I

Introduction to Measurement and Data Analysis
Introduction to Measurement

In physics lab the activity in which you will most frequently be engaged is measuring things. Using a wide variety of measuring instruments you will measure times, temperatures, masses, forces, speeds, frequencies, energies, and many more physical quantities. Your tools will span a range of technologies from the simple (such as a ruler) to the complex (perhaps a digital computer). Certainly it would be worthwhile to devote a little time and thought to some of the details of “measuring things” that may have not yet occurred to you.

True Value - How Tall?

At first thought you might suppose that the goal of measurement is a very straightforward one: find the true value of the thing being measured. Alas, things are seldom as simple as we would like. Consider the following “case study.”

Suppose you wished to measure how your lab partner’s height. One way might be to simply look at him or her and estimate, “Oh, about five-nine,” meaning five feet, nine inches tall. Of course you couldn’t be sure that five-eight or five-ten, or even five-eleven might be a better estimate. In other words, your measurement (estimate) is uncertain by some amount, perhaps an inch or two either way. The “true value” lies somewhere within a range of uncertainty and one way to express this notion is to say that your partner’s height is five feet, nine inches plus or minus two inches or $69 \pm 2$ inches.

It should begin to be clear that at least one of the goals of measurement is to reduce the uncertainty to as small an amount as is feasible and useful. Feasible and useful are important adjectives here.

In some texts the uncertainty may be referred to as the error in the measurement, but our ordinary understanding of the word “error” implies some sort of a mistake or blunder. That is not the intended meaning in science. Uncertainties occur regardless of the amount of care and attention paid to the measuring process. We shall discuss ways to estimate and classify these unavoidable uncertainties later. Mistakes and blunders can (and must) be ruthlessly hunted down and eliminated.

In your quest to determine your partner’s height you might get a tape measure marked off every eighth of an inch. A new measurement may allow you to state that his or her height is “five feet nine and three-quarters inches,” but since there are marks on the tape only every eighth of an inch you are really saying that the height is closer to five nine and three-quarters than it is to five nine and five-eighths or five nine and seven-eighths. It is reasonable to expect that your measurement is “good” to within half of the smallest division on the tape—in this case half of an eighth, or a sixteenth. You now express the height as five feet, nine inches plus or minus one-sixteenth inch or $69.75 \pm 0.06$ inches.

Note that we have rounded off one-sixteenth (0.0625) to 0.06 (more about that later). Now this is clearly a “better” measurement. The uncertainty is quite a bit smaller, but is it feasible to make it even smaller still? If you were to continue this obsession with measuring your partner (and your partner consented) you might purchase a precision stainless steel tape that is marked off every one hundredth of an inch, thus reducing the uncertainty to 0.005 inches. Perhaps a research grant from MA (Measurers Anonymous) would fund the purchase of a laser interferometer, capable of measuring to within a wavelength of light (about 0.00002 inches). Would you have at last found the “true height” of you partner? Well, you would be able to express his or her height as
69.74843 ± 0.00001 inches.

You note, to your dismay, that the measurement is still uncertain. All the King’s horses and all the King’s men (or, to use a more modern idiom, all the money in the National Science Foundation) cannot give you the means to find the “true height” of your partner. In fact, some modern findings in quantum physics place some fundamental limits on our ability to measure things. (Look up the Heisenberg Uncertainty Principle in your physics text.)

Things get worse yet. As you carried out this exercise you probably discovered that your tape measure (the cheap one, made of mylar plastic) would stretch somewhat, depending on how tightly you pulled it. When you got the expensive stainless steel tape you hoped to eliminate that problem, but no, stainless steel can stretch too. Because you can measure more carefully, you probably also notice that the steel tape expands and contracts as the temperature changes.

As if the problem wasn’t ugly enough, your partner remembers reading in a physiology textbook that a person’s height actually varies over the course of the day, being greater in the morning (after horizontal sleep) and less after gravity has done its work of compressing the vertebrae for a few hours. A few quick checks over a several hour period reveals the awful truth—there is no “one true height” for you partner.

Before you throw up your hands and run screaming from the lab, recall the words above: feasible and useful. While it is certainly feasible to measure a person’s height with a laser interferometer, it is hardly useful. One reason to measure height might be to get the right size for an article of clothing. For this purpose your original, cheap, mylar tape measure is perfectly adequate. Were you measuring rocket engine parts for the space shuttle, the interferometer might be an absolute essential.

All measurements are uncertain to some degree. That is an inescapable fact. Some of the uncertainty is due to the limitations of the measuring instrument (such as stretchy tapes, fuzzy markings, etc) and some is due to natural variations in the thing being measured (people who shrink after they awaken or steel balls that expand when heated).

The drawing at right shows your ruler (marked every eighth of an inch), and the arrow indicates the height of your partner. Note that you must make an estimate of which eighth-inch mark is closest to the arrow.

