{i}{r}$

  Magnetic field at the center of a circular arc: $B = \dfrac{\mu_0}{4\pi} \dfrac{i}{r} \phi$

  Force between parallel wires carrying currents: $F_{ab} = \dfrac{\mu_0 i_a i_b}{2\pi d} L$ (parallel currents attract, opposite repel)

  Ampere's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$

  Magnetic field of a solenoid: $B = \mu_0 i n$

  Magnetic field of a dipole (on axis): $\vec{B} = \dfrac{\mu_0}{2\pi} \dfrac{\vec{\mu}}{z^3}$

- **Induction:**

  Magnetic Flux: $\Phi = \int \vec{B} \cdot d\vec{A}$

  Faraday's law: $\mathcal{E} = -\dfrac{d\Phi}{dt}$ $\left(= -N \dfrac{d\Phi}{dt}\right.$ for a coil with N turns)

  Induced Electric Field: $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi}{dt}$

  Motional emf: $\mathcal{E} = BLv$

  Definition of Self-Inductance: $L = \dfrac{N\Phi}{i}$

  Inductance of a solenoid: $L = \mu_0 n^2 A l$

  EMF (Voltage) across an inductor: $\mathcal{E}_L = -L \dfrac{di}{dt}$

  RL Circuit: rise of current: $i = \dfrac{\mathcal{E}}{R}(1 - e^{-\frac{tR}{L}})$ decay of current: $i = i_0 e^{-\frac{tR}{L}}$

  Magnetic Energy: $U_B = \dfrac{1}{2} L i^2$ Magnetic energy density: $u_B = \dfrac{B^2}{2\mu_0}$

- **LC circuits:**

  Electric energy in capacitor: $U_E = \dfrac{q^2}{2C} = \dfrac{CV^2}{2}$

  Magnetic energy in an inductor: $U_B = \dfrac{Li^2}{2}$

  LC circuit oscillations: $q = Q\cos(\omega t + \phi)$ $\left(i = \dfrac{dq}{dt} \quad q = CV\right)$ $\omega = \dfrac{1}{\sqrt{LC}}$

- **Circuits with generators**

  RLC in series with AC generator: $\mathcal{E}(t) = \mathcal{E}_m \sin \omega_d t$

  Current $i(t) = I \sin(\omega_d t - \phi)$, $I = \dfrac{\mathcal{E}_m}{Z}$

  Impedance: $Z = \sqrt{R^2 + (X_L - X_C)^2}$

  phase constant: $\tan \phi = \dfrac{X_L - X_C}{R}$

  power: $P_{av} = I_{rms}^2 R = \mathcal{E}_{rms} I_{rms} \cos \phi$

single circuit elements:	resistor	inductor	capacitor
resistance or reactance:	R	$X_L = \omega_d L$	$X_C = \frac{1}{\omega_d C}$
phase constant ϕ:	0^o	$+90^o$	-90^o
voltage drop amplitude:	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

- **Transformers:**
  Transformation of voltage: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ Turns ratio: $\frac{N_p}{N_s}$ Energy conservation: $I_p V_p = I_s V_s$
- **Maxwell's Equations**
  $$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \qquad \oint \vec{B} \cdot d\vec{A} = 0 \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$
  Displacement current: $i_d = \epsilon_0 \frac{d\Phi_E}{dt}$
- **Electromagnetic Waves:** Wave traveling in +x direction: $E = E_m \sin(kx - \omega t)$ $B = B_m \sin(kx - \omega t)$, where $\vec{E} \perp \vec{B}$, and the direction of travel is $\vec{E} \times \vec{B}$
  $E/B = c$, $f\lambda = c$, $\lambda = \frac{2\pi}{k}$, Velocity of light in vacuum $= c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$
  Energy flow: $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ $I = \frac{1}{c\mu_0} E_{rms}^2$ $E_{rms} = E_m/\sqrt{2}$ $I = \frac{P_s}{4\pi r^2}$
  Radiation force and pressure: total absorption : $F_r = \frac{IA}{c}$ $p_r = \frac{I}{c}$ total reflection : $F_r = \frac{2IA}{c}$ $p_r = \frac{2I}{c}$
- **Polarizing Sheets:** unpolarized−> polarized : $I = \frac{1}{2} I_0$, polarized−> polarized : $I = I_0 \cos^2 \theta$
- **Reflection/refraction:** (Angles are measured from the normal to the interface)
  Law of reflection: $\theta_i = \theta_r$ Law of refraction: $n_2 \sin \theta_2 = n_1 \sin \theta_1$
  Total internal reflection (critical angle): $\theta_c = \sin^{-1} \frac{n_2}{n_1}$
- **Images from mirrors and thin lenses**
  Thin lens and mirror formula: $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$ Spherical mirror: $f = r/2$
  Magnification equation: $|m| = \frac{h'}{h}$, $m = -\frac{i}{p}$, $m > 0$ means image upright, $m < 0$ means image inverted.
  Total magnification of a two-lens system: $M = m_1 m_2$
- **Interference:**
  Constructive interference: Phase difference= $(m)2\pi$ (path length difference $m\lambda$)
  Destructive interference: Phase difference = $(m + \frac{1}{2})2\pi$ (path length difference $(m + \frac{1}{2})\lambda$) $m = 0, 1, 2, ...$
  Index of refraction: $\lambda_n = \frac{\lambda}{n}$, $v = \frac{c}{n}$, v = velocity of light in a medium.
  Phase difference (in wavelengths) when traveling through two different media: $N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$
- **Two-slit interference:** bright fringes : $d \sin \theta = (m)\lambda$, dark fringes : $d \sin \theta = (m + \frac{1}{2})\lambda$ $m = 0, 1, 2, ...$
  Intensity in Two-Slit Interfence: $I = 4I_0 \cos^2 \frac{1}{2}\phi$, $\phi = \frac{2\pi d}{\lambda} \sin \theta$
- **Thin-Film Interference**
  $n_1 < n_2 < n_3$: Destructive interference when $2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2}$, $m = 0, 1, 2, ...$
  $n_1 < n_2,\ n_2 > n_3$: Destructive interference when $2L = m\frac{\lambda}{n_2}$, $m = 0, 1, 2, ...$
- **Diffraction :**
  Single Slit diffraction: Minima : $a \sin \theta = m\lambda$, $m = 1, 2, ...$ Intensity : $I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2$, $\alpha = \frac{\pi a}{\lambda} \sin \theta$
  Circular-Aperture diffraction: 1st minimum : $\sin \theta = 1.22\frac{\lambda}{d}$
  Rayleigh's criterion for resolvability: $\sin \theta_R = 1.22\frac{\lambda}{d}$
  Double Slit diffraction: $I(\theta) = I_m(\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha}\right)^2$, $\alpha = \frac{\pi a}{\lambda} \sin \theta$ $\beta = \frac{\pi d}{\lambda} \sin \theta$