Whenever you record a measurement you should make note of the uncertainty inherent in that measurement. The “plus or minus” notation is a good way to do so. In this laboratory we will adhere to the following rules:

**RULE 1.** Uncertainties shall be rounded off to one significant digit.

**RULE 2.** Measurements shall be rounded off to the digit in which they are uncertain.
For example, if you had measured a time interval as 2.2475 seconds with an uncertainty of 0.0166 seconds then you should record the measurement as

$$2.25 \pm 0.02 \text{ seconds.}$$

First you rounded off the uncertainty to 1 significant digit (0.0166 → 0.02), thus the uncertainty lies in the “hundredths” digit. You then rounded off the measured value to the nearest hundredth (2.2475 → 2.25). By this you are stating that the “true value” lies somewhere between 2.23 and 2.27 seconds. At first glance this seems pessimistic in that it “throws away” useful information, but in fact any other expression is just false security, leading others to believe that the measurement was better than it really was. As a practical matter in teaching labs such as this course, the tendency is to underestimate rather than overestimate uncertainties, so the above rules are realistic ones.

**Precision and Accuracy**

Common use of our language assigns roughly similar meaning to the terms “precise” and “accurate,” but in scientific terms the words have quite different implications.

**Precision** in measurement implies the ability to distinguish between closely spaced values. An electronic digital stopwatch that “reads out” to 0.01 seconds is much more precise than a wristwatch with a sweep second hand, which can at best be “read” to a half second. The number of significant digits thus tells us something about the precision of the measurement.

**Special Note:** Following the 1973 gas crisis, Detroit automakers were faced with the marketing problem of getting carbuyers to accept smaller cars. Advertisers decided that rather than call them “small,” “compact,” “economy,” or “personal sized,” they coined the term “precision sized.” The English language is a wonderful thing, is it not?

All instruments have a smallest increment that can be detected. This smallest increment is called the least-count of the instrument. A meter stick has markings every millimeter (0.001 meter) so its least-count is 0.001 meter. A medical “fever thermometer” is usually marked off in tenths of a degree, so its least-count is 0.1 degree. The more precise an instrument is, the smaller will be the least count.

**Accuracy** refers to the ability of an instrument to give a reading that compares favorably with generally accepted values. It really has little or nothing to do with the precision of the instrument.

We know that water freezes at zero degrees Celsius. A thermometer placed in a mixture of ice and water should, if it is accurate, read very nearly zero degrees. In fact, you would expect it to read within one “division” of zero. That is, zero, within its least-count. If it does not read zero within its least-count, then the precision of the markings is not useful because the thermometer gives inaccurate readings.

Accuracy of instruments can only be checked by comparing the instrument against a comparison standard. You can purchase a “standard kilogram mass” from a science supply company and use it to check the accuracy of scales. The “ice-water” bath is a common check for thermometers, as is a “boiling water” bath. A “standard battery cell” is used to check voltmeters.

In labs doing very careful and intricate measurements the instruments will be routinely calibrated and checked against comparison standards. The instrument will usually have a tag or sticker telling when it was calibrated last, by whom, and how closely it agreed with the comparison standard.
(a) Target Shooting. Neither precise nor accurate.

(b) Target Shooting. Accurate, but not very precise.

(c) Target Shooting. Precise, but not very accurate.

(d) Target Shooting. Both precise and accurate.
In a teaching lab such as this course you have no recourse but to presume that the instruments are accurate to within their least-count. Should you suspect that an instrument is inaccurate due to a malfunction you should ask your Instructor to check it for you. Because you can judge the precision of an instrument by its marked divisions, but must presume its accuracy, the following rule applies in this class:

**RULE 3.** The minimum uncertainty of a single measured value is presumed to be one-half of the least count of the instrument used to make the measurement.

Were you to measure the length of a sheet of tablet paper with a ruler marked off every millimeter the measurement could correctly be recorded as

\[
279.0 \pm 0.5 \text{ millimeters}
\]

**Digital Instruments**

We must slightly modify the rule above when the measuring instrument indicates the measured value with a digital (numerical) display. In this case you are not given the opportunity to estimate whether the indicated value is closer to one reading or another. The instrument may be “rounding off” or it may simply be truncating (discarding extra digits) to fit the size of its display.

Even the NBA (and now the NCAA) recognized this physics problem in basketball when they required game clocks to be able to display fractions of a second during the last minute of play. Previously, if the game clock showed 4 seconds remaining, there was no way to tell if there were 4.9 seconds or 4.0 seconds because the clock simply truncated (cut off) the tenths of seconds. Nine-tenths of a second can be an eternity for a defensive player facing former LSU Tiger Shaquille O’Neal “in the paint.” Today’s game clock will show tenths of a second during the last minute, so for example the clock might now display 4.3 seconds. This display is still uncertain and could be as much as 4.39 s or as little as 4.30 s due to the hundredths digit being truncated, but the uncertainty is of a magnitude that is negligible as far as designing basketball strategy is concerned. All of this discussion leads us to the following corollary to Rule 3:

**RULE 3(A).** Typically, the minimum uncertainty of a single measurement made with an instrument incorporating a digital readout is equal to the value of the least significant digit (least-count) of the display, but check with the instrument’s documentation to make sure.

For example, suppose you thought you were sick and had a fever and you took your temperature with both a conventional mercury thermometer and a digital electronic thermometer. Perhaps both indicated 101.4 °F. The correct representation of each measurement, assuming that the least-counts of both thermometers were 0.1 °F, would be

- Mercury thermometer: \(101.40° \pm 0.05° \text{ F}\)
- Digital Thermometer: \(101.4° \pm 0.1° \text{ F}\)

As you can see, just because the instrument is modern, electronic, and digital doesn’t always mean it’s better.

**Multiple Measurements**

One way to improve your confidence in a measurement is to repeat the measurement several times. This is especially valuable when measuring things that are themselves somewhat variable. You would expect that repeating the measurement would allow you do determine a value that is somehow “better” than that given
by a single reading.

A baseball is not a perfect sphere. It has raised seams, the leather is not of uniform thickness, and so on. If you set out to determine the “true diameter” of a baseball you would probably measure its diameter at several different places. Suppose that the following were measurements of the diameter of a baseball, taken with a ruler whose least-count is one millimeter.

(each ± 0.5 mm) 72 mm, 74 mm, 75 mm, 72 mm, 73 mm, 73 mm, 75 mm, 73 mm

After inspecting these data you could reasonably conclude that the diameter lies somewhere between the lowest value (72 mm) and the highest (75 mm). If forced to settle on one single number to report you might calculate the mean value (simply the average) as 73.375 mm, but you are now aware that such a statement would be claiming an awful lot of precision that’s probably not justifiable given the least-count of the ruler and the variability of the baseball.

What you really want to do is to make a claim of the most likely diameter of the baseball as well as an estimate of the uncertainty of that claim. Intuitively, the most likely value is just the mean value, but the uncertainty deserves some deeper thought. Let us make a table of data, showing each of the measured values and the amount by which each measurement differs from the mean of all the measurements (the deviation from the mean).

<table>
<thead>
<tr>
<th>Measured value, $x_i$</th>
<th>Deviation, $d_i = x_i - x_{avg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>72 mm</td>
<td>$72 - 73.375 = -1.375$ mm</td>
</tr>
<tr>
<td>74 mm</td>
<td>$74 - 73.375 = 0.625$ mm</td>
</tr>
<tr>
<td>75 mm</td>
<td>$75 - 73.375 = 1.625$ mm</td>
</tr>
<tr>
<td>72 mm</td>
<td>$72 - 73.375 = -1.375$ mm</td>
</tr>
<tr>
<td>73 mm</td>
<td>$73 - 73.375 = -0.375$ mm</td>
</tr>
<tr>
<td>75 mm</td>
<td>$75 - 73.375 = 1.625$ mm</td>
</tr>
<tr>
<td>73 mm</td>
<td>$73 - 73.375 = -0.375$ mm</td>
</tr>
</tbody>
</table>

Perhaps the average of the deviations from the mean would provide a good estimate of the uncertainty. Upon close inspection however, you will find that the sum of the deviations—and hence their average—is exactly zero. That should come as no surprise, for by definition, some of the values are a bit larger than the mean and some are a bit smaller, so the sum of the deviations will always be zero for any collection of data.

The equation for the mean, where the bar over the symbol indicates the mean value and N is the number of individual measurements, is

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Mean Average (1)

What is of interest is the amount of deviation, not whether it is above or below the mean. If all of the deviations were converted to positive numbers before averaging, a more useful estimate of the uncertainty might be made. One way to convert the deviations to positive numbers is to take the absolute value of each. Another ultimately more useful way is to square them. The data table now looks like this:
The mean of the squares of the deviations from the mean is 1.23 mm^2. If you take the square root of this result (in effect undoing the squaring done earlier), you obtain the square root of the mean of the squares of the deviations from the mean, which turns out to be 1.11 mm. That mouthful of a name is usually shortened to just root-mean-square (or even shorter to R.M.S.) and the result is called the standard deviation of the set of values \(x_i\), usually represented by the greek letter sigma (\(\sigma\)). In equation form:

\[
\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} d_i^2}
\]

Population Standard Deviation

Using the standard deviation as an estimate of the uncertainty and applying rules 1 and 2, then

baseball diameter = 73 ± 1 millimeters

Unfortunately, statisticians cannot leave well enough alone and insist that there are very good arguments for computing the standard deviation by dividing by one fewer than the number of measurements. In effect the \(N\) in the equation for the standard deviation above is simply replaced by \((N-1)\).

\[
\sigma_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}
\]

Sample Standard Deviation (2)

We shall use this latter expression. The earlier expression (using \(N\)) is usually called the population standard deviation, while the latter (using \(N - 1\)) is called the sample standard deviation. For the data in the table above the population standard deviation is 1.11 mm, and the sample standard deviation is 1.19 mm. After applying Rules 1 and 2, you can see that the correct expression for the diameter of the baseball is the same in either case.

If you inspect the two equations for standard deviations it should be obvious that as the number of measurements, \(N\), becomes quite large, the two types of standard deviation become very nearly equal. Statistically speaking, unless there are at least five or six measurements, standard deviation analysis of any kind is not a very good predictor of uncertainty. Within this laboratory guide, when the symbol \(\sigma\) or the term standard deviation is used, you should presume that the meaning is that of sample standard deviation. Furthermore the following rules will apply:

**RULE 4.** The most likely value of a quantity that has been measured several times is the mean value of the series of measurements.

**RULE 5.** The uncertainty in the value of a quantity that has been measured several times is the sample standard deviation of the series of measurements.

<table>
<thead>
<tr>
<th>(x_i)</th>
<th>(d_i)</th>
<th>((d_i)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>72 mm</td>
<td>-1.375 mm</td>
<td>1.891 mm^2</td>
</tr>
<tr>
<td>74 mm</td>
<td>0.625 mm</td>
<td>0.391 mm^2</td>
</tr>
<tr>
<td>75 mm</td>
<td>1.625 mm</td>
<td>2.641 mm^2</td>
</tr>
<tr>
<td>72 mm</td>
<td>-1.375 mm</td>
<td>1.891 mm^2</td>
</tr>
<tr>
<td>73 mm</td>
<td>-0.375 mm</td>
<td>0.141 mm^2</td>
</tr>
<tr>
<td>73 mm</td>
<td>-0.375 mm</td>
<td>0.141 mm^2</td>
</tr>
<tr>
<td>75 mm</td>
<td>1.625 mm</td>
<td>2.641 mm^2</td>
</tr>
<tr>
<td>73 mm</td>
<td>-0.375 mm</td>
<td>0.141 mm^2</td>
</tr>
</tbody>
</table>
Absolute and Fractional Uncertainty

The uncertainties which you have been calculating thus far are more precisely called the absolute uncertainty. The absolute uncertainty is expressed using the same units as the measurement. In the data above, the absolute uncertainty in the baseball diameter is 1 mm.

If you divide the absolute uncertainty by the most likely value of the measurement you obtain the fractional uncertainty. Using the baseball data

\[
\text{fractional uncertainty} = \frac{\sigma_D}{D}
\]

In this case \( \frac{\sigma_D}{D} = \frac{1 \text{ mm}}{73 \text{ mm}} = 0.014 \).

The fractional uncertainty is a ratio of two numbers of the same units, and since the units (mm) cancel out, it is a dimensionless quantity. Multiplying by 100 turns the fractional uncertainty into the percent uncertainty. In this case, the percent uncertainty in the baseball diameter is 1.4%.

Both fractional and absolute uncertainties play very important roles as we investigate the effect of measurement uncertainty on calculations which use those measurements.

Reporting uncertainties in laboratory reports

Lab reports should always include consideration of uncertainty, both in raw data and in the final results of calculations. The table below will help to clarify which are the reportable uncertainties for various kinds of reportable results. Here we use the term direct measurement to mean a quantity read straight from the dial of the instrument, such as the length or width of a table. An indirect measurement is a value arrived at as the result of a calculation, such as the area of a table top.

<table>
<thead>
<tr>
<th>Kind of Measurement</th>
<th>Uncertainty that should be reported</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single, direct</td>
<td>Instrument Uncertainty</td>
</tr>
<tr>
<td>Multiple, direct measurements</td>
<td>Sample standard deviation or instrument uncertainty (Whichever is larger)</td>
</tr>
<tr>
<td>Indirect</td>
<td>Propagation of uncertainty according to formulae</td>
</tr>
</tbody>
</table>

The propagation of uncertainty (also known as the propagation of error) will be the focus of an upcoming lab activity.

Sources of Uncertainty

You should now have a fairly good idea of why measurements are uncertain. Part of the uncertainty is due to the limitations in the precision and accuracy of the measuring instrument and part is due to actual variations in the thing being measured. It is important that you realize and understand that, in many cases, you cannot know the individual contributions. You can only analyze your data and determine the overall uncertainty.

There are a few exceptions to that statement. For instance, if you used a ruler marked every one mm to measure the diameters of all of the apples in a crate and found the uncertainty (sample standard deviation) to be over 10 mm, you should rightfully attribute the variation to the apples, not to changes in the ruler.
useful “rule of thumb” is that if the uncertainty is significantly larger than the least-count of the instrument (and the instrument is not malfunctioning) then the uncertainty is mostly attributable to the thing being measured.

**Random Uncertainties**

Random uncertainties are as completely unpredictable as the roll of a pair of dice. They may be due to instrument fluctuations or to variations in the measured quantity, *and you cannot know which*. It is possible to estimate the magnitude of random uncertainty. Such methods as the standard deviation and the techniques used when adding, subtracting, multiplying, and dividing measurements will allow you to make good estimates of random uncertainties. Better instruments, more care in measurement, and consistent procedures will all help to minimize random uncertainty, *but it can never be eliminated.*

**Consistent Bias**

*Bias* refers to a discrepancy in a measured quantity due to a flaw in the measuring instrument. *Consistent* means the flaw is always present and is present in a uniform (though not necessarily known) fashion. If you can find out what the consistent bias is you can compensate for it as you analyze your data. There are a number of types of consistent bias, here are just a few.

*Zero bias.* If an instrument does not read zero when there is no input, then it exhibits a zero bias. For example, suppose the first 3 mm of your meter stick had broken off. Any measurement made with that stick would always be 3 mm too long. But if you noticed it you could just subtract 3 mm from each measurement and there would be no problem. Another example might be a light intensity meter. It should read zero in the dark. If it doesn’t, there is a zero bias to contend with.

*Proportional bias.* If an instrument reads high or low by a constant percentage of its reading, then it exhibits proportional bias. Consider a clock that runs 1 minute fast per day. If you set it at midnight, by the following midnight it would be 1 minute fast, or about 0.07% fast. After one more day it would be 2 minutes fast, but that would still be 0.07% of the total time. At the end of a full year it would be 356 minutes fast, or about 6 hours. Compared to a year, 6 hours is still just 0.07%. As before, if you notice the problem you can compensate for it. As you will see when we discuss graphs of data, proportional bias shows up in the slope of a graph, while zero bias shifts the whole graph a fixed amount (up, down, left, or right).

*Nonlinear bias.* A discrepancy which is repeatable but non-uniform on either an absolute or a percentage basis exhibits a nonlinear bias. An example might be a spring-wound clock that gradually slows down as the spring tension decreases. On a graph such a discrepancy will show up as a distortion of the shape of the graph. For instance, a straight line is distorted into a curve.

**Mistakes and Blunders**

They do happen, but are neither acceptable nor excusable. When you catch yourself in a mistake you must repeat the measurements or calculations. If that is not possible you must eliminate the mistaken data from any consideration. **Under no circumstances may you submit a report which includes data or calculations which are known to be mistaken.** Such action is completely contrary to the principles of ethics in science. If eliminating the mistaken data means that you cannot complete the experimental procedure then you must accept that fact and its consequences. In class that will mean a lower grade on an assignment. In your professional career it may have far more serious ramifications, **so let’s be careful out there!**
Measurement of Mass, Length, and Time

The magnitudes of physical quantities are expressed in various systems of units. The English System is one example, with feet for distance and pounds for force. The metric system is another, with meters for distance and newtons for force. All systems of units have one thing in common. They must all refer to a standard set of fundamental units for the quantities mass, length (i.e. distance), and time. Your ability to make careful, accurate measurements of these quantities is crucial to your success in this laboratory course. You will use the metric or MKS (for meter, kilogram, second) system of units in this laboratory. In the metric system the standard units are as follows:

**TIME**

The second (abbreviated s) — The standard second is the duration of 9,192,631,770 vibrations of the radiation given off by a cesium-133 atom undergoing transitions between the two hyperfine levels of the fundamental state.

**LENGTH**

The meter (abbreviated m) — The standard meter is the distance that light will travel in a vacuum in 1/299,792,458 of a second.

**MASS**

The kilogram (abbreviated kg) — The standard kilogram is the mass of a certain cylinder of platinum-iridium alloy, stored in a double-walled vacuum jar in a vault at the International Bureau of Weights and Measures (BIPM) in Sevres, France.

Implicit in these definitions of standards is that the speed of light is exactly 299,792,458 meters per second.

When you report measurements you should use units appropriate to the thing being measured. For example, if you take a trip on an airplane you will probably cover many kilometers (10^3 meters) over a time interval of a fraction of an hour to several hours. You would be more comfortable expressing the speed of the plane in kilometers per hour (km/hr) than in meters per second (m/s). You should learn the principal set of metric prefixes (such as centi, milli, kilo, micro, etc.) which allow the fundamental units to be modified to account for very large or very small measurements.

Because you have probably grown up using the English system of pounds, feet, and seconds, it is worthwhile to spend a little time considering the relative sizes of things measured in English and metric quantities. For example, a yard is just slightly shorter than a meter.

**Measurement of Length**

You will use two basic instruments for measuring length. In order of increasing precision, they are the meter ruler and the vernier caliper. You will learn to use each of these in the activities that follow.

**The Meter Ruler.** Your meter ruler may take the form of the familiar meter-stick or perhaps a retractable metric tape measure. In either case the meter ruler is used as a comparison device. That is, the thing being measured is compared to the ruler and its markings. The length of an object is found by placing it adjacent to the ruler, noting the scale markings closest to each end of the object, and subtracting to determine the distance from one marking to the other.

Meter rulers usually have a mark for each millimeter (10^{-3} m) along their length. Thus the least-count for a meter ruler is usually expressed as 1 mm, and the uncertainty of a single measurement as 0.5 mm.

When making multiple measurements with a meter ruler you should not simply lay the object along the ruler in the same place over and over again. Rather, move the object to various places along the length of
the ruler, noting the difference in readings from one end to the other each time.

**The Vernier Caliper.** The vernier caliper is another comparison device. It improves on the meter ruler by incorporating an additional set of scale markings (called a *vernier*) which have the effect of *mechanically magnifying* the space between adjacent 1 mm markings. A diagram is below.

![Vernier Caliper Diagram](image)

There are three modes of measurement possible. The *outside jaws* can be used to measure the outer dimensions of an object, such as the thickness of a sheet of plastic or the outside diameter of a pipe. The *inside jaws* are used to measure such things as the inside diameter of a pipe or the diameter of a hole. The *depth gauge* can be inserted into a recessed space to measure such things as the depth of a hole or a groove.

To make a measurement with a vernier caliper two observation must be made. For the first, you note the position of the main index mark along the main scale. The main scale is marked every 1 mm, thus the main index mark gives the reading to the nearest mm. When reading the main scale always note the lesser of the two marks when the index is between marks.

Next you must read the vernier scale. The vernier itself is a special scale that has ten division spanning the same distance as nine divisions of the main scale. Each division on the vernier thus represents nine/tenths of a main division. Look for the main scale mark and the vernier scale mark which are most nearly in perfect alignment. The value of that mark on the vernier scale represents the *number of tenths of a mm* which should be added to the main scale reading to obtain the composite measurement.

![Vernier Scale Example](image)
This process of mechanical magnification allows the vernier caliper to measure lengths to the nearest 0.1 mm, so the uncertainty of a single measurement is half of that least-count, or 0.05 mm. Your instructor will probably use a large model of the vernier scale to show you how to make vernier readings.

**The Digital Caliper.** Just like the vernier caliper, a digital caliper allows you to make very precise measurements over small distances. The greatest advantage to using the digital caliper however is that the instrument takes care of the process of “reading” the vernier scale. In this lab you will use a 6” digital caliper to make numerous small length measurements throughout the course. The particular digital caliper you will use in lab can measure distances from 0 to 150 mm (15.0 cm). As a digital readout instrument, we need to consult with the manufacturer to determine the uncertainty in any measurement you make with the caliper. The caliper’s documentation says that the measurement uncertainty for the caliper is 0.03 mm.

To use the digital calipers, push the ON/OFF button and check the display to see what units the scale is set for. If the scale is set for inches, press the blue “inch/mm” button to switch to measuring in millimeters. Now close the jaws of the calipers completely and press the yellow ZERO button if the scale does not read “0.00 mm”. You can now separate the jaws of the caliper (or lengthen the depth blade) enough to fit the object you wish to measure. Once you are close to the distance you need, you can adjust the jaws or depth blade with the thumb wheel until there is gentle contact with your object and read off the measurement on the LCD screen.

**Measurement of Mass**

You will measure mass by using a **balance** scale usually referred to simply as a balance. Both mechanical balances and electronic balances will be available.

**SPECIAL NOTE**

You will be using a balance to measure the **mass** of objects, not the weight. An object’s mass is an intrinsic physical property of the object (the amount of “stuff” in it) and does not depend on location in space or the action of any gravitational or other forces.

The **weight** of an object is the measure of the force exerted on the object by gravity, and thus can depend on the location of the object in space. For example, though your mass would remain constant, you would weigh less on the Moon.

Mass and weight are linearly related by a constant which is the acceleration due to gravity. Near the surface of the Earth at sea level, this acceleration is approximately 9.8 m/s$^2$, thus

\[
\text{weight in Newtons} = (9.8 \text{ m/s}^2)(\text{mass in kilograms})
\]

The confusion between mass and weight is a common one amongst students. Be sure you devote enough thought to the distinction. You might start by forcing yourself to think, as you use the balances, “I am **massing** this object, not **weighing** it.”

A balance is a comparison device also. The unknown object is placed on the balance and a set of known masses are adjusted until the forces of gravity are equal on the unknown and the known. The mass of the unknown is then determined by comparison with the known.

In contrast, a **spring scale** (such as a roadside shrimp vendor might use) directly measures the pull of gravity by measuring the elongation of a spring. It really does **weigh** objects since it reads the force (weight) directly. Which is better? You might recall seeing a sticker on the butcher’s scale in the supermarket that
reads, “No Springs, Honest Weight.” Of course, the butcher really means mass, but never argue with anyone carrying a cleaver...

**Triple Beam Balance.** The triple beam balance, shown in the picture, is the instrument you will most frequently use for mass measurements. It is designed such that instead of varying the amount of the comparison mass used, a fixed amount of mass has its position varied in order to make measurements.

There are three beams, each carrying a movable mass called a poise. The unknown is placed on the pan and the poise are adjusted until the balance indicator is centered. The mass of the unknown is then read from the marks along the beams.

It is important that the balance be used on a level surface. Your lab table should be sufficiently level. It is also important to check the zero condition of the balance. With nothing on the pan and all the poises at the zero position, the balance indicator should be centered. If not, the zero adjustment screw can be turned to bring the indicator to the balance point. Always check the zero at the beginning of any lab procedure that involves use of the balance.

You will note that the beam carrying the smallest poise is marked off every 0.1 gram. Thus the least-count of the triple beam balance is 0.1 gram. The uncertainty in mass determinations using the triple beam balance is ±0.05 gram. The maximum capacity of the balance is 610 grams. Auxiliary masses may be added to the balance to extend its range to 1110 or 1610 grams.

**Single pan electronic analytical balance.** Much more accurate mass measurements may be made using the electronic analytical balance. An example is the Ohaus E-120 in your laboratory. The E-120 balance has a range of from 0 to 120 grams and the digital display reads to the nearest 0.001 gram (1 milligram). According to the manufacturer, the uncertainty in the measurement is ±0.001 gram. Note that in this case the uncertainty and the least-count are the same, which is a characteristic of most digital-readout instruments.
Using the balance is very simple. It zeroes itself and displays the mass directly in grams, but the balance is very sensitive. Even breathing on it can cause inaccuracy. To make a measurement be sure that the balance is turned ON by pressing the ON/TARE button. After a few seconds the display will stabilize. With the sample pan empty press the ON/TARE button once again to zero the balance. Within a couple of seconds the display should read zero or very nearly so. Now place the object to be measured on the pan and read the display once it has stabilized.

The TARE feature can be used to determine the mass of the contents of containers. Place an empty container on the pan and press the ON/TARE button. After the balance has zeroed itself remove the container and fill it with the substance to be measured. Place the filled container on the pan and the mass of the contents of the container will be displayed. The TARE feature in effect subtracts off the mass of the container.

Measurement of Time

You will use your computer to measure time. During some timing operations you will start and stop the timing manually by pressing a key, much like using a stopwatch. At other times electronic means will be used to start and stop the timing operation. Photogates (infrared light beam sensors) will be used in several of your experiments.

After you have turned on your computer and started it up your Instructor will explain how to “boot up” the system) select the Photogate timer and keyboard stopwatch option from the program menu by pressing PT and then the ENTER key. Follow the program menu to use the Keyboard Timing Modes. The program displays times to the nearest 0.0001 second (0.1 millisecond). Because of variations in the electronics from one computer to another, the uncertainty in the measurement is about 0.0005 seconds.

Volume of an Irregular Object

The volume of an irregular object may be found by one of several methods. Which method to use will depend upon the nature of the object and the precision needed.

- **Liquid displacement** - the object is completely immersed in a liquid and the amount of liquid displaced is measured. This is the only practical method (for small objects) when the object is irregular in all dimensions.

- **Graphical summation** - for objects with one dimension of uniform measure (such as thickness) the object may be traced onto graph paper then the number of “covered” squares can be counted to find the area of one of the flat surfaces. This might be called “eyeball integration.”

- **Paper doll method** - also for objects with one uniform dimension. The object is traced onto a sheet of paper, then the shape of the object is carefully cut out and weighed. A full sheet of paper is weighed also, and the ratio of the weights will be the same as the ratio of the areas.
Introduction to Measurement

APPARATUS  cm ruler, meter-stick, digital caliper, triple beam balance, set of slotted masses, metal cylinder, plastic cylinder, plastic shape, computer

PROCEDURE The following procedures are designed to familiarize you with several of the measuring instruments you will use in this class. The order in which you do these is not really important, so you and your partner can work independently if you wish, using different measuring tools.

BE SURE THAT YOU TRY EACH OF THE INSTRUMENTS YOURSELF. Learn how to use them and how to interpret their readings. Do not rely on your partner’s “expertise.”

Read each of the following numbered instructions fully and carefully before beginning the measurements called for. This way you will know what is to be measured, which instrument should be used, and what data should be recorded.

Special note about units

Though you probably recorded some of your data in grams, centimeters, millimeters, etc. because it was convenient to do so, you should express your results in MKS units. That means masses in kilograms, distances in meters, areas in square meters, volumes in cubic meters, times in seconds, and forces (weights) in Newtons, densities in Newtons per cubic meter of kilograms per cubic meter.

You may perform unit conversion either before or after calculations, though conversions before calculations are probably the method you will find most comfortable.

1. Use the meter-stick and/or ruler to measure the length and width of the laboratory table. Record several (5) measurements for each dimension and the unit for each measurement in the first column of the table. The position of these measurements should be spaced to give representative values covering the entire table top. Note that the meter-stick is not long enough for the length of the table. Use your creativity. Record the uncertainty in each individual measurement for both the length and width of the table.

2. Use the digital caliper to measure the diameter and length of the metal cylinder. You only need to make one measurement of the length of the cylinder, but make several (5) measurements of the diameter at various positions along the cylinder’s length. Record the unit for each measurement in the first column of the table. Also, record the uncertainty in your length measurement and in each individual measurement of the diameter.

3. Measure the mass of the metal cylinder using the triple beam balance. Record both the mass and the uncertainty associated with this measurement. (Make sure you include the units of your measurement.)

4. Use the digital caliper to measure the diameter and length of the red cylinder. You only need to make one measurement of the length of the cylinder, but make several (5) measurements of the diameter at various positions along the cylinder’s length. Record the uncertainty in your length measurement and in each individual measurement of the diameter.
5. Measure the mass of the red cylinder using the triple beam balance. Record both the mass and the uncertainty associated with this measurement. (Make sure you include the units of your measurement.)

6. We would also like to measure the reaction times for you and your partner. To take the time measurements, you will use a digital timer on the computer. Your TA will tell you which program on the computer to use to make several (5) measurements of your reaction time. Record reaction time measurements for both you and your partner, along with the units and uncertainty for each individual measurement.

7. Carefully measure and record the length, width, and thickness of the plastic shape. Measure at several places so that you can determine the mean value of each measurement. Measure and record the mass of the object.

**ANALYSIS**

1. Is the uncertainty of a single length measurement for the table the same as the uncertainty of a single width measurement? If not, which one is larger and why? What could you do to decrease this uncertainty?

2. Compute the mean (average) and standard deviation (uncertainty) of both the length and width of the table. Refer to boxed equations (1) and (2) in the *Multiple Measurements* section of the *Introduction to Measurement* chapter of this week’s lab for these calculations. Upon completion of these calculations you will have obtained a result to be entered into both the mean length and mean width blanks of your data table. Be sure to enter the uncertainties as well. An example of such an entry is (note that the uncertainty is rounded off to the digit of the measurement which is uncertain:

   Mean width 0.667 ± 0.002 m

3. Using the same procedure that you used in step 1, calculate the mean and standard deviation of the diameter of both cylinders, your reaction time, your partner’s reaction time, and the length, width, and thickness of the plastic object.

4. If you know the absolute uncertainty of a quantity (whether for a single measurement or a series of measurements), you can calculate the fractional uncertainty of the quantity. For a quantity that is being reported as value ± uncertainty, the fractional uncertainty can be calculated by:

   \[
   \text{fractional uncertainty} = \frac{\text{uncertainty}}{\text{value}}
   \]

   Calculate the fractional uncertainty of the following quantities: length of table, width of table, your reaction time, your partner’s reaction time, mass of metal cylinder, mass of red cylinder, mass of plastic object.

5. You recorded the length of a single measurement of the table with its uncertainty and the mean length of the table with its statistical uncertainty. Which uncertainty should you keep when you report your measurement? Why?
6. Why is the statistical uncertainty in your reaction time measurement so much larger than what you might quote as the measurement uncertainty?

7. Compare your reaction time to your lab partner’s reaction time. Are your reaction times the same? Justify your answer.

8. As a general rule, what do you lose in taking multiple measurements? So why do you want to take multiple measurements of a quantity when you can?
Measurement of Mass, Length, and Time

Laboratory Table: *(Show your work)*

**Length of Table**

Uncertainty of Individual Length Measurement: ± ______

<table>
<thead>
<tr>
<th>Meas</th>
<th>Length ( )</th>
<th>Deviation $(L - \bar{L})$ ( )</th>
<th>$(L - \bar{L})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>3</td>
<td></td>
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<td></td>
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<tr>
<td>4</td>
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<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average Length $(\bar{L})$: ____________  Standard Deviation of Length: ____________

Mean Length: ____________ ± ____________  Fractional Uncertainty of Length: ____________

**Width of Table**

Uncertainty of Individual Width Measurement: ± ______

<table>
<thead>
<tr>
<th>Meas</th>
<th>Width ( )</th>
<th>Deviation $(W - \bar{W})$ ( )</th>
<th>$(W - \bar{W})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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</tr>
<tr>
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</tr>
<tr>
<td>Sum</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Average Width $(\bar{W})$: ____________  Standard Deviation of Width: ____________

Mean Width: ____________ ± ____________  Fractional Uncertainty of Width: ____________
### Metal Cylinder:

<table>
<thead>
<tr>
<th>Diameter ( )</th>
<th>Mass of Cylinder: _________ ± _________</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fractional Uncertainty of Mass: _________</td>
</tr>
<tr>
<td></td>
<td>Length: _________ ± _________</td>
</tr>
<tr>
<td></td>
<td>Mean Diameter: _________ ± _________</td>
</tr>
</tbody>
</table>

### Red Cylinder:

<table>
<thead>
<tr>
<th>Diameter ( )</th>
<th>Mass of Cylinder: _________ ± _________</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fractional Uncertainty of Mass: _________</td>
</tr>
<tr>
<td></td>
<td>Length: _________ ± _________</td>
</tr>
<tr>
<td></td>
<td>Mean Diameter: _________ ± _________</td>
</tr>
</tbody>
</table>

### Reaction Times:

<table>
<thead>
<tr>
<th>Your’s ( )</th>
<th>Your Partner’s ( )</th>
<th>Uncertainty of Your Times: ± _________</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Uncertainty of Partner’s Times: ± _________</td>
</tr>
</tbody>
</table>

Your Mean Time: _________ ± _________  Partner’s Mean Time: _________ ± _________

Fractional Uncertainty of Your Reaction Time: _________  Fractional Uncertainty of Your Partner’s Time: _________

### Plastic Object:

<table>
<thead>
<tr>
<th>Length ( )</th>
<th>Width ( )</th>
<th>Thickness ( )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Mass: _________ ± _________  Fractional Uncertainty of Mass: _________

Mean Length: _________ ± _________  Mean Width: _________ ± _________

Mean Thickness: _________ ± _